Transmit Antenna Selection for Decision Feedback Detection in MIMO Fading Channels
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Abstract—In this paper, we investigate a transmit antenna selection (TAS) approach for the decision-feedback detector (DFD) over Rayleigh fading channels. In particular, for a multiple-input multiple-output (MIMO) channel with $M$ transmit and $N$ ($N \geq M$) receive antennas, we derive a lower bound on the outage probability for the TAS approach. The selected transmit antennas are those that maximize the post-processing signal-to-noise ratio (SNR) at the receiver end. It is shown that the proposed TAS approach achieves a performance close to optimal selection based on exhaustive search, introduced in the literature, but at a lower complexity. Simulation results are presented to validate and demonstrate the performance gain of the proposed TAS approach.

Index Terms—Transmit antenna selection (TAS), decision-feedback detector (DFD), multiple-input multiple-output (MIMO), outage probability.

I. INTRODUCTION

RECENTLY, the authors in [1] and [2] have demonstrated that using a multiple-input multiple-output (MIMO) system one can drastically increase the system capacity, and improve the reliability of wireless transmission relative to a single-input single-output (SISO) system. Motivated by their performance gain, many schemes have been proposed to exploit the high spectral efficiency of MIMO systems, among which is the decision-feedback detector (DFD), which is also known as the vertical Bell labs layered space-time (VBLAST) [3]. The DFD is relatively simple and can reap a large portion of the high spectral efficiency of a MIMO system. However, a major factor limiting the use of multiple-antenna systems arises from the deployment of $M$ transmit and $N$ receive radio frequency (RF) chains, which normally comprise low-noise amplifiers (LNAs), analog-to-digital converters (ADCs), etc. This complexity problem can be mitigated by antenna selection at the transmitter and/or receiver. With antenna selection, a small number of analog RF chains are multiplexed between a much larger number of transmit/receive antennas. Therefore, antenna selection reduces the computational complexity as well as hardware cost. See [4] and [5] for a general review of various transmit and receive antenna selection schemes.

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Transmit antenna selection (TAS) for spatial multiplexing systems was first presented in [6]. More specifically, the authors show that feeding back an optimal subset of transmit antennas often increases system capacity over the case of no feedback. However, the selection criterion proposed therein is based on Shanon capacity and is not specialized to a specific receiver structure. With this motivation, the authors in [7] and [8] propose several TAS approaches that aim at minimizing the system error rate for spatial multiplexing systems employing linear receivers. Also, various receive antenna selection schemes for MIMO systems have been studied in the recent literature. In [9], a detailed analysis on suboptimal capacity-maximizing receive antenna selection schemes is presented. Recently, the authors in [10] present a comprehensive analysis of the outage probability for MIMO systems with receive antenna selection. And, the authors show that the full diversity order is maintained with receive antenna selection. The influence of joint transmit/receive antenna selection upon the fundamental diversity-multiplexing (D-M) tradeoff gain [11] is presented in [12]. In which, the authors show that a MIMO system with antenna selection has the same D-M tradeoff as the full system if the multiplexing gain is less than some threshold $P_{th}$. Other works related to antenna selection for MIMO systems can be found in [13] and [14]. We refer to [15] for results on the effect of detection ordering on the diversity gain per layer and D-M tradeoff gain of DFD in a MIMO Rayleigh fading channel. In this paper, we study the performance of a TAS scheme for the DFD over flat Rayleigh fading channels. We present a TAS criterion that maximizes the post-processing signal-to-noise ratio (SNR) at the receiver. A lower bound on the outage probability is also derived. It is worth mentioning that the TAS is facilitated by a low-rate feedback channel. The sole purpose of this feedback channel is to indicate antenna combination that maximizes the post-processing SNR. Finally, our work is a natural extension of the existing literature on transmit antenna subset selection for spatial multiplexing systems [6]–[8]. However, our work is different in that it deals with DFD receiver. Moreover, the authors present a detailed analysis on outage probability for the proposed TAS scheme.

The remainder of the paper is outlined as follows. System and channel model is introduced in Section II. The TAS approach and outage probability of the DFD with TAS are analyzed in Section III. Section IV presents simulation results to demonstrate the gain achieved using the proposed TAS approach and to assess the accuracy of our analytical results. Finally, conclusions are given in Section V.
II. SYSTEM AND CHANNEL MODEL

We consider a MIMO spatial multiplexing (MIMO-SM) system that employs \( M \) transmit and \( N \) (\( N \geq M \)) receive antennas, and a \( 1 : K \) (\( K < M \)) spatial multiplexer as shown in Fig. 1. The system works as follows. At one symbol time, \( K \) input symbols are multiplexed to produce the \( K \)-dimensional symbol vector \( x \) for transmission over \( K \) active transmit antennas out of \( M \) possible ones. The optimal subset \( p \), which constitutes of the \( K \) transmit antennas, is determined by a selection algorithm operating at the receiver. The latter indicates, at each fading state, to the transmitter through a low-bandwidth, zero-delay and error-free feedback channel, the optimal subset \( p \) of size \( K \). Note that \( p \) is the set of all possible subsets of selected transmit antennas given by

\[
P = \left\{ \left( \frac{M}{K} \right) \right\} \quad \text{for a given } K \left( K < M \right).
\]

At the receiver end, we have a DFD to cancel interference and obtain estimates of the transmitted data.

Let \( H \) denote the \( N \times M \) channel matrix (without TAS), and \( H_p \) denote the \( N \times K \) channel submatrix corresponding to the selected transmit antennas in \( p \). The corresponding sampled received baseband signal is then given by

\[
y = H_p \Pi_p x + n,
\]

where \( y \in \mathbb{C}^{N \times 1} \) is the received signal vector, \( \Pi_p \in \mathbb{R}^{K \times K} \) is a channel-dependent permutation matrix corresponding to the detection ordering, \( H_p \in \mathbb{C}^{N \times K} \) consists of independent and identically distributed (i.i.d.) circularly symmetric Gaussian random variables with zero-mean and unit-variance, i.e., \( h_{i,j} \sim \mathcal{CN}(0,1) \) for \( 1 \leq i \leq N, \ 1 \leq j \leq K \). We assume that the fading coefficients are constant over the entire frame and vary independently from one frame to another. The receiver has a perfect knowledge of the channel matrix \( H \), and the information symbol vector \( x \in \mathbb{C}^{K \times 1} \) consists of independent and uniform power transmitted substreams. The receiver noise \( n \sim \mathcal{CN}(0, N_0 I_N) \) consists of independent circularly symmetric zero-mean complex Gaussian entries of variance \( N_0 \), where \( I_N \) is an identity matrix of size \( N \).

III. TRANSMIT ANTENNA SELECTION AND ANALYSIS

A. Transmit Antenna Selection Approach

The DFD algorithm was shown to suppress the interference by either zero-forcing (ZF) or minimum mean-square error (MMSE) criterion. However, here we constrain our discussion to the ZF case. The reason is that ZF nulling criterion has lower implementation complexity which keeps the analysis more tractable than the MMSE. Furthermore, the performance of the ZF receiver approaches that of MMSE at high SNR. In this section, we use the full system channel matrix \( H \) since no selection is yet performed. It is well-known that the DFD can be concisely represented by the QR decomposition [16], [17], i.e., \( H = QR \), where \( Q \) is an \( N \times M \) semi-unitary matrix \( (Q^H Q = I_M, \text{where } I_M \text{ is an identity matrix of size } M) \) with its orthonormal columns being the ZF nulling vectors, and \( R \) is an \( M \times M \) upper triangular matrix with real-valued positive diagonal entries. Correspondingly, the ordered DFD can be represented by applying the QR decomposition to \( H \) with its columns permuted, i.e., \( H \Pi = QR \), where \( \Pi \) is the full-channel-dependent permutation matrix (i.e., function of \( H \)). The receiver performs a QR factorization of \( H \), and then it implements two operations: nulling and cancellation.

We have

\[
y = QR x + n.
\]

The transmitted symbols are detected as follows. Multiplying both sides of (3) by \( Q^H \) yields

\[
\tilde{y} = Rx + \tilde{n},
\]

where \( \tilde{n} = Q^H n \) and \( \tilde{y} = Q^H y \).

The received vector \( \tilde{y} \), in matrix form, can be written as

\[
\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_M \end{bmatrix} = \begin{bmatrix} r_{1,1} & r_{1,2} & \ldots & r_{1,M} \\ 0 & r_{2,2} & \ldots & r_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & r_{M,M} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ \vdots \\ \tilde{n}_M \end{bmatrix}.
\]

The general sequential signal detection, which involves cancellation (decision feedback), is given by

\[
\hat{x}_i = Q \left[ \frac{1}{ \gamma_{i} } \left( \tilde{y}_i - \sum_{j=i+1}^{M} r_{i,j} \hat{x}_j \right) \right],
\]

with \( i = M, M-1, \ldots, 1 \), and \( Q \) stands for the mapping to the nearest point in the symbol constellation, \( (\cdot) \) is the hard/soft estimate, \( r_{i,j} \) is the \((i,j)\)th entry of \( R \). Inspection of (6) reveals that, to estimate \( x_M \), the receiver needs to multiply by the inverse of \( r_{M,M} \). Thus \( \tilde{y}_M \) constitutes a virtual subchannel that has no interference from other subchannels. Hence, the decision statistic for the \( M \)th received symbol is

\[
\hat{x}_M = Q \left[ \frac{1}{ r_{M,M} } \tilde{y}_M \right] = Q \left[ x_M + \left( \frac{1}{ r_{M,M} } \tilde{n}_M \right) \right].
\]

However, \( \tilde{y}_{M-1} \) is subject to interference from the \( M \)th subchannel through the off-diagonal entry \( r_{M-1,M} \) and so on. Assume that the previous decisions are correct (i.e., no propagation of error), the DFD decouples the MIMO channel into a set of \( M \) independent, parallel SISO virtual subchannels, and the different substreams can be expressed as

\[
\tilde{y}_i = r_{i,i} x_i + \tilde{n}_i, \quad \text{for } i = 1, 2, \ldots, M.
\]

Since \( \mathbb{E} [ \tilde{n} \tilde{n}^H ] = N_0 I_N \) with \( \mathbb{E} [ \cdot ] \) stands for expectation and \( (\cdot)^H \) is the conjugate transpose, the output SNR of the \( i \)th substream is given by

\[
\gamma_i = r_{i,i}^2 \gamma_0,
\]

where \( \gamma_0 = \mathbb{E} [ x^H x ] / N_0 \) is the average normalized received SNR at each receive antenna. Thus, the output SNRs
of the substreams are determined by the diagonal entries of the matrix \( \mathbf{R} \) which in turn depends on \( \mathbf{H} \). Based on (9), a pragmatic TAS criterion would be to choose the subset of transmit antennas with the highest \( r_{i,i} \)’s values. Now, it is essential to mention that our AS criterion is also applicable to the case where propagation of error exists. In this case, one can show that the general sequential decision statistic can be written as

\[
\hat{x}_{M-i} = Q \left[ x_{M-i} + \frac{1}{r_{M-i,M-i}} \left( \tilde{n}_{M-i} + \sum_{j=M-i+1}^{M} r_{M-i,j} \Delta e_j \right) \right],
\]

with \( 0 \leq i \leq M-1 \), and \( \Delta e_j \) denotes the error term resulting from the hard/soft estimate made on the \( x_j \) symbol. Hence to minimize the error term, we have to select the largest \( r_{i,i} \)’s values. The reason is that the error term is inversely proportional to \( r_{i,i} \)’s values. Thus, Clearly, the adopted transmit selection criterion is optimal in the sense that it maximizes the post-processing SNR at the receiver side.

Now, neglecting the propagation of error and using (9), we can see that the channel capacity is now equivalent to the capacity of a MIMO-SM system with linear receiver employed. Thereby the channel is now decoupled into \( M \) parallel substreams, for which the capacity with DFD is given by [18]

\[
C = \sum_{i=1}^{M} \log_2 (1 + \gamma_i),
\]

where \( \gamma_i \) is the post-processing SNR for the \( i \)th substream. Thus the capacity of the transmit antenna selected system is given by

\[
C_{TAS} = \sum_{i=1}^{K} \log_2 (1 + r_{i,i}^2 \gamma_i),
\]

where \( r_{i,i} \) is the \( (i,i) \)th entry of \( \mathbf{R} \) with \( \mathbf{H}_p \mathbf{H}_p = Q \mathbf{R} \).

### B. Analysis on Outage Probability

In this section, we present a comprehensive analysis of the outage probability for the TAS scheme over independent Rayleigh fading channels. A lower bound on the outage probability at high SNR regime is presented.

Recall that the instantaneous capacity expression of a MIMO fading channel (without performing TAS) is given by [1]

\[
C = \log_2 \det \left[ \mathbf{I}_M + \frac{\zeta}{M} \mathbf{H}^H \mathbf{H} \right], \text{ bits/s/Hz},
\]

where \( \zeta = \mathbb{E} [\mathbf{x}^H \mathbf{x}] / N_0 \) is the total average energy over a symbol period (i.e., total input power when no TAS is performed), and \( \det(\cdot) \) is the determinant.

Now, an outage event occurs when the information transmission rate (i.e., overall input data rate of the full system), denoted by \( R_c \), is greater than the instantaneous capacity \( C \). Hence, the outage probability is given by [2]

\[
\mathcal{P}_{\text{outage}} = \Pr (C < R).
\]

The overall system performance of the DFD is limited by the first detected layer (i.e., equal rates are allocated across layers). As a result, the overall outage probability of the DFD is dominated by that of the \( K \)th substream (i.e., first detected layer). Using the fact that equal rates are allocated across the layers, a lower bound on the outage probability can be derived by considering the case of single selected transmit antenna. According to our approach, this single selected transmit antenna, denoted by \( \nu \), is determined by

\[
\nu = \arg \max_{1 \leq i \leq M} \{ r_{i,i}^2 \}.
\]

The outage probability of the \((K = 1, N)\) system can then be written as

\[
\mathcal{P}_{\text{outage},K=1} = \Pr \left\{ \log_2 \left( 1 + r_{\nu,\nu}^2 \frac{\zeta}{R/M} \right) < R/M \right\} = \Pr \left\{ r_{\nu,\nu}^2 < \frac{(2R/M - 1)}{\zeta} \right\} = \mathcal{F}_{r_{\nu,\nu}} \left( \frac{(2R/M - 1)}{\zeta} \right),
\]

where \( \mathcal{F}_{r_{\nu,\nu}} (\cdot) \) is the cumulative distribution function (CDF) of the random variable \( r_{\nu,\nu}^2 \).

Thus the outage probability for the TAS scheme is lower bounded by

\[
\mathcal{P}_{\text{outage},\text{TAS}} \geq \mathcal{F}_{r_{\nu,\nu}} \left( \frac{(2R/M - 1)}{\zeta} \right).
\]

Using this fact, the CDF of \( r_{\nu,\nu}^2 \) can be written as

\[
\mathcal{F}_{r_{\nu,\nu}} \left( \frac{(2R/M - 1)}{\zeta} \right) = \mathcal{F}_{r_{1,1}} \left( \frac{(2R/M - 1)}{\zeta} \right) \times \ldots \times \mathcal{F}_{r_{M,M}} \left( \frac{(2R/M - 1)}{\zeta} \right).
\]

Now substituting (18) in (17), we get

\[
\mathcal{P}_{\text{outage, TAS}} \geq \mathcal{F}_{r_{1,1}} \left( \frac{(2R/M - 1)}{\zeta} \right) \times \ldots \times \mathcal{F}_{r_{M,M}} \left( \frac{(2R/M - 1)}{\zeta} \right).
\]

It is important to keep in mind the fact that the entries of \( \mathbf{R} \) are independent of each other. Moreover, with fixed \( \mathbf{H} \), the square of the \( i \)th diagonal element of \( \mathbf{R} \), \( r_{i,i}^2 \), is of central chi-square distribution with \( 2(N-i+1) \) degrees of freedom [20], [21], i.e., \( r_{i,i}^2 \sim \chi^2(N-i+1) \). Consequently, \( \mathcal{F}_{r_{i,i}} \left( \frac{(2R/M - 1)}{\zeta} \right) \) with \( 1 \leq i \leq M \), is the CDF of a central chi-square distribution \( \sim \chi^2(N-i+1) \). Using this fact, the CDF can be expressed as [22]

\[
\mathcal{F}(x,k) = P \left( \frac{k}{2} \frac{x}{\zeta} \right),
\]
where $P(k, x)$ denotes a normalized incomplete Gamma function (regularized Gamma function) defined as [22]

$$P(k, x) = \frac{1}{\Gamma(k)} \int_0^x e^{-t} t^{k-1} dt,$$

where $k (k \geq 0)$ denotes the degrees of freedom. Now substituting (20) in (19), we get

$$P_{\text{outage,TAS}} \geq \left[ P \left( N, \frac{(2^{R/M} - 1)}{2\zeta} \right) \right] \times \ldots \times \left[ P \left( N - M + 1, \frac{(2^{R/M} - 1)}{2\zeta} \right) \right].$$

The power series expansion of $P(k, x)$ is given by [22]

$$P(k, x) = x^k \gamma^*(k, x) = x^k e^{-x} \sum_{n=0}^{\infty} \frac{x^n}{\Gamma(k + n + 1)},$$

where $\gamma^*(k, x)$ is the incomplete Gamma function. In order to get $P_{\text{outage,TAS}}$ at high SNR, we substitute (22) in (21). Now, using the fact that $\Gamma(z) = (z-1)\Gamma(z)$, where $z$ is a positive integer, $P_{\text{outage,TAS}}$ at high SNR can be written as

$$P_{\text{outage,TAS}} \geq \left( \frac{2^{R/M} - 1}{2\zeta} \right)^{M} \times \left( \frac{1}{\prod_{i=1}^{M} (N - i + 1)!} \right) \times \left( \frac{2^{R/M} - 1}{2\zeta} \right)^{M N - \frac{1}{2}(M^2 - M)} \times \zeta^{-(M N - \frac{1}{2}(M^2 - M))}.$$  

The expressions in (16) and (23) suggest that the diversity order of the outage probability when the best transmit antenna is selected, according to (15), is $M N - \frac{1}{2}(M^2 - M)$. It is worth pointing out that the diversity order of the outage probability for the TAS scheme is upper bounded by

$$D_{\text{TAS}} \leq M N - \frac{1}{2}(M^2 - M),$$

where equality (i.e., $D_{\text{TAS}} = M N - \frac{1}{2}(M^2 - M)$) holds only for the $(K = 1, N)$ system employing DFD and performing the proposed TAS. Whereas the diversity gain of the $(M = 1, N)$ system employing DFD and without performing TAS is only $N$.

IV. SIMULATION RESULTS

In this section, we present both analytical and simulation results for the proposed TAS scheme in independent Rayleigh flat fading channels. In the following, a system with $M$ transmit and $N$ receive antennas out of which $K$ transmit antennas are chosen, is referred to as an $(M, N; K)$ system.

In Fig. 2, we evaluate the performance of the proposed TAS approach. The performance is measured in terms of the bit-error rate (BER) for a frame of 100 symbols from quaternary phase-shift keying (QPSK) complex constellations averaged over 10,000 frames. As shown, Fig. 2 depicts the BER performance of the $(M = 4, N = 4; K = 2)$ system employing DFD and performing the proposed TAS scheme. As a benchmark, the performance of the same system performing optimal capacity-based TAS approach [6] is shown. Also, for reference, we plot along the performance of the $(M = 2, N = 2; K = 0)$ system employing ML detector without performing TAS. It is clear from the figure that the proposed TAS with $(M = 2, N = 4; K = 2)$ achieves a performance very close to optimal capacity-based TAS. Note that optimal capacity-based TAS involves an exhaustive search over all possible $(\binom{M}{K})$ subsets of transmit antennas, requiring around $(\binom{M}{K}) K^3$ complex additions/multiplications, which grows exponentially with $M$ for $K \approx M/2$ [23]. However, it can be easily shown that our proposed TAS has a $O(M^3)$ complexity. It can be noticed that both approaches outperform the $(M = 2, N = 2; K = 0)$ system employing ML detector. Note that here all systems have the same bandwidth efficiency.

In Fig. 3, we evaluate the performance of the proposed TAS scheme from the capacity point of view in a full $(M = 3, N = 3)$ MIMO system. We observe from the figure that the capacity of the proposed $(M = 3, N = 3; K = 2)$ system is close to optimal capacity-based TAS introduced in [6] but with much lower complexity.

Fig. 4 displays the outage probability for a $(M = 2, N = 2; K = 1)$ system performing the proposed TAS. For the same system, we plot along the closed-form expression given in (23). As a benchmark, we plot along the outage probability curves for the $(M = 2, N = 2; K = 0)$ and $(M = 1, N = 2; K = 0)$ systems, respectively. The
Fig. 3. Capacity v/s SNR for different antenna-selection schemes, $M = 3$, $N = 3$.

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