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Brief paper

Robust adaptive control of a class of nonlinear systems with unknown dead-zone☆

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Abstract

This paper deals with the adaptive control of a class of continuous-time nonlinear dynamic systems preceded by an unknown dead-zone. By using a new description of a dead-zone and by exploring the properties of this dead-zone model intuitively and mathematically, a robust adaptive control scheme is developed without constructing the dead-zone inverse. The new control scheme ensures global stability of the adaptive system and achieves desired tracking precision. Simulations performed on a typical nonlinear system illustrate and clarify the validity of this approach.

Keywords: Dead-zone; Robust adaptive control; Nonlinear system; Stability analysis

1. Introduction

Generally, each industrial motion control system has the structure of a dynamical system, usually of the Lagrangian form, preceded by some nonsmooth nonlinearities in the actuator, either dead-zone, backlash, saturation, etc. Furthermore, these nonsmooth nonlineárities in such actuators (e.g. hydraulic servo valves, electric servomotors) are often unknown and time-variant. For example, a common source of nonsmooth nonlinearities arises from friction, which vary with temperature, speed and wear, or even differ significantly between mass-produced components. Thus, the study of nonsmooth nonlinearities involved has been of great interest to control researchers for a long time. The control of such systems needs to be robust, in order to compensate the time-variant effects of these nonlinearities. The problems are particularly important when the expected accuracy of the motion system is high.

Dead-zone, which can severely limit system performances, is one of the most important nonsmooth nonlinearities arisen in actuators, such as servo valves and DC servo motors. In most practical motion systems, the dead-zone parameters are poorly known, and robust control techniques are being sought. Proportional-derivative (PD) controllers have been observed to result in limit cycles. Due to the nonanalytic nature of dead-zone in actuators and the fact that the exact parameters (e.g. width of dead-zone) are unknown, systems with dead-zones present a challenge for the control design engineers. An immediate method for the control of dead-zone is to construct an adaptive dead-zone inverse. This approach was pioneered by Tao and Kokotovic (1994, 1995). Continuous- and discrete-time adaptive dead-zone inverses for linear systems with unmeasurable dead-zone outputs were built by Tao and Kokotovic (1994, 1995), respectively. Simulations indicated that the tracking performance is greatly improved by using dead-zone inverse. The work by Cho and Bai (1998) continued the above research and a perfect asymptotical adaptive cancellation of an unknown dead-zone was achieved analytically to systems in which the output of a dead-zone is measurable. To simplify the controller design, Kim, Park, Lee, and Chong (1994) proposed a two-layered fuzzy logic controller for the control of systems with dead-zones. In which, a fuzzy precompensator and a normal PD type fuzzy controller were introduced to control systems with dead-zones. Most recently, Lewis, Tim, Wang, and Li (1999) proposed a fuzzy precompensator in

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nonlinear industrial motion system and Selmic and Lewis (2000) employed neural networks to construct a dead-zone precompensator. Such approaches promise to improve the tracking performance of motion system in presence of unknown dead-zones.

A common feature for the above mentioned approaches is the construction of an inverse dead-zone nonlinearity to minimize the effects of dead-zone. However, different approaches may also be pursued. Based only on the intuitive concept and piece-wise description of dead-zones (Section 2), in this paper, a new approach for adaptive control of linear or nonlinear systems with dead-zones is introduced without constructing the inverse of the dead-zone. The new control law ensures a global stability of the entire adaptive system and asymptotical tracking (Section 4). Computer simulations were carried out to illustrate the effectiveness of the approach (Section 5).

2. Dead-zone model and its intuitive properties

The dead-zone with input $v(t)$ and output $w(t)$, as shown in Fig. 1, is described by

$$w(t) = D(v(t)) = \begin{cases} m_r(v(t) - b_r) & \text{for } v(t) \geqslant b_r, \\ 0 & \text{for } b_l < v(t) < b_r, \\ m_l(v(t) - b_l) & \text{for } v(t) \leqslant b_l. \end{cases} \quad (1)$$

As stated by Tao and Kokotovic (1994), this dead-zone model is a static simplification of diverse physical phenomena with negligible fast dynamics. Eq. (1) is a good model for a hydraulic servo valve or a servo motor.

The key features of the dead-zone in the control problems investigated in this paper are

(A1) The dead-zone output $w(t)$ is not available for measurement.
(A2) The dead-zone slopes in positive and negative region are same, i.e. $m_r = m_l = m$.
(A3) The dead-zone parameters b_r, b_l, and m are unknown, but their signs are known: $b_r > 0$, $b_l < 0$, $m > 0$.

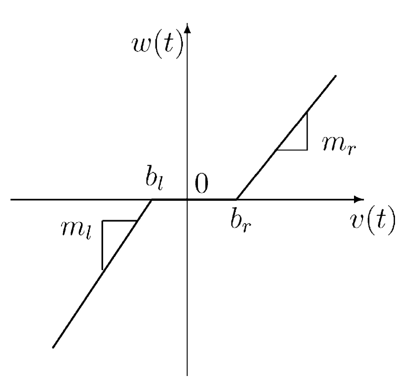

Fig. 1. Dead-zone model.

(A4) The dead-zone parameters b_r, b_l, and m are bounded, i.e. there exist known constants $b_{r\min}$, $b_{r\max}$, $b_{l\min}$, $b_{l\max}$, $m_{\min}$, $m_{\max}$ such that $b_r \in [b_{r\min}, b_{r\max}]$, $b_l \in [b_{l\min}, b_{l\max}]$, and $m \in [m_{\min}, m_{\max}]$.

Remark. Assumption (A1) is common in practical systems, such as servomotors and servovalves. If $w(t)$ is measurable, the control of dead-zone will be relatively easy and will not be discussed in this paper. Assumption (A2) is generally adopted in the literature (see, for example, Kim et al., 1994; Lewis et al., 1999) and can commonly be met in the industrial systems. Assumptions (A3) and (A4) are physically satisfied in real plants.

From a practical point of view, we can re-define model (1) as

$$w(t) = D(v(t)) = mv(t) + d(v(t)), \quad (2)$$

where m is called the general slope of the dead-zone, $d(v(t))$ can be calculated from (1) and (2) as,

$$d(v(t)) = \begin{cases} -mb_r & \text{for } v(t) \geqslant b_r, \\ -mv(t) & \text{for } b_l < v(t) < b_r, \\ -mb_l & \text{for } v(t) \leqslant b_l. \end{cases} \quad (3)$$

From Assumptions (A2) and (A4), one can conclude that $d(v(t))$ is bounded, and satisfies

$$|d(v(t))| \leqslant \rho,$$

where ρ is the upper-bound, which can be chosen as

$$\rho = \max\{m_{\max} b_{r\max}, -m_{\max} b_{l\min}\}, \quad (4)$$

where $b_{l\min}$ carries a negative value.

3. Control problem statement

In this paper, the system to be controlled consists of nonlinear plants preceded by actuators with dead-zone. That is, the dead-zone is present in series as the input of the nonlinear plant.

A dead-zone nonlinearity can be denoted as an operator

$$w(t) = D(v(t)) \quad (5)$$

with $v(t)$ as input and $w(t)$ as output. The operator $D(v(t))$ has been described in the previous section. The nonlinear dynamic system preceded by the above dead-zone is described as

$$x^{(n)}(t) + \sum_{i=1}^{r} a_i Y_i(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)) = bw(t). \quad (6)$$

We have the following assumptions about system (6):

(A5) Y_i are known continuous, linear or nonlinear functions.
(A6) Parameters a_i are unknown but constant.
(A7) The control gain b is unknown but constant. And further the sign of b is known. From now on, without losing generality, we assume $b > 0$.

Remark. Assumptions (A5)–(A7) confine the type of systems to be considered in this paper. It should be noted that this type of system represents a large class of physical nonlinear systems. In the spirit of the nonlinear control literature (Isidori, 1989), these systems are in normal form and have the relative degree equal to n without finite zero dynamics. The system treated in this paper can be extended to an even more general class of nonlinear systems (Mareels, Penfold, & Evans, 1992). However, as will be clear later, the goal of this paper is to show the controller design strategy without construction of an inverse function in a simple setting that reveals its essential features.

The control objective is to design a control law for $v(t)$ in (5) to let the plant state vector, $\mathbf{x}=[x,\dot{x},\ldots,x^{(n-1)}]^{\mathrm{T}}$, follow a specified desired trajectory, $\mathbf{x}_{\mathrm{d}}=[x_{\mathrm{d}},\dot{x}_{\mathrm{d}},\ldots,x_{\mathrm{d}}^{(n-1)}]^{\mathrm{T}}$, i.e., $\mathbf{x}\to\mathbf{x}_{\mathrm{d}}$ within a desired accuracy as $t\to\infty$.

4. Adaptive controller design

In this section, we shall propose an adaptive controller for plants of the form in (6), preceded by a dead-zone described in (1), which will guarantee global system stability and yields the system output tracking to a desired trajectory within a desired accuracy.

Using expression (2), system (6) becomes

$$x^{(n)}(t)+\sum_{i=1}^{r}a_iY_i(x(t),\dot{x}(t),\ldots,x^{(n-1)}(t)) =bmv(t)+bd(v(t)). \tag{7}$$

In which, the state variables of the control problem become linear to the input signal $v(t)$. It is very important to note that $d(v(t))$ is uniformly bounded.

From (7), the signal $w(t)$ is expressed as a linear function of input signal $v(t)$ plus a bounded term. In such a case, the currently available robust control techniques can be utilized for the controller design. This explains the reason for defining the intuitive simplified dead-zone model (3). In the following development, we shall adopt a robust adaptive approach to illustrate the controller development. It should be mentioned that other control approaches can also be exploited for the controller development, see, for example, Mareels et al. (1992).

For the development of an adaptive control law, the following additional assumption regarding the desired trajectory is made.

(A8) The desired trajectory, $\mathbf{x}_{\mathrm{d}}=[x_{\mathrm{d}},\dot{x}_{\mathrm{d}},\ldots,x_{\mathrm{d}}^{(n-1)}]^{\mathrm{T}}$ is continuous and available. Furthermore $[\mathbf{x}_{\mathrm{d}}^{\mathrm{T}},x_{\mathrm{d}}^{(n)}]^{\mathrm{T}}\in\Omega_{\mathrm{d}}\subset R^{n+1}$ with Ω_{d} being a compact set.

Remark. Assumption (A8) depicts a restriction on the types of reference signals which may be used.

To achieve the above stated control objective, a filtered tracking error is defined as Slotine and Coetsee, 1986

$$s(t)=\left(\frac{\mathrm{d}}{\mathrm{d}t}+\lambda\right)^{n-1}\tilde{x}(t)\quad\text{with}\quad\lambda>0, \tag{8}$$

which can be rewritten as

$$s(t)=\Lambda^{\mathrm{T}}\tilde{\mathbf{x}}(t)$$

with

$$\Lambda^{\mathrm{T}}=[\lambda^{n-1},(n-1)\lambda^{n-2},\ldots,1],$$

where $\tilde{\mathbf{x}}=\mathbf{x}-\mathbf{x}_{\mathrm{d}}$

From (8) and by defining $\Lambda_v^{\mathrm{T}}=[0,\lambda^{n-1},(n-1)\lambda^{n-2},\ldots,(n-1)\lambda]$, it follows:

$$\begin{aligned}\dot{s}(t)&=\Lambda_v^{\mathrm{T}}\tilde{\mathbf{x}}(t)+\tilde{x}^{(n)}(t)\\&=\Lambda_v^{\mathrm{T}}\tilde{\mathbf{x}}(t)-\sum_{i=1}^{r}a_iY_i(\mathbf{x}(t))\\&\quad+bmv(t)+bd(v(t))-x_{\mathrm{d}}^{(n)}(t).\end{aligned} \tag{9}$$

Remark. It has been shown by Slotine and Coetsee (1986) and Slotine (1984) that the definition (8) has the following properties: (i) the equation $s(t)=0$ defines a time-varying hyperplane in R^n, on which the tracking error vector $\tilde{\mathbf{x}}(t)$ decays exponentially to zero, (ii) if $\tilde{\mathbf{x}}(0)=0$ and $|s(t)|\leqslant\varepsilon$ with constant ε, then $\tilde{\mathbf{x}}(t)\in\Omega_\varepsilon\triangleq\{\tilde{\mathbf{x}}(t)\|\tilde{\mathbf{x}}_i|\leqslant 2^{i-1}\lambda^{i-n}\varepsilon,\ i=1,\ldots,n\}$ for $\forall t\geqslant 0$, and (iii) if $\tilde{\mathbf{x}}(0)\neq 0$ and $|s(t)|\leqslant\varepsilon$, then $\tilde{\mathbf{x}}(t)$ will converge to Ω_ε within a time-constant $(n-1)/\lambda$.

To keep the state variables on $s(t)=0$, the condition $s(t)\dot{s}(t)\leqslant -M|s(t)|$ should be satisfied, which reasonably leads to an ideal control law

$$v(t)=-k_{\mathrm{d}}s(t)+\frac{1}{bm}(x_{\mathrm{d}}^{(n)}(t)-\Lambda_v^{\mathrm{T}}\tilde{\mathbf{x}}^{\mathrm{T}}(t))+\sum_{i=1}^{r}\frac{a_i}{bm}Y_i(\mathbf{x}(t))-\frac{d(v(t))}{m}-Msgn(s(t)), \tag{10}$$

where k_{d} is a positive constant and M is a constant. However, in (10) some system parameters such as a_i/bm are unknown and a robust control law should be considered in the controller design. Before introducing the control law, some preparations are needed. Firstly, rather than driving the adaptive law with the filtered error $s(t)$ a tuning error, s_ε, is introduced as follows:

$$s_\varepsilon=s-\varepsilon\,sat\left(\frac{s}{\varepsilon}\right), \tag{11}$$

where ε is an arbitrary positive constant and $sat(\cdot)$ is the saturation function defined as:

$$sat(z)=\begin{cases}1 & for\ z\geqslant 1,\\ z & for\ -1<z<1,\\ -1 & for\ z\leqslant -1.\end{cases} \tag{12}$$

Secondly, in presenting the robust adaptive control law, we define

$$\tilde{\theta} = \hat{\theta} - \theta, \quad \tilde{\phi} = \hat{\phi} - \phi, \tag{13}$$

where $\hat{\theta}$ is an estimate of θ, defined as $\theta \triangleq [a_1/bm, \ldots, a_r/bm]^{\mathrm{T}} \in R^r$, and $\hat{\phi}$ is an estimate of ϕ, which is defined as $\phi \triangleq (bm)^{-1}$.

Based on the given plant and dead-zone models under the assumptions described above, the following control and adaptation laws are presented:

$$v(t) = -k_{\mathrm{d}} s(t) + \hat{\phi} u_{f\mathrm{d}}(t) + Y^{\mathrm{T}}(\mathbf{x})\hat{\theta} - k^* sat\left(\frac{s}{\varepsilon}\right), \tag{14}$$

$$\dot{\hat{\theta}}_i = -\gamma Y_i(\mathbf{x}) s_\varepsilon, \tag{15}$$

$$\dot{\hat{\phi}} = -\eta u_{f\mathrm{d}} s_\varepsilon, \tag{16}$$

where

$$u_{f\mathrm{d}}(t) = x_{\mathrm{d}}^{(n)}(t) - \Lambda_v^{\mathrm{T}} \tilde{\mathbf{x}}(t) \tag{17}$$

$Y \triangleq [Y_1, \ldots, Y_r]^{\mathrm{T}} \in R^r$; k^* is a control gain, satisfying

$$k^* \geqslant \rho / m_{\min} \tag{18}$$

therein, ρ is defined in (4); γ and η are positive constants, determining the rates of adaptations;

Remarks. (1) It should be noticed that in (14)–(16), the exact values mentioned in Section 2 are not required. This will simplify the control design in practical systems.

(2) The tuning error, s_ε, will disappear when the filtered error, s, is less than ε, which shall be the equivalent of creating an adaptation deadband.

(3) The term $k^* sat(s/\varepsilon)$ actually reflects the component for compensation of the bounded function $d(v)$. And it gives the robust property of the adaptive law. It also should be noticed that if ε is chosen too small, the linear region of function $sat(s/\varepsilon)$ will be too thin, which will cause a risk of exciting high-frequency fluctuations. As $\varepsilon \to 0$, the function $sat(s/\varepsilon)$ eventually becomes discontinuous. In such a case, the controller becomes a typical variable structure control scheme, which may cause chattering phenomena. This suggests that a trade-off must be made between the value of ε and trajectory-following requirements.

The stability of the closed-loop system described by (1), (6) and (14)–(16) is established in the following theorem:

Theorem. *For the plant in Eq.* (6) *with dead-zone* (1) *at the input subject to assumptions* (A1)–(A8), *the robust adaptive controller specified by Eqs.* (14)–(16) *ensures that all the closed-loop signals are bounded and the state vector* $\mathbf{x}(t)$ *converges to* $\Omega_\varepsilon = \{\mathbf{x}(t) \| \tilde{\mathbf{x}}_i| \leqslant 2^{i-1}\lambda^{i-n}\varepsilon,\ i = 1, \ldots, n\}$ *for* $\forall t \geqslant t_0$.

The proof of this theorem is shown in the Appendix.

5. Simulation studies

In this section, we will illustrate the above method on a nonlinear systems described as (Zhang & Feng, 1997)

$$\ddot{x} = a_1 \frac{1 - e^{-x}}{1 + e^{-x}} - a_2(\dot{x}^2 + 2x)\sin\dot{x}$$
$$- 0.5 a_3 x \sin 3t + bw(t), \tag{19}$$

where $w(t)$ is an output of a dead-zone. The parameters to be simulated are $b=1$ and $a_1=a_2=a_3=1$. Without control, i.e., $w(t)=0$, as stated by Zhang and Feng (1997), the system (19) is unstable (i.e. the linearization of the system at the origin is unstable).This also has been verified by simulation. The control objective is to let the system state $[x, \dot{x}]^{\mathrm{T}}$ follow the desired trajectory $[x_{\mathrm{d}}, \dot{x}_{\mathrm{d}}]^{\mathrm{T}}$.

To verify the robustness of the method, the term $0.5a_3 x \sin 3t$ in the above system is treated as an unmodeled dynamics or disturbance, which will not be considered in the controller design.

In the simulation, parameters of the dead-zone are $b_r=0.5$, $b_l=-0.6$, $m=1$. And their bounds are chosen as $b_{r\min}=0.1$, $b_{r\max}=0.6$, $b_{l\min}=-0.7$, $b_{l\max}=-0.1$, $m_{\min}=0.85$, $m_{\max}=1.25$, and $k^*=2.5$. In the robust adaptive control law (14)—(16), the control parameters are chosen as $k_{\mathrm{d}}=10$, $\gamma=0.5$, $\eta=0.5$, $\varepsilon=0.01$, and the sample rate as 0.005.

Choosing the desired trajectory $x_{\mathrm{d}}(t) = 2.5\sin t$, simulation results, with initial values as $\mathbf{x}(0) = [-2.5, 3.5]^{\mathrm{T}}$, $\theta_1(0) = 0.85$, $\theta_2(0) = 0.85$, $\phi(0) = 0.85$, are shown in Figs. 2–6. Fig. 2 shows the position tracking performance and Fig. 3 shows the corresponding tracking error. Fig. 4 shows the input control signal of dead-zone $v(t)$. From Fig. 3, it clearly shows that the proposed robust controller results in excellent tracking performance. We should emphasize that the unmodeled dynamics $0.5a_3 x \sin 3t$ has not been accounted in the controller.

For the purpose of comparison, Fig. 5 shows the tracking error of the same system with same parameters and desired trajectory except that there was no dead-zone (i.e., $b_r=b_l=0$ and $m=1$). Fig. 6 shows the corresponding input control signal to the system. Comparing Figs. 4 and 6, it is obvious that $v(t)$ plays a role when it crosses zero, which implies that

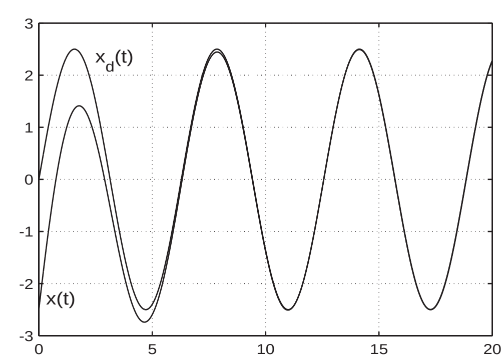

Fig. 2. Tracking performance of the system with dead-zone.

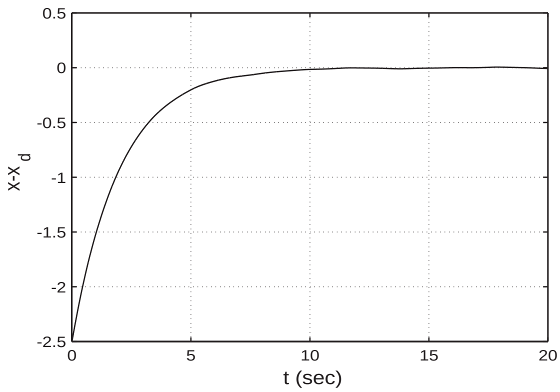

Fig. 3. Tracking error of the system state with dead-zone.

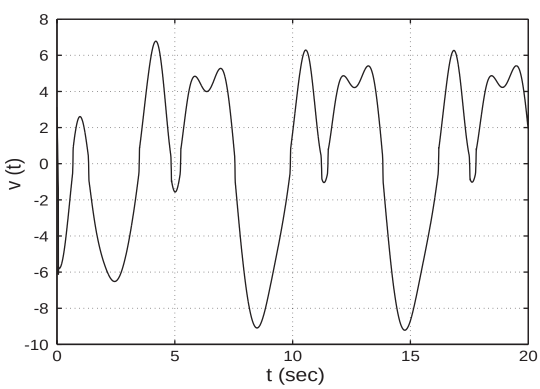

Fig. 4. Control signal $v(t)$ acting as the input of dead-zone.

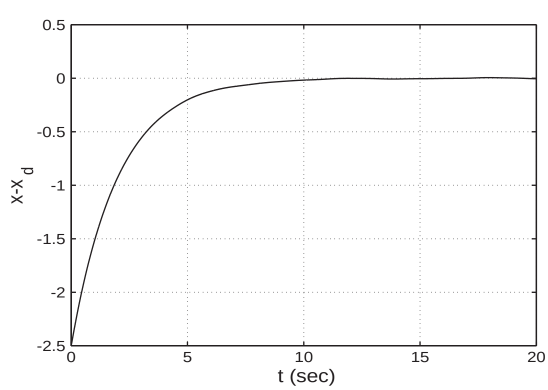

Fig. 5. Tracking error of the system state without dead-zone.

the developed control algorithm can overcome the effects of the dead-zone.

As yet, no analytical approach has been developed for the selection of the control constants. The approach to select their values was through iterative simulation. For example, in the simulation, ε has been tried from 0.005 to 0.5, the best tracking result was achieved when it was in the range

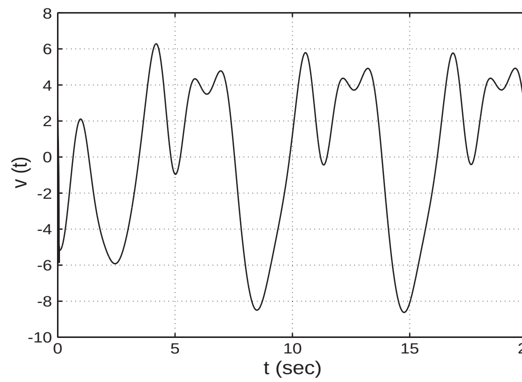

Fig. 6. Control signal $v(t)$ without dead-zone.

$\varepsilon = 0.015$–0.05. All the simulation results in the paper correspond to $\varepsilon = 0.02$.

6. Conclusion

In practical control systems, dead-zones with unknown parameters in physical components may severely limit the performance of control. In this paper, a robust adaptive control architecture is proposed for a class of continuous-time nonlinear dynamic systems preceded by a dead-zone. By using a new description of a dead-zone and by showing the properties of this dead-zone model intuitively, this robust adaptive control scheme is developed without constructing a dead-zone inverse. The new control law ensures global stability of the entire system and achieves both stabilization and tracking within a desired precision. Simulations performed on a unstable nonlinear system illustrate and clarify the approach.

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Appendix.

Proof. Using expression (7), the time derivative of filtered error (8) can be written as

$$\dot{s}(t) = -u_{f\mathrm{d}}(t) - \sum_{i=1}^{r} a_i Y_i(\mathbf{x}(t)) + bmv(t) + bd(v). \quad \text{(A.1)}$$

Using the control law (14)–(16), the above equation can be rewritten as

$$\dot{s}(t) = -u_{f\mathrm{d}}(t) - \sum_{i=1}^{r} a_i Y_i(\mathbf{x}(t)) + bm\left[-k_\mathrm{d}s(t) + \hat{\phi}u_{f\mathrm{d}}(t) + Y^\mathrm{T}(\mathbf{x})\hat{\theta} - k^* sat\left(\frac{s}{\varepsilon}\right)\right] + bd(v). \tag{A.2}$$

To establish global boundedness, we define a positive function as

$$V(t) = \frac{1}{2}\left[\frac{1}{bm}s_\varepsilon^2 + \frac{1}{\gamma}\tilde{\theta}^\mathrm{T}\tilde{\theta} + \frac{1}{\eta}\tilde{\phi}^2\right]. \tag{A.3}$$

Using (A.2) and the fact $s_\varepsilon \dot{s}_\varepsilon = s_\varepsilon \dot{s}$, it follows:

$$\begin{aligned}\dot{V}(t) &= \frac{1}{bm}s_\varepsilon \dot{s} + \frac{1}{\gamma}\tilde{\theta}^\mathrm{T}\dot{\hat{\theta}} + \frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}} \\ &= -k_\mathrm{d}s_\varepsilon s + s_\varepsilon\left[\hat{\phi}u_{f\mathrm{d}}(t) + Y^\mathrm{T}(\mathbf{x})\hat{\theta} - k^* sat\left(\frac{s}{\varepsilon}\right)\right] \\ &\quad + \frac{1}{bm}s_\varepsilon\left[-u_{f\mathrm{d}}(t) - \sum_{i=1}^{r} a_i Y_i(\mathbf{x}(t)) + bd(v)\right] \\ &\quad + \frac{1}{\gamma}\tilde{\theta}^\mathrm{T}\dot{\hat{\theta}} + \frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}} \\ &= -k_\mathrm{d}s_\varepsilon s + s_\varepsilon\left[\hat{\phi}u_{f\mathrm{d}}(t) + Y^\mathrm{T}(\mathbf{x})\hat{\theta} - k^* sat\left(\frac{s}{\varepsilon}\right)\right] \\ &\quad + s_\varepsilon[-\phi u_{f\mathrm{d}}(t) - Y^\mathrm{T}\theta + d(v)/m] \\ &\quad + \frac{1}{\gamma}\tilde{\theta}^\mathrm{T}\dot{\hat{\theta}} + \frac{1}{\eta}\tilde{\phi}\dot{\hat{\phi}}.\end{aligned} \tag{A.4}$$

By applying adaptation law (15), (16), one has

$$\begin{aligned}\dot{V}(t) &= -k_\mathrm{d}s_\varepsilon s - k^* s_\varepsilon sat\left(\frac{s}{\varepsilon}\right) + \frac{d(v)}{m}s_\varepsilon \\ &= -k_\mathrm{d}s_\varepsilon\left(s_\varepsilon + \varepsilon\, sat\left(\frac{s}{\varepsilon}\right)\right) - k^* s_\varepsilon sat\left(\frac{s}{\varepsilon}\right) + \frac{d(v)}{m}s_\varepsilon \\ &= -k_\mathrm{d}s_\varepsilon^2 - (k_\mathrm{d}\varepsilon + k^*)|s_\varepsilon| + \frac{d(v)}{m}s_\varepsilon \\ &\leqslant -k_\mathrm{d}s_\varepsilon^2 - k_\mathrm{d}\varepsilon|s_\varepsilon| - \left(k^* - \frac{|d(v)|}{m}\right)|s_\varepsilon| \\ &\leqslant -k_\mathrm{d}s_\varepsilon^2.\end{aligned} \tag{A.5}$$

In the above procedure (A.5), the relationship equation (18), $|s_\varepsilon| = 0$ for $|s| \leqslant \varepsilon$ and $|s_\varepsilon| = s_\varepsilon sat(s/\varepsilon)$ for $|s| > \varepsilon$, have been used.

Integrating both sides of Eq. (A.5) shows that

$$\int_0^t k_\mathrm{d}s_\varepsilon^2\,\mathrm{d}\tau \leqslant V(0) - V(t) \leqslant V(0) < \infty, \quad \forall t \geqslant 0.$$

Therefore, $s_\varepsilon \in L_2 \cap L_\infty$ and $\tilde{\theta}, \tilde{\phi} \in L_\infty$. From the definition of s_ε, one can conclude that $s(t) \in L_\infty$. Considering the filtered error dynamics (A.2), we see that $\dot{s} \in L_\infty$, which implies that $\dot{s}_\varepsilon \in L_\infty$. Because $s_\varepsilon \in L_2 \cap L_\infty$ and $\dot{s}_\varepsilon \in L_\infty$, it follows from Barbalat's lemma that $s_\varepsilon(t) \to 0$ as $t \to \infty$. The remark following Eq. (8) indicates that $\tilde{\mathbf{x}}(t)$ will converge to Ω_ε. □

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