Model reference adaptive control of continuous-time systems with an unknown input dead-zone

X.-S. Wang, H. Hong and C.-Y. Su

Abstract: The adaptive control of continuous-time linear dynamic systems preceded by an unknown dead-zone in state space form is discussed. A lemma to simplify the error equation between the plant and the matching reference model is introduced which allows the development of a robust adaptive control scheme by involving the dead-zone inverse terms. This adaptive control law ensures global stability of the entire system and achieves the desired tracking precision even when the slopes of the dead-zone are unequal. Simulations performed on a typical linear system illustrate and clarify the validity of this approach.

1 Introduction

A dead-zone, which can severely limit system performance, is one of the most important nonsmooth nonlinearities that arise in actuators such as, servo valves and DC servo motors. In most practical motion systems, the dead-zone parameters are only poorly known, and robust adaptive control techniques are required. Proportional-derivative (PD) controllers have been observed to result in limit cycles, Due to the nonanalytic nature of the dead-zone in actuators and the fact that the exact parameters (e.g. the width of the dead-zone) are unknown, systems with dead-zones present a challenge for control design engineers.

An immediate method for the control of the dead-zone is to construct an adaptive dead-zone inverse. This approach was pioneered by Tao and Kokotovic [1, 2]. Continuous-time and discrete-time adaptive dead-zone inverses for linear systems were built in [1] and [2], respectively. Simulations indicated that the tracking performance is significantly improved by using a dead-zone inverse. This works was extended in [3] and [4] and a perfect asymptotical adaptive cancellation of an unknown dead-zone was achieved with the condition that the output of a dead-zone is measurable.

Alternative methods to produce an approximate dead-zone inverse include trying fuzzy logic or neural network precompensators. Kim *et al.* [5], Jang [6] and Lewis *et al.* [7] have proposed fuzzy precompensators in nonlinear industrial motion systems and Selmic and Lewis [8] employed neural networks to construct a dead-zone precompensator. Corradini and Orlando [9] separated an unknown dead-zone into a known part and a bounded unknown part, and used direct compensation of the known part and a variable structure controller for the whole system to overcome the effect of the unknown part.

We now extend the approach of constructing an adaptive dead-zone inverse in transfer function form by considering systems in a state space form. By given a matching condition to the reference model, an adaptive controller with an adaptive dead-zone inverse can be introduced. Benefiting from the matching condition of the reference model, the global convergence is guaranteed even when the dead-zone slopes are unequal and the output of the dead-zone is not measurable as needed in [1] and [4], which may be a valuable choice for a number of practical problems that can be simplified in the proposed system structures.

2 Dead-zone model and its properties

The dead-zone with input $v(t)$ and output $w(t)$ is shown in Fig. 1 and can be described by:

$$w(t) = D(v(t)) = \begin{cases} m_r(v(t) - b_r) & \text{for } v(t) \geq b_r \\ 0 & \text{for } b_l < v(t) < b_r \\ m_l(v(t) - b_l) & \text{for } v(t) \leq b_l \end{cases} \quad (1)$$

As stated in [1], this dead-zone model is a static simplification of diverse physical phenomena with negligible fast dynamics. Equation (1) is a good model for a hydraulic servo valve or a servo motor.

The key features of the dead-zone in the control problems currently investigated are:

(A1) The dead-zone output $w(t)$ is not available for measurement.
(A2) The dead-zone parameters b_r, b_l, m_r, m_l are unknown, but their signs are known as: $b_r > 0$, $b_l < 0$, $m_r > 0$, $m_l > 0$.
(A3) The dead-zone parameters b_r, b_l, m_r, m_l are bounded, i.e. there exist known constants $b_{r\,min}$, $b_{r\,max}$, $b_{l\,min}$, $b_{l\,max}$, $m_{r\,min}$, $m_{r\,max}$, $m_{l\,min}$, $m_{l\,max}$ such that $b_r \in [b_{r\,min}, b_{r\,max}]$, $b_l \in [b_{l\,min}, b_{l\,max}]$, and $m_r \in [m_{r\,min}, m_{r\,max}]$, $m_l \in [m_{l\,min}, m_{l\,max}]$

Assumptions (A1) and (A2) are common in practical systems, such as servo motors and servo valves. If $w(t)$ is

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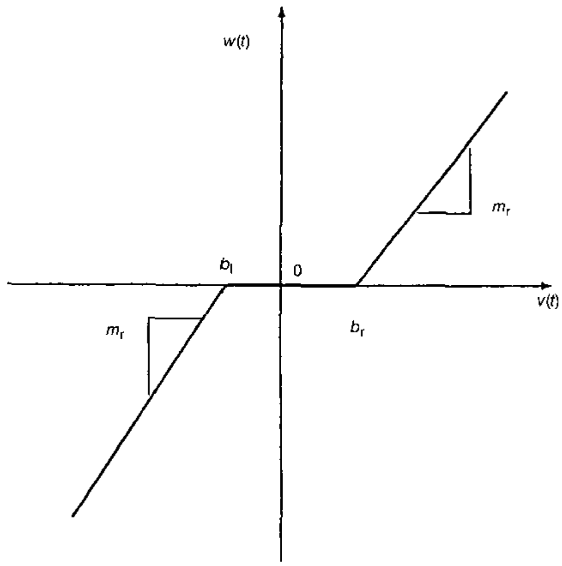

Fig. 1 *Dead-zone model*

measurable, the control of the dead-zone will be relatively easy. Assumption (A3) is also common for linear systems with dead-zones, which is reasonable in real systems.

3 Statement of the problem

A dead-zone nonlinearity can be denoted as an operator

$$w(t) = D(v(t)) \tag{2}$$

with $v(t)$ being the input and $w(t)$ the output. The operator $D(v(t))$ has been discussed in detail in the preceding Section. The dynamic system in a state space preceded by the above dead-zone can be described in canonical form as:

$$\dot{X}_p(t) = A_p X_p(t) + B w(t) \tag{3}$$

The control objective is to let $X_p(t)$ in (3) follow a reference signal $X_m(t)$ defined as:

$$\dot{X}_m(t) = A_m X_m(t) + B r(t) \tag{4}$$

in which, $r(t)$ is a specified desired trajectory input and A_m is an asymptotically stable matrix in $R^{n \times n}$ with

$$\det(sI - A_m) \equiv R_m(s) = (s+k)R(s) \quad k > 0 \tag{5}$$

and $R(s)$ being a Hurwitz polynomial.

We have the following assumptions about system (3) and the reference modal (4):

(A4) $A_p \in R^{n \times n}$ is unknown, $B \in R^n$ is known, while (A_p, B) is controllable with:

$$A_p + B\alpha^T = A_m \tag{6}$$

for some unknown vector $\alpha \in R^n$.

Assumption (A4) confines the type of systems to be considered to those that represent a number of systems of practical interest. An example system is discussed in Section 5.

4 Adaptive controller design

In presenting the adaptive control law, we define the state difference (error) between the plant and the reference model as:

$$E = X_p - X_m \tag{7}$$

By using (3), (4) and (6), we have

$$\dot{E} = A_m E + B(w(t) - r - \alpha^T X_p) \tag{8}$$

To design the controller, we simplify the vector equation to a scalar error form by introducing the following lemma as used and proved in [10].

Lemma 1: Let

$$\dot{X} = AX + bv \quad a(s) = \det(sI - A) = (s+k)R(s) \tag{9}$$

where A is asymptotically stable with a characteristic polynomial $a(s)$, $k > 0$ and (A, b) is controllable. Then

1. There exist h, such that

$$h^T(sI - A)^{-1} b = \frac{1}{s+k} \tag{10}$$

2. if $x = h^T X$, then: (i) $x \in L^\infty \Rightarrow X \in L^\infty$; and (ii) if $\lim_{t\to\infty} x(t) = 0$, then $\lim_{t\to\infty} X(t) = 0$

Based on lemma 1, it is obviously from (10) that there exist a h, so that:

$$h^T(sI - A_m)^{-1} B = \frac{1}{s+k} \tag{11}$$

Then, a scalar error is defined as:

$$e_c = h^T E \tag{12}$$

Form the Laplace transform of (8), we have:

$$E(s) = (sI - A_m)^{-1} B(w(s) - r(s) - \alpha X_r(s))$$

Multiply both sides with h^T and applying (11) and (12), we have:

$$e_c(s) = \frac{1}{s+k}(w(s) - r(s) - \alpha X_p(s))$$

Thus (8) is changed to:

$$\dot{e}_c = -ke_c + (w(t) - r - \alpha^T X_p) \tag{13}$$

4.1 A known system without a dead-zone

In this case, $v(t) = w(t)$, in (13), if we let:

$$v(t) = w(t) = r + \alpha^T X_p \tag{14}$$

then we have

$$\dot{e}_c = -ke_c$$

because $k > 0$, which implies that $e_c \to 0$ as $t \to \infty$. According to lemma 1, we have $E \to 0$, that is to say that as $t \to \infty$, we have $X_p \to X_m$.

4.2 A known system with a known dead-zone

We define the desired output of dead-zone $w_d(t)$ as:

$$w_d(t) = r + \alpha^T X_p$$

It is obvious from (14), that this $w_d(t)$ will lead $X_p \to X_m$. Therefore, the task is to find $v(t)$ so that the output of the dead-zone satisfies $w(t)=w_d(t)$. For this purpose, we let the input of the known dead-zone be

$$v(t)=\begin{cases}\dfrac{w_d+m_r b_r}{m_r} & \text{if } w_d>0\\ 0 & \text{if } w_d=0\\ \dfrac{w_d+m_l d_l}{m_l} & \text{if } w_d<0\end{cases} \tag{15}$$

which will lead $w(t)=w_d(t)$ To demonstrate the point, we introduce χ_r, χ_l and N as:

$$\chi_r=\begin{cases}1 & \text{for } w_d(t)\geq b_r\\ 0 & \text{otherwise}\end{cases}$$

$$\chi_l=\begin{cases}1 & \text{for } w_d(t)\leq b_l\\ 0 & \text{otherwise}\end{cases}$$

$$\boldsymbol{N}=[\chi_r,\chi_l]$$

And define

$$\boldsymbol{\theta}=[\theta_r,\theta_l]^T=[m_r b_r, m_l b_l]^T \tag{16}$$
$$\boldsymbol{m}=[m_r,m_l]^T$$

Thus the dead-zone of (1) can be written as:

$$w(t)=D(v(t))=\boldsymbol{Nm}v(t)-\boldsymbol{N\theta} \tag{17}$$

And we can rewrite (15) as

$$v(t)=\frac{1}{\boldsymbol{Nm}}(r+\boldsymbol{\alpha}^T\boldsymbol{X}_p+\boldsymbol{N\theta}) \tag{18}$$

Substituting (15) into (17) clearly shows $w(t)=w_d(t)$. Thus, the effect of the known dead-zone can be completely compensated, and the same tracking performance as the preceding Section will he achieved.

4.3 Unknown system with an unknown dead-zone

As stated in assumption (A4), $\boldsymbol{A}_p\in\boldsymbol{R}^{n\times n}$ is unknown, $\boldsymbol{B}\in\boldsymbol{R}^n$ is known, while $(\boldsymbol{A}_p,\boldsymbol{B})$ is controllable. What we do not know, is the unknown vector $\boldsymbol{\alpha}\in\boldsymbol{R}^n$ in the matching condition (6).

In this case, we will use an adaptive controller to control the unknown system with the task of compensating the effect of the unknown dead-zone.

By defining the estimated value of $\boldsymbol{\alpha}$ as $\hat{\boldsymbol{\alpha}}$, we have the estimate error of $\boldsymbol{\alpha}$

$$\tilde{\boldsymbol{\alpha}}=\hat{\boldsymbol{\alpha}}-\boldsymbol{\alpha} \tag{19}$$

Assume $\hat{\boldsymbol{\theta}}=[\hat{\theta}_r,\hat{\theta}_l]^T$ is the estimated value of $\boldsymbol{\theta}$, and the estimate error is:

$$\tilde{\boldsymbol{\theta}}=\hat{\boldsymbol{\theta}}-\boldsymbol{\theta}\triangleq[\tilde{\theta}_r,\tilde{\theta}_l]^T$$
$$=[\widehat{m_r b_r},\widehat{m_l b_l}]^T-[m_r b_r, m_l b_l]^T \tag{20}$$

Define a slope ratio as

$$\boldsymbol{\phi}=[\phi_r,\phi_l]^T=\left[\frac{m_r}{m_r},\frac{m_l}{m_l}\right]^T=[1,1]^T$$

And the estimated slope ratio is defined as

$$\hat{\boldsymbol{\phi}}=[\hat{\phi}_r,\hat{\phi}_l]^T=\left[\frac{m_r}{\widehat{m}_r},\frac{m_l}{\widehat{m}_l}\right]^T$$

we have the estimate error of the slope ratio as

$$\tilde{\boldsymbol{\phi}}=[\tilde{\phi}_r,\tilde{\phi}_l]^T=\left[\frac{m_r}{\widehat{m}_r},\frac{m_l}{\widehat{m}_l}\right]^T-[1,1]^T \tag{21}$$

From (17), the estimated dead-zone can be expressed as:

$$\widehat{w(t)}=\boldsymbol{N}\hat{\boldsymbol{m}}v(t)-\boldsymbol{N}\hat{\boldsymbol{\theta}}$$

Based on the given plant and reference model as well as the dead-zone model subjecting to the assumptions described above, the following control and adaptation laws are presented:

$$v(t)=\frac{1}{\boldsymbol{N}\hat{\boldsymbol{m}}}\left(-k_d e_c+r+\hat{\boldsymbol{\alpha}}^T\boldsymbol{X}_p+\boldsymbol{N}\hat{\boldsymbol{\theta}}^T-k^*\mathrm{sat}\left(\frac{e_c}{\epsilon}\right)\right) \tag{22}$$

$$\dot{\hat{\boldsymbol{\theta}}}=\mathrm{proj}(\hat{\boldsymbol{\theta}},-\gamma e_\epsilon) \tag{23}$$

$$\dot{\hat{\boldsymbol{\alpha}}}=-\lambda e_\epsilon\boldsymbol{X}_p \tag{24}$$

$$\dot{\hat{\boldsymbol{\phi}}}=\mathrm{proj}(\hat{\boldsymbol{\phi}},-\eta e_\epsilon n_v) \tag{25}$$

where

$$n_v=-k_d e_c+r+\hat{\boldsymbol{\alpha}}^T\boldsymbol{X}_p+\boldsymbol{N}\hat{\boldsymbol{\theta}}-k^*\mathrm{sat}\left(\frac{e_c}{\epsilon}\right)$$

and for $(i=r,l)$,

$$\mathrm{proj}(\hat{\boldsymbol{\theta}},-\gamma e_\epsilon)=\begin{cases}0, & \text{if } \hat{\theta}_i=\theta_{i\max} \text{ and } \gamma e_\epsilon<0\\ & \text{if } [\theta_{i\min}<\hat{\theta}_i<\theta_{i\max}]\\ -\gamma e_\epsilon, & \text{or } [\hat{\theta}_i=\theta_{i\max} \text{ and } \gamma e_\epsilon\geq 0]\\ & \text{or } [\hat{\theta}_i=\theta_{i\min} \text{ and } \gamma e_\epsilon\leq 0]\\ 0, & \text{if } \hat{\theta}_i=\theta_{i\min} \text{ and } \gamma e_\epsilon>0\end{cases} \tag{26}$$

$$\mathrm{proj}(\hat{\boldsymbol{\phi}},-\eta e_\epsilon n_v)=\begin{cases}0, & \text{if } \hat{\phi}_i=\phi_{i\max} \text{ and } \eta e_\epsilon n_v<0\\ & \text{if } [\phi_{i\min}<\hat{\phi}_i<\phi_{i\max}]\\ -\eta e_\epsilon n_v, & \text{or } [\hat{\phi}_i=\phi_{i\max} \text{ and } \eta e_\epsilon n_v\geq 0]\\ & \text{or } [\hat{\phi}_i=\phi_{i\min} \text{ and } \eta e_\epsilon n_v\leq 0]\\ 0, & \text{if } \hat{\phi}_i=\phi_{i\min} \text{ and } \eta e_\epsilon n_v>0\end{cases} \tag{27}$$

In the above adaptive laws:

$$e_\epsilon=e_c-\epsilon\,\mathrm{sat}\left(\frac{e_c}{\epsilon}\right) \tag{28}$$

where ϵ is an arbitrary positive constant and sat(·) is the saturation function defined as:

$$\mathrm{sat}(z)=\begin{cases}1 & \text{for } z\geq 1\\ z & \text{for } -1<z<1\\ -1 & \text{for } z\leq -1\end{cases} \tag{29}$$

In control law (22), $k_d>0$ and $k^*>0$ are two control constants which affect the error convergence rate. In (23), (24) and (25), $\gamma>0$, $\lambda>0$ and $\eta>0$ are constants, which determine the rates of adaptations.

We note the following:

1. The term $-k_d e_c$ in the control law is a feedback term, while $\hat{\boldsymbol{\alpha}}^T\boldsymbol{X}_p$ can be considered as a feedforward term and r is the specified desired trajectory input. The term $-k^*$ $\mathrm{sat}(e_c/\epsilon)$ has a similar function as to that when used in variable structure control.
2. The term $\boldsymbol{N}\hat{\boldsymbol{\theta}}^T$ and $\boldsymbol{N}\hat{\boldsymbol{m}}$ correspond to the compensation of the dead-zone width and slope, respectively, which will be adaptively adjusted during the control process.

3. The term e_ϵ instead of e_c, is employed in the adaptive law, which introduces a dead-band and gives the robust property of the adaptive law. It also should be noticed that if ϵ is chosen too small, the linear region of function $\mathrm{sat}(s/\epsilon)$ will be too thin, which will cause a risk of exciting high frequency fluctuation due to the disturbance. As $\epsilon \to 0$, the function $\mathrm{sat}(s/\epsilon)$ eventually becomes discontinuous. In such a case, the controller becomes a typical adaptive control scheme, which may cause chattering phenomena. It should also be noticed that this term will affect the tracking precision ϵ of the plant.

4. In the above control law, two projection operators have been used. It can be found that the projection operator for $\hat{\boldsymbol{\theta}}$ has the following properties: (i) if $\hat{\boldsymbol{\theta}}(0) \in \Omega_{\boldsymbol{\theta}}$ then $\hat{\boldsymbol{\theta}}(t) \in \Omega_{\boldsymbol{\theta}}$; (ii) $\|\mathrm{proj}(p, y)\| \leq \|y\|$; (iii) $-(p-p^*)\Lambda \mathrm{proj}(p, y) \geq -(p-p^*)\Lambda y$, where Λ is a positive defined symmetric matrix. And these three properties are also valid for the projection operator defined of $\hat{\boldsymbol{\phi}}$.

The stability of the closed-loop system described by (3), (4) and (22–25) is established in the following theorem.

Theorem 1: For the plant in (3) with the dead-zone (1) at the input subject to assumptions, (A1–A4), the robust adaptive controller specified by (22–25) ensures that: (i) $\hat{\boldsymbol{\theta}}(t) \in \Omega_{\boldsymbol{\theta}}$; (ii) the state vectors are bounded; and (iii) the state vector $\boldsymbol{X}_\mathrm{p}(t)$ converges to $\boldsymbol{X}_\mathrm{m}(t)$ with precision ϵ for $\forall t \geq t_0$.

Proof: To establish global boundedness, we define a Lyapunov function candidate as:

$$V(t) = \frac{1}{2}\left[e_\epsilon^2 + \frac{1}{\gamma}\tilde{\boldsymbol{\theta}}^T\tilde{\boldsymbol{\theta}} + \frac{1}{\eta}\tilde{\boldsymbol{\phi}}^T\tilde{\boldsymbol{\phi}} + \frac{1}{\lambda}\tilde{\boldsymbol{\alpha}}^T\tilde{\boldsymbol{\alpha}}\right] \quad (30)$$

Since the discontinuity at $|e_c| = \epsilon$ is of the first kind and since $e_\epsilon = 0$ when $|e_c| \leq \epsilon$ it follows that the derivative $\dot{V}$ exists for all e_c, and given by:

$$\dot{V}(t) = 0 \quad \text{when } |e_c| \leq \epsilon \quad (31)$$

When $|e_c| > \epsilon$, the fact $e_\epsilon \dot{e}_\epsilon = e_\epsilon \dot{e}_c$, by applying adaptive law (22), we have:

$$\begin{aligned}
\dot{V}(t) &= e_\epsilon \dot{e}_c + \frac{1}{\gamma}\tilde{\boldsymbol{\theta}}^T\dot{\hat{\boldsymbol{\theta}}} + \frac{1}{\eta}\tilde{\boldsymbol{\phi}}^T\dot{\hat{\boldsymbol{\phi}}} + \frac{1}{\lambda}\tilde{\boldsymbol{\alpha}}^T\dot{\hat{\boldsymbol{\alpha}}} \\
&= e_\epsilon(-ke_c + w(t) - r - \boldsymbol{\alpha}^T\boldsymbol{X}_\mathrm{p}) \\
&\quad + \frac{1}{\gamma}\tilde{\boldsymbol{\theta}}^T\dot{\hat{\boldsymbol{\theta}}} + \frac{1}{\eta}\tilde{\boldsymbol{\phi}}^T\dot{\hat{\boldsymbol{\phi}}} + \frac{1}{\lambda}\tilde{\boldsymbol{\alpha}}^T\dot{\hat{\alpha}} \\
&= e_\epsilon(-ke_c + \boldsymbol{N}\boldsymbol{m}v(t) - \boldsymbol{N}\boldsymbol{\theta} - r - \boldsymbol{\alpha}^T\boldsymbol{X}_\mathrm{p}) \\
&\quad + \frac{1}{\gamma}\tilde{\boldsymbol{\theta}}^T\dot{\hat{\boldsymbol{\theta}}} + \frac{1}{\eta}\tilde{\boldsymbol{\phi}}^T\dot{\hat{\boldsymbol{\phi}}} + \frac{1}{\lambda}\tilde{\boldsymbol{\alpha}}^T\dot{\hat{\alpha}} \\
&= e_\epsilon\left(-ke_c + \frac{\boldsymbol{N}\boldsymbol{m}}{\boldsymbol{N}\hat{\boldsymbol{m}}}\left(-k_\mathrm{d}e_c + r + \hat{\boldsymbol{\alpha}}^T\boldsymbol{X}_\mathrm{p} + \boldsymbol{N}\hat{\boldsymbol{\theta}}\right.\right. \\
&\quad \left.\left. + k^*\mathrm{sat}\left(\frac{e_c}{\epsilon}\right)\right) - \boldsymbol{N}\boldsymbol{\theta} - r - \boldsymbol{\alpha}^T\boldsymbol{X}_\mathrm{p}\right) + \frac{1}{\gamma}\tilde{\boldsymbol{\theta}}^T\dot{\hat{\boldsymbol{\theta}}} \\
&\quad + \frac{1}{\eta}\tilde{\boldsymbol{\phi}}^T\dot{\hat{\boldsymbol{\phi}}} + \frac{1}{\lambda}\tilde{\boldsymbol{\alpha}}^T\dot{\hat{\boldsymbol{\alpha}}} \\
&= e_\epsilon\left(-ke_c + (1 + \boldsymbol{N}\tilde{\boldsymbol{\phi}})\left(-k_\mathrm{d}e_c + r + \hat{\boldsymbol{\alpha}}^T\boldsymbol{X}_\mathrm{p}\right.\right. \\
&\quad \left.\left. + \boldsymbol{N}\hat{\boldsymbol{\theta}} - k^*\mathrm{sat}\left(\frac{e_c}{\epsilon}\right)\right) - \boldsymbol{N}\boldsymbol{\theta} - r - \boldsymbol{\alpha}^T\boldsymbol{X}_\mathrm{p}\right) \\
&\quad + \frac{1}{\gamma}\tilde{\boldsymbol{\theta}}^T\dot{\hat{\boldsymbol{\theta}}} + \frac{1}{\eta}\tilde{\boldsymbol{\phi}}^T\dot{\hat{\boldsymbol{\phi}}} + \frac{1}{\lambda}\tilde{\boldsymbol{\alpha}}^T\dot{\hat{\boldsymbol{\alpha}}}
\end{aligned} \quad (32)$$

To get the above equation, the following relation is used:

$$\begin{aligned}
\frac{\boldsymbol{N}\boldsymbol{m}}{\boldsymbol{N}\hat{\boldsymbol{m}}} &= \chi_\mathrm{r}\frac{m_\mathrm{r}}{\hat{m}_\mathrm{r}} + \chi_\mathrm{l}\frac{m_\mathrm{l}}{\hat{m}_\mathrm{l}} \\
&= \chi_\mathrm{r}(1 + \tilde{\phi}_\mathrm{r}) + \chi_\mathrm{l}(1 + \tilde{\phi}_\mathrm{l}) \\
&= 1 + \boldsymbol{N}\tilde{\boldsymbol{\phi}}
\end{aligned} \quad (33)$$

By using the properties of the projection operators:

$$(\hat{\theta}_i - \theta_i)^T\mathrm{proj}(\hat{\theta}_i, -\gamma e_\epsilon) \leq -(\hat{\theta}_i - \theta_i)^T\gamma e_\epsilon$$
$$(\hat{\phi}_i - \phi_i)^T\mathrm{proj}(\hat{\phi}_i, -\eta e_\epsilon n_v) \leq -(\hat{\phi}_i - \phi_i)^T\eta e_\epsilon n_v$$

we have

$$\begin{aligned}
\dot{V}(t) &= e_\epsilon\left(-ke_c + \boldsymbol{N}\tilde{\boldsymbol{\phi}}\left(-k_\mathrm{d}e_c + r + \hat{\boldsymbol{\alpha}}^T\boldsymbol{X}_\mathrm{p}\right.\right. \\
&\quad \left. + \boldsymbol{N}\hat{\boldsymbol{\theta}} - k^*\mathrm{sat}\left(\frac{e_c}{\epsilon}\right)\right) - k_\mathrm{d}e_c + r + \hat{\boldsymbol{\alpha}}^T\boldsymbol{X}_\mathrm{p} \\
&\quad \left. + \boldsymbol{N}\hat{\boldsymbol{\theta}} - k^*\mathrm{sat}\left(\frac{e_c}{\epsilon}\right) - \boldsymbol{N}\boldsymbol{\theta} - r - \boldsymbol{\alpha}^T\boldsymbol{X}_\mathrm{p}\right) \\
&\quad + \frac{1}{\gamma}\tilde{\boldsymbol{\theta}}^T\dot{\hat{\boldsymbol{\theta}}} + \frac{1}{\eta}\tilde{\boldsymbol{\phi}}^T\dot{\hat{\boldsymbol{\phi}}} + \frac{1}{\lambda}\tilde{\boldsymbol{\alpha}}^T\dot{\hat{\boldsymbol{\alpha}}} \\
&\leq e_\epsilon\left(-ke_c + \boldsymbol{N}\tilde{\boldsymbol{\phi}}n_v - k_\mathrm{d}e_c + \hat{\boldsymbol{\alpha}}^T\boldsymbol{X}_\mathrm{p}\right. \\
&\quad \left. + \boldsymbol{N}\hat{\boldsymbol{\theta}} - k^*\mathrm{sat}\left(\frac{e_c}{\epsilon}\right) - \boldsymbol{N}\boldsymbol{\theta} - \boldsymbol{\alpha}^T\boldsymbol{X}_\mathrm{p}\right) \\
&\quad - \boldsymbol{N}\tilde{\boldsymbol{\theta}}^T e_\epsilon - \boldsymbol{N}\tilde{\boldsymbol{\phi}}^T e_\epsilon n_v + \frac{1}{\lambda}\tilde{\boldsymbol{\alpha}}^T\dot{\hat{\boldsymbol{\alpha}}}
\end{aligned} \quad (34)$$

By applying adaptation law (23), (24) and (25) we have:

$$\begin{aligned}
\dot{V}(t) &\leq e_\epsilon\left(-ke_c - k_\mathrm{d}e_c - k^*\mathrm{sat}\left(\frac{e_c}{\epsilon}\right)\right) \\
&= -(k + k_\mathrm{d})e_\epsilon\left(e_\epsilon + \epsilon\,\mathrm{sat}\left(\frac{e_c}{\epsilon}\right)\right) - k^*e_\epsilon\mathrm{sat}\left(\frac{e_c}{\epsilon}\right) \\
&= -(k + k_\mathrm{d})e_\epsilon^2 - (k + k_\mathrm{d})\epsilon|e_\epsilon| - k^*|e_\epsilon| \\
&\leq 0
\end{aligned} \quad (35)$$

To reach the above result, the relation $|e_\epsilon| = e_\epsilon\mathrm{sat}(e_c/\epsilon)$ for $|e_c| > \epsilon$ and $k > 0$, $k_\mathrm{d} > 0$, $k^* > 0$ are used.

Equations (30), (31) and (35) imply that V is a Lyapunov function which leads e_ϵ and $\tilde{\boldsymbol{\theta}}$ global bounded and convergence to zero. From the definition of e_ϵ, we can conclude that $e_c(t)$ is bounded, and convergence to $\epsilon\mathrm{sat}\ (e_c(t)/\epsilon)$, which implied that, if $\boldsymbol{X}(0)$ is bounded, then $\boldsymbol{X}(t)$ is also bounded for all $t > 0$, and tracking bounded $\boldsymbol{X}_m(t)$ with the precision of ϵ. □

It should be noticed that, in practical use of the proposed method, the state variables should be available either by estimate or measurement.

5 Simulation studies

In this Section, we illustrate the above method on a practical linear system, a 0.54 m long flexible beam with total inertial 0.07 kgm^2, whose first-order eigen frequency is 69.57 rad/s, damping ratio is 0.05 and the first-order vibration shape at the tip is −2.91.

Consider only the rigid motion and first-order vibration. In this case, the four state variables can be directly measured. Indeed we only need to measure the speed of the shaft and the vibration signal of the first natural frequency at the beam tip. We have the system model as:

$$\begin{cases} \dot{X}_p = A_p X_p + B w(t) \\ y = C X_p \end{cases} \tag{36}$$

where $w(t)$ is an output of a dead-zone. In the simulation, the beam is actuated by a DC motor with the parameters of the dead-zone being $b_r = 2.4$, $b_l = -2.3$, $m_r = m_l = 1.0$. And the bounds of them are chosen as $b_{r\,min} = 0$, $b_{r\,max} = 3.0$, $b_{l\,min} = -3.0$, $b_{l\,max} = 0$, $m_{min} = 0.7$, $m_{max} = 1.3$. Thus $\phi_{max} = m_{max}/m_{min} = 1.3/0.7$ and $\phi_{min} = m_{min}/m_{max} = 0.7/1.3$. In practice, all these limit values can be obtained from field experiment.

By applying the beam theory [11], we have:

$$A_p = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -4840 & -6.96 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & \frac{1}{0.07} & 0 & \frac{2}{0.07} \end{bmatrix}^T$$

$$C = [0.54 \quad 0 \quad -2.91 \quad 0]$$

This system is controllable and observable. We consider the close-loop pole placement system as the stable reference model. The poles and the zeros of the original system are poles $= [-3.48 + 69.48i, -3.48 - 69.48, 0, 0]'$, zeros $= [-21.90, 22.61]'$, while the gain is -75.43. We want to put the poles at $[-2, -20, -25 + 40i, -25 - 40i]$, which generate the reference system and corresponding α as

$$A_m = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -18.4 & -10.5 & 783.2 & -27.3 \\ 0 & 0 & 0 & 1 \\ -36.8 & -21 & -3273.6 & -61.5 \end{bmatrix}$$

$$\alpha = [-1.28 \quad -0.74 \quad 54.83 \quad -1.91]'$$

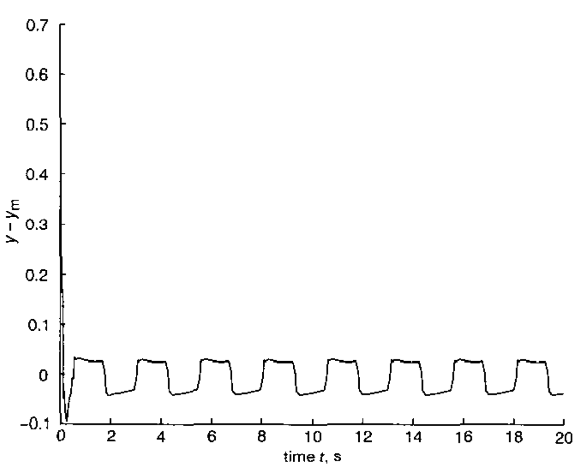

Fig. 2 *Tracking error between the plant and reference model outputs*

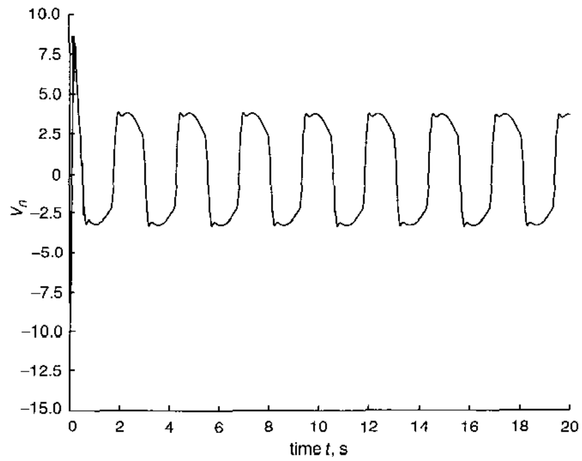

Fig. 3 *Control input of a system with a dead-zone*

This implies that in (5), $k = 2$, $R(s) = s^3 + 70s^2 + 3225s + 44\,500$. Based on the construction of the vector $\boldsymbol{h}$ in the proof of lemma 1 in [10], we have

$$\boldsymbol{h} = [0.65 \quad 0.05 \quad 3.94 \quad 0.02]'$$

In the adaptive control law (22)–(25), we take $k_d = 15$, $k^* = 1.8$, and choose $\gamma = 10.0$, $\eta = 0.05$, $\lambda = 0.1$, $\epsilon = 0.05$.

Choosing the specific input signal as $r(t) = 5.5\sin(2.5t)$ which is rather small signal compared with the dead-zone parameters mentioned above. That is the dead-zone is vital to the plant.

For the initial values of $b_{r0} = 0$, $b_{l0} = 0$, $m_{r0} = 1.0$, $m_{l0} = 1.0$, $X_{p0} = [1.0 \;\; 0.2 \;\; 0 \;\; 1]^T$, $X_{m0} = [0 \;\; 0 \;\; 0 \;\; 0]$ and a sample rate of 0.005, the simulation results are shown in Figs. 2–5.

Fig. 2 shows the tracking error between the plant and the reference model and Fig 3 shows the input control signal $v(t)$ to the motor, while the desired dead-zone output $w(t)$ is shown in Fig. 4. We see from Fig. 2 that the proposed adaptive controller clearly results in a good tracking performance.

We should mention that it is desirable to compare the control inputs with and without the dead-zone. This can be done by comparing Fig. 3 with Fig. 5. It can be easily seen that the input signals are quite different during crossing zeros due to the existence of the dead-zone. It should also

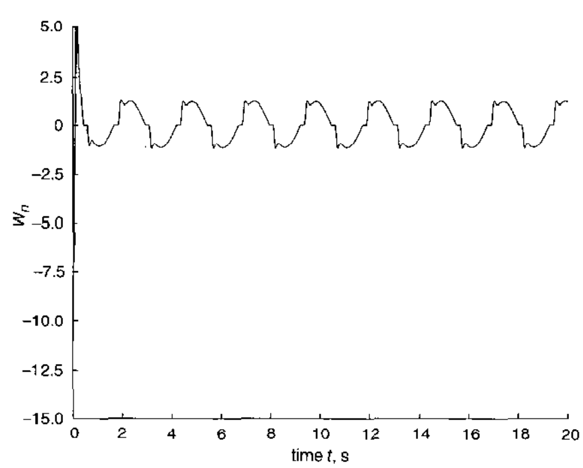

Fig. 4 *Output signal w(t) of the dead-zone*

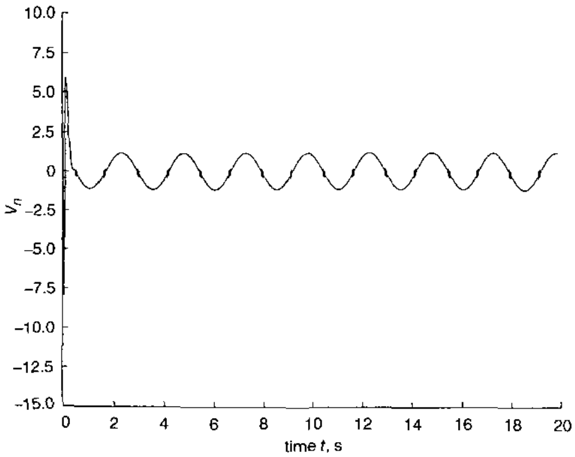

Fig. 5 *Control input of a system without a dead-zone*

be mentioned that from Figs. 4 and 5 one can see that the amplitude of the output signal of the dead-zone is almost the same as the input signal of a system without a dead-zone, which also verifies the validity of the proposed method.

6 Conclusions

In practical control systems, dead-zones with unknown parameters in physical components severely limit the control performance. A robust adaptive control architecture has been proposed for a class of continuous-time linear dynamic systems preceded by an unknown dead-zone. The properties of the dead-zone model have been discussed and a robust adaptive control scheme has been developed. The proposed control law ensures the global stability of the entire system and achieves both stabilisation and tracking within a desired precision. Simulations performed on a flexible beam system illustrate and clarify the validity of this approach.

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