Finally, we have shown that the above developments can be easily applied to discrete systems.

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Force/Motion Control of Constrained Robots Using Sliding Mode

Chun-Yi Su, Tin-Pui Leung, and Qi-Jie Zhou

Abstract—**A sliding mode control algorithm is presented for trajectory tracking of an end-effector on a constrained surface with specified constraint forces by using the theory of variable structure systems. The development of the algorithm is based on a new formulation of the dynamic model and the expansion of sliding surfaces to include the constraint force error. The proposed sliding controller is explicit which ensures the occurrence of the sliding mode on the intersection of the surfaces. A detailed numerical example is presented to illustrate the developed method.**

I. INTRODUCTION

In many industrial applications of robots, the robot end-effector is in contact with a constrained surface. A long list of such applications could be given, including contour following, deburring, grinding, and assembling. In such cases, the constraint force due to the contact with the constrained surface has to be taken into consideration. Therein, when the constrained surface is described by a holonomic smooth manifold, the constraint forces are implicitly defined as the forces required to satisfy the constraints [1]-[3]. The control of such systems, as opposed to pure motion control in free space, is called constrained robot control [1]. The objective of constrained robot control is to determine the input torques to achieve trajectory tracking on the constrained surface with specified constraint forces.

A number of papers have been presented to address the issues of the control constrained robot, for example, the nonlinear decoupling control [4], adaptive control [3], [5], computed torque control [1], and others [2]. Sliding mode control, as a robust control method, has been successfully applied to the pure motion control of robot manipulators [6]. The main feature of sliding mode control is to allow the sliding mode to occur on a prescribed switching surface, so that the system is only governed by the sliding equation and remains insensitive to a class of disturbances and parameter variations [7]. However, due to the complexity of the control problem of constrained robots, sliding mode control strategies have not been adequately developed. Recently, Young [8] has proposed a sliding mode control scheme for constrained robot motion, however, control of the constraint force is not included in his approach.

In this note, a sliding control algorithm to achieve trajectory tracking of an end-effector on the constrained surface with specified constraint forces is proposed for rigid, nonredundant constrained robots. By assuming complete knowledge of the constrained surfaces, and recognizing that the degrees of freedom of robot manipulators decrease while the end-effector is constrained, a new dynamic model suitable for motion and constraint force control is derived. Then by exploiting the particular structure of its dynamics, the fundamental properties of the dynamics are obtained to facilitate controller design. Finally, by expanding the dimensions of the sliding surface to include the constraint force error, a joint space sliding mode control algorithm is derived, using only the measurements of joint position, velocity, and constraint force.

This note is organized as follows: a new dynamic model of the constrained robot is derived in Section II; Section III presents the proposed sliding control algorithm based on the dynamic model derived. Section IV provides illustrative examples using the proposed approach. In Section V, some conclusions are presented.

II. CONSTRAINED ROBOT DYNAMICS

Based on Euler-Lagrangian formulation, in the absence of friction, the motion equation of an n-link rigid constrained robot can be expressed in joint space as [9]

$$D(q)\ddot{q} + B(q,\dot{q})\dot{q} + G(q) = u + f \qquad (1)$$

where $q \in \mathbb{R}^n$ is the vector of joint displacements; $u \in \mathbb{R}^n$ is the vector of applied joint torques; $f \in \mathbb{R}^n$ is the vector of constraint forces in joint space. $D(q)$ is the $n \times n$ inertia matrix, which is symmetric and positive definite for each $q \in \mathbb{R}^n$; $B(q,\dot{q})\dot{q} \in \mathbb{R}^n$ is the vector of Coriolis and centrifugal torques; $G(q) \in \mathbb{R}^n$ is the vector of gravitational torques.

Two simplifying properties should be noted about this dynamic structure.

Property 1 [10]: The individual terms on the left-hand side of (10), and therefore the whole dynamics, are linear in terms of a suitably selected set of equivalent manipulator and load parameters, i.e.,

$$D(q)\ddot{q} + B(q,\dot{q})\dot{q} + G(q) = Y(q,\dot{q},\ddot{q})\alpha \qquad (2)$$

where $Y(q,\dot{q},\ddot{q})$ is an $n \times r$ matrix of known functions of q, $\dot{q}$, and $\ddot{q}$; and $\alpha \in \mathbb{R}^r$ is equivalent parameters.

Property 2 [11]: Given a proper definition of the matrix B, $\dot{D}(q) - 2B(q,\dot{q})$ is skew-symmetric.

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Let $p \in \mathbb{R}^n$ denote the generalized position vector of the end-effector in Cartesian space. If the constraints imposed are described by a holonomic smooth manifold, then the algebraic equation for the constraints can be written as

$$\phi(p) = 0 \tag{3}$$

where the mapping $\phi: \mathbb{R}^n \to \mathbb{R}^m$ is twice continuously differentiable.

Assuming that the vector p can be expressed in joint space by the relation

$$p = H(q) \tag{4}$$

where the mapping $H: \mathbb{R}^n \to \mathbb{R}^n$ is invertible and twice continuously differentiable, then the constrained equation in joint space can be written as

$$\psi(q) = \phi(H(q)) = 0. \tag{5}$$

The Jacobian matrix of the constrained equation (5) is

$$J(q) = \frac{\partial \psi}{\partial q} = \frac{\partial \phi}{\partial p}\frac{\partial H(q)}{\partial q} \tag{6}$$

which is nonsingular due to the assumption that the robot is nonredundant.

Since $\psi(q) = 0$ is identically satisfied, it is evident that $J\dot{q} = 0$. Thus, the effect of the constraints on the end-effector can be viewed as restricting the robot dynamics to the manifold Ω defined by

$$\Omega = \{(q, \dot{q}): \psi(q) = 0; J(q)\dot{q} = 0\}$$

rather than the space $\mathbb{R}^{2n}$.

When the end-effector is moving along the constrained surface, the constraint force in joint space is then given by

$$f = J^T(q)\lambda \tag{7}$$

where $\lambda \in \mathbb{R}^m$ is the associated Lagrangian multiplier [1], [12].

Since the presence of m constraints causes the manipulator to lose m degrees of freedom, the manipulator is left with only $n - m$ degrees of freedom. In this case, $n - m$ lineary independent coordinates are sufficient to characterize the constrained motion. Choosing $n - m$ out of n joint variables, denoted by

$$q^1 = [q_1^1 \cdots q_{n-m}^1]^T \tag{8}$$

to be the generalized coordinates describes the constrained motion of the manipulator. The remaining joint variables are denoted by

$$q^2 = [q_1^2 \cdots q_m^2]^T. \tag{9}$$

By the implicit function theorem, the constraint equation (5) can always be expressed explicitly as [1]

$$q^2 = \sigma(q^1). \tag{10}$$

It is assumed that the elements of q^1 are chosen to be the first $n - m$ components of q. If this is not the case, (1) can always be reordered so that the first $n - m$ equation correspond to q^1 and the last m equation to q^2.

Defining

$$L(q^1) = \begin{bmatrix} I_{n-m} \\ \dfrac{\partial \sigma(q^1)}{\partial q^1} \end{bmatrix}. \tag{11}$$

Then, from (10),

$$\dot{q} = L(q^1)\dot{q}^1 \tag{12}$$

$$\ddot{q} = L(q^1)\ddot{q}^1 + \dot{L}(q^1)\dot{q}^1. \tag{13}$$

Therefore, the dynamic model (1) of robots, when restricting to the constraint surface, can be expressed in a reduced form as

$$D(q^1)L(q^1)\ddot{q}^1 + B_1(q^1, \dot{q}^1)\dot{q}^1 + G(q^1) = u + J^T(q^1)\lambda \tag{14}$$

where B_1 is defined as $B_1(q^1, \dot{q}^1) = D(q^1)\dot{L}(q^1) + B(q^1, \dot{q}^1)L(q^1)$.

Remark: Equation (14) is suitable for control purposes which forms the basis for the subsequent development. This is because the equality constraint equations are embedded into the dynamic equation, resulting in an affine nonlinear system without constraints.

By exploiting the structure of the equation (14), three properties could be obtained.

Property 3: Motion equation (14) is still linear in terms of a suitably selected set of parameters, i.e.,

$$D(q^1)L(q^1)\ddot{q}^1 + B_1(q^1, \dot{q}^1)\dot{q}^1 + G(q^1) = Y_1(q^1, \dot{q}^1, \ddot{q}^1)\alpha.$$

This property can easily be proved from the derivation of Property 1 given in [9], [10].

Property 4: Define the matrix $A(q^1) = L^T(q^1)D(q^1)L(q^1)$, then $\dot{A}(q^1) - 2L^T(q^1)B_1(q^1, \dot{q}^1)$ is skew symmetric.

Proof:
$$\dot{A} - 2L^TB_1 = \dot{L}^TDL + L^T\dot{D}L + L^TD\dot{L} - 2L^TB_1 = L^T(\dot{D} - 2B)L.$$

From the knowledge of Property 2 that $\dot{D} - 2B$ is skew-symmetric, it is easy to know that $\dot{A} - 2L^TB_1$ is also skew-symmetric.

Property 5:

$$J(q^1)L(q^1) = L^T(q^1)J^T(q^1) = 0.$$

This property can also be proved by premultiplying (12) by $J(q^1)$, using $J\dot{q} = 0$, and noting that q^1 is linearly independent.

The above properties are fundamental for designing the force/motion sliding control laws.

III. Sliding Controller for Constrained Robot

In this section, a general tracking problem for constrained robots is considered. The objective of the control is that given a desired joint trajectory q_d and desired constraint force f_d, or identically desired multiplier λ_d, which satisfy the imposed constraints, i.e., $\psi(q_d) = 0$ and $f_d = J^T(q_d)\lambda_d$, to determine a sliding control law such that for all $(q(0), \dot{q}(0)) \in \Omega$, that $q \to q_d$, and $f \to f_d$ as $t \to \infty$.

It should be noted that, since $q^2 = \sigma(q^1)$, it is only required to find a sliding control law to satisfy $q^1 \to q_d^1$ as $t \to \infty$.

Defining

$$e_m = q^1(t) - q_d^1(t) \tag{15}$$

$$e_f = \int_0^t (f - f_d)\, dt \tag{16}$$

$$\dot{q}_r^1 = \dot{q}_d^1 - \Lambda_1 e_m - \Lambda_2 e_f \tag{17}$$

where e_m is the tracking error; e_f is the accumulated force error; q_r^1 is the reference trajectory; Λ_1 and Λ_2 are tunable matrices.

Defining α as a constant r-dimensional vector, containing the unknown elements in the suitably selected set of equivalent dynamic parameters, then the linear parametrizability of the dynamics (Property 3) leads to

$$DL\ddot{q}_r^1 + B_1\dot{q}_r^1 + G = Y_1(q^1, \dot{q}^1, \dot{q}_r^1, \ddot{q}_r^1)\alpha \tag{18}$$

where $Y_1(q^1, \dot{q}^1, \dot{q}_r^1, \ddot{q}_r^1)$ is an $n \times r$ matrix of known functions of q^1, $\dot{q}^1$, $\dot{q}_r^1$, and $\ddot{q}_r^1$.

The sliding surface is defined as

$$s_1 = \dot{q}^1 - \dot{q}_r^1 = \dot{e}_m + \Lambda_1 e_m + \Lambda_2 e_f. \tag{19}$$

The sliding controller is defined as

$$u = Y_1(q^1, \dot{q}^1, \dot{q}_r^1, \ddot{q}_r^1)\varphi - L(q^1)s_1 - J^T(q^1)\lambda_d \tag{20}$$

where Y_1 is defined in (18), L is defined in (11), and $\varphi = [\varphi_1 \cdots \varphi_r]^T$ is the switching function designed according to the variable structure theory [7] as explained below.

Based on the sliding surface (19), using (14), (18), (19), (20), and after some calculations, the following is obtained:

$$DL\dot{s}_1 = Y_1\varphi - Y_1\alpha - B_1 s_1 - Ls_1 - J^T(\lambda - \lambda_d).$$

According to Property 5, the above equation becomes

$$A\dot{s}_1 = L^T DL\dot{s}_1 = L^T Y_1\varphi - L^T Y_1\alpha - L^T B_1 s_1 - L^T L s_1. \tag{21}$$

To derive the control algorithm, the generalized Lyapunov function is considered

$$V = \tfrac{1}{2}s_1^T A s_1. \tag{22}$$

Differentiating V with respect to time and using Property 4 yields

$$\begin{aligned} \dot{V} &= \tfrac{1}{2}(\dot{s}_1^T A s_1 + s_1^T \dot{A} s_1 + s_1^T A \dot{s}_1) \\ &= s_1^T A \dot{s}_1 + s_1^T L^T B_1 s_1 \\ &= s_1^T(L^T Y_1\varphi - L^T Y_1 \alpha - L^T B_1 s_1 - L^T L s_1) + s_1^T L^T B_1 s_1 \\ &= s_1^T(L^T Y_1 \varphi - L^T Y_1 \alpha - L^T L s_1). \end{aligned} \tag{23}$$

Choosing

$$\varphi_1 = -\bar{\alpha}_1 \operatorname{sgn}\left(\sum_{j=1}^{n-m} s_{1j}(L^T Y_1)_{ji}\right); \qquad i = 1, \cdots, r \tag{24}$$

where $\bar{\alpha}_i > |\alpha_i|$, $\forall i$, gives the result

$$\begin{aligned} \dot{V} &= -s_1^T L^T L s_1 - \sum_{i=1}^{r} \bar{\alpha}_i \left| \sum_{j=1}^{n-m} s_{1j}(L^T Y_1)_{ji} \right| \\ &\quad - \sum_{i=1}^{r} \alpha_i \sum_{j=1}^{n-m} s_{1j}(L^T Y_1)_{ji} \\ &\le s_1^T L^T L s_1 < 0. \end{aligned} \tag{25}$$

From (22) and (25), it is evident that $\|s_1\|$ at least converges exponentially to zero, i.e., $e_m \to 0$, and $e_f \to 0$ as $t \to \infty$. Also $q_d^2 = \sigma(q_d^1)$, which implies that $q^2 \to q_d^2$ if $q^1 \to q_d^1$, therefore, we propose the following theorem.

Theorem: Consider the robot system described by (1), using control law (20) and (24). The closed-loop system is globally asymptotically stable in the sense that

$$q \to q_d \quad \text{as } t \to \infty$$
$$f \to f_d \quad \text{as } t \to \infty$$

for any $(q(0), \dot{q}(0)) \in \Omega$.

The following remarks should be noted.

In the theorem, the control law is related to the parameter bounds in a simple fashion so that the parameter variations in the plant can be taken into account easily.

Since the control law is discontinuous across the sliding surface, such a control law leads to control chattering. Chattering is undesirable in practice because it involves high control activity and further may excite high frequency dynamics which was neglected in the course of modeling. This can be remedied by approximating these discontinuous control laws by continuous ones inside the boundary layer [13]. To do this, the sgn$(\cdot)$ in (24) is replaced by sat$(\cdot/\epsilon)$, where ϵ is the boundary layer thickness. This leads to tracking to within a guaranteed precision.

IV. Simulated Example

A two-link robotic manipulator with a circular path constraint, as given in [8], is used to verify the validity of the control approach outlined in this note. The matrices of the original model, in the form of (1), can be written as

$$D(q) = \begin{bmatrix} \alpha + \beta + 2\eta\cos q_2 & \beta + \eta\cos q_2 \\ \beta + \eta\cos q_2 & \beta \end{bmatrix}$$

$$B(q, \dot{q}) = \begin{bmatrix} -\eta\dot{q}_2 \sin q_2 & -\eta(\dot{q}_1 + \dot{q}_2)\sin q_2 \\ \eta\dot{q}_1 \sin q_2 & 0 \end{bmatrix}$$

$$G(q) = \begin{bmatrix} \alpha e_1 \cos q_2 + \eta e_1 \cos(q_1 + q_2) \\ \eta e_1 \cos(q_1 + q_2) \end{bmatrix}$$

where $e_1 = g/l_1$, g is acceleration of gravity; and the three unknown parameters α, β, and η are functions of the unknown physical parameters

$$\alpha = (m_1 + m_2)l_1^2$$
$$\beta = m_2 l_2^2$$
$$\eta = m_2 l_1 l_2.$$

The constraint is a circle in the work space (the x-y plane) whose center coincides with the axis of rotation of the first link. Fig. 1 depicts the two-link manipulator and the constraint. The constraint surface is expressed mathematically as

$$\phi(p) = x^2 + y^2 - r^2 = 0, \qquad P = [x\ y]^T. \tag{26}$$

The transformation from work space to joint space is given by

$$H(q) = \begin{bmatrix} l_1\cos(q_1) + l_2\cos(q_1 + q_2) \\ l_1\sin(q_1) + l_2\sin(q_1 + q_2) \end{bmatrix}. \tag{27}$$

The constraint, when expressed in terms of joint space, is

$$\psi(q) = l_1^z + l_2^2 + 2l_2 l_2 \cos q_2 - r^2 = 0 \tag{28}$$

which has an unique constant solution for q_2

$$q_2 = \cos^{-1}\left(\frac{r^2 - (l_1^2 + l_2^2)}{2l_1 l_2}\right) = q_2^*. \tag{29}$$

The Jacobian matrix of (28) is

$$J(q) = \begin{bmatrix} 0 \\ -2l_1 l_2 \sin q_2 \end{bmatrix}^T \tag{30}$$

therefore the matrix defined in (11) is

$$L(q^1) = [1 \quad 0]^T. \tag{31}$$

The constrained robot motion equation (14), when restricted to the circle, can be expressed as

$$\begin{aligned} &\begin{bmatrix} \alpha + \beta + 2\eta\cos q_2^* \\ \alpha + \eta\cos q_2^* \end{bmatrix}\ddot{q}_1 + \begin{bmatrix} 0 \\ \eta\dot{q}_1 \sin q_2^* \end{bmatrix}\dot{q}_1 \\ &\quad + \begin{bmatrix} \alpha e_1 \cos q_2^* + \eta e_1 \cos(q_1 + q_2^*) \\ \eta e_1 \cos(q_1 + q_2^*) \end{bmatrix} \\ &= \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -2l_1 l_2 \sin q_2^* \end{bmatrix}\lambda. \end{aligned} \tag{32}$$

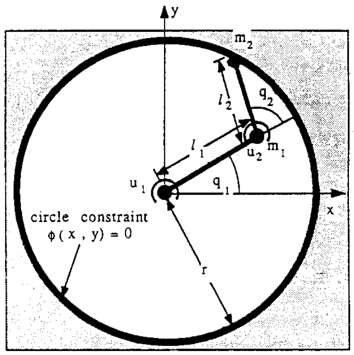

Fig. 1. A two-link manipulator and the circle constraint.

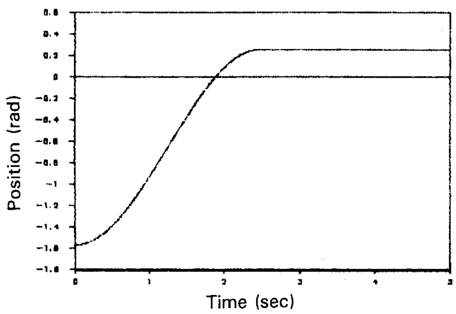

Fig. 2. Desired trajectory.

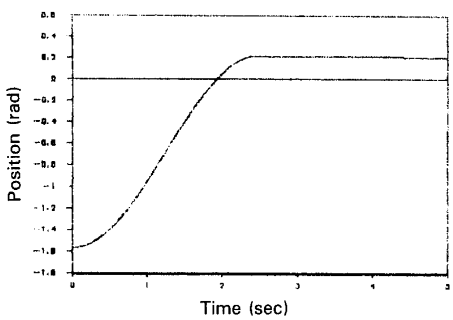

Fig. 3. Actual trajectory.

The constraint forces are

$$f_1 = 0$$

$$f_2 = -2l_1l_2(\sin q_2^*)\lambda \tag{33}$$

The control objective is to determine a feedback control so that the joint q_1 tracks the desired trajectory q_{1d} and maintains the constraint force f_2 to the desired f_d, where q_{1d} and f_d are assumed to be consistent with the imposed constraint.

Since $\lambda \to \lambda_d$ means $f_2 \to f_d$, hence in this simulation, q_{1d} and λ_d are chosen as

$$q_{1d} = \begin{cases} -90 + 52.5(1 - \cos(1.26t)) \\ 15 \end{cases} \tag{34}$$

$$\lambda_d = 10. \tag{35}$$

The true values of α, β, and η are $\alpha = 0.8$, $\beta = 0.32$, and $\eta = 0.4$. Thus, $\bar{\alpha}_1 = 1$, $\bar{\alpha}_2 = 0.5$, $\bar{\alpha}_3 = 0.5$, and the two tunable parameters Λ_1 and Λ_2 are chosen as $\Lambda_1 = 30$, $\Lambda_2 = 1$.

Since trajectory tracking on the constrained surface with specified constraint force is of interest, the initial position and the velocity of manipulator is chosen on the desired trajectory.

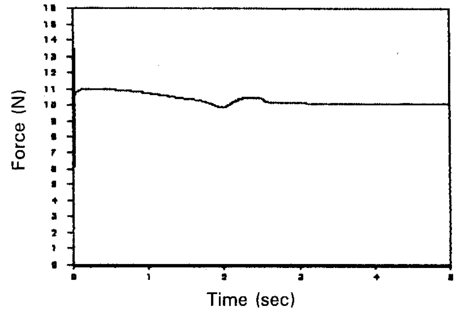

Fig. 4. Actual contact force.

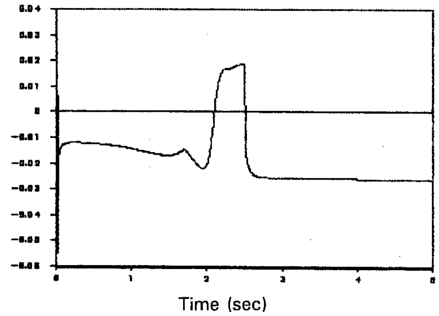

Fig. 5. Sliding surface.

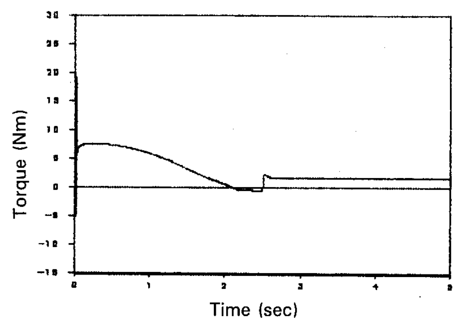

Fig. 6. Torque exerted at joint one.

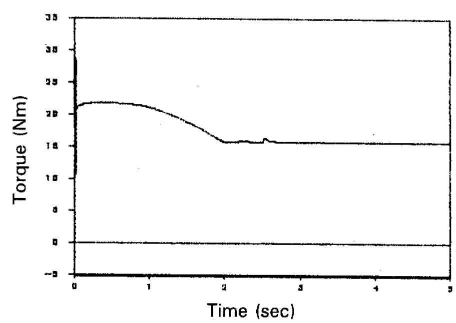

Fig. 7. Torque exerted at joint two.

$$q_1(0) = -90;\ q_2(0) = 80;\ \dot{q}_1(0) = \dot{q}_2(0) = 0.$$

The initial constraint force is assumed as $f_2 = 0$, i.e., $\lambda = 0$. In order to reduce the control chattering, the boundary layer is chosen as $\epsilon_1 = \epsilon_2 = \epsilon_3 = 0.05$.

The results of the simulation are shown in Figs. 2-7. Fig. 2 shows the desired joint trajectory, Fig. 3 shows the actual trajectory of joint 1 with maximum tracking error 0.06 rad, and Fig. 4 shows contact force λ. The final maximum error with λ_d is 0.1. The sliding surface s_1 is shown in Fig. 5, and Figs. 6 and 7 show the torques exerted at manipulator joints. These results show that the control objective is achieved successfully.

V. Conclusion

A sliding mode control algorithm to achieve trajectory tracking of end effector on a constrained holonomic smooth surface with specified constraint force is presented by using the theory of variable structure system. The major contributions of this note lie in the establishment of a new dynamic model to describe the constrained robot motion, which makes it possible to seek a sliding mode control law. By expanding the dimension of the sliding surface to include the constraint force, an explicit sliding mode control formulation is obtained which ensures the occurrence of the sliding mode on the intersection of the surfaces without necessarily stabilizing each individual one. A simple two link manipulator and a circle constraint has been used to illustrate the methodology developed in this note, and the simulation results are quite satisfactory.

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Synthesis of Proportional-Plus-Derivative Feedbacks for Descriptor Systems

Viet X. Le

Abstract—**Construction for proportional-plus-derivative (PD) feedbacks for descriptor (or singular) systems is given. It is shown that, under the assumption of controllability of the open-loop finite and (dynamic) infinite modes, feedbacks employing states and state derivatives may be constructed to accomplish shifting all open-loop modes (finite as well as infinite) to desired finite points while ensuring regularity of the closed-loop pencils. Further, the construction reveals that the required state-derivative feedbacks are specified completely by the open-loop dynamic infinite-mode structures of the descriptor systems. This class of PD feedbacks circumvents certain difficulties associated with the construction for constant state-feedbacks as given by Armentano, by Lewis, and by Fletcher and co-workers.**

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I. Introduction

Since the pioneering works of Rosenbrock [1] and of Luenberger [2], which introduced descriptor-system concepts into control theory, interest in developing solution techniques for descriptor-system problems has continued to grow. The topic of stabilization or pole-placement for descriptor systems—aided by the work of Verghese *et al.* [3] which refines certain key structural results of Rosenbrock [1]—has been much studied in recent years. Cobb [4], who probably was the first to investigate the problem, gives a construction for state feedbacks from a geometric viewpoint. However, in [4] the requirement for regularity of closed-loop pencils under state feedbacks is assumed to hold and is not investigated further. In Armentano [5], in deriving certain results that strengthen those of [4], the author gives also a geometric procedure for the construction for state feedbacks which place open-loop poles at desired finite points in the complex plane *while* ensuring regularity of the closed-loop pencils. Later, Lewis [6], Fletcher *et al.* [7], and Kautsky and Nichols [8] derive nongeometric algorithms for constructing state feedbacks that satisfy both the pole-shifting and the closed-loop pencil regularity requirements.

Although the results of [5]–[8] provide algorithms for the construction of state feedbacks for descriptor systems, a close scrutiny of these techniques reveals certain nontrivial difficulties, which motivate the writing of this note. In brief, the chief cause of the difficulties, which we will examine in detail in the next section, is due to the constraint of constant state feedbacks. The main objective of this note is to present a construction for an alternative, less restrictive class of feedbacks for descriptor systems, i.e., proportional-plus-derivative (PD) feedbacks employing both states and state derivatives, which circumvents nicely the aforementioned difficulties.

We note that the use of PD controllers has had a long history in industrial practice where derivative controls are employed to provide anticipatory action for overshoot reduction in the responses; see, e.g., [9, p. 509]. Likewise, the use of PD feedbacks in descriptor systems has been reported previously in the literature, e.g., in [10]–[12]. In [10] and [12] (we defer until Remark 3.4 a comparison of the approach and results of [11] to those of this note), PD feedbacks are used to accomplish the objective of shifting all controllable open-loop finite and dynamic infinite modes of descriptor systems to desired finite points. However, in these works, an explicit construction for such a PD feedback is not given and, more important, the satisfaction of the requirement for closed-loop pencil regularity, as in Cobb [1], has been hypothesized. For the main result of this note (Section III), we describe an explicit construction for a class of PD feedbacks which shift all open-loop poles (finite as well as infinite) to desired finite points while ensuring the requirement for regularity of the closed-loop pencils.

The organization for the paper is as follows. In Section II, we examine in detail the difficulties associated with existing constructive techniques for constant state feedbacks. In Section III we give the main result of the note. The conclusions follow in Section IV.