Performance of multi-relay coded cooperative diversity in asynchronous code-division multiple-access over fading channels

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Abstract: In this study, multi-relay decode-and-forward (DAF) cooperative networks employing convolutional coding are studied for asynchronous direct-sequence code-division multiple-access (DS-CDMA) systems over frequency-selective slow fading channels. The authors show that the full benefits of coded cooperative diversity cannot be achieved if no multi-user interference suppression is employed at the cooperative end. The authors consider two scenarios; perfect and imperfect inter-user channels. In that, the bit-error-rate performance of the cooperative system is investigated for an uplink transmission where a decorrelator detector is used at both the relay and base station receivers. Both simulation and analytical results are presented to demonstrate the diversity gains of the convolutionally coded cooperative network.

1 Introduction

Cooperative diversity systems [1] allow the receiver to see independent versions of the information which yields to spatial diversity and/or coding gain compared to non-cooperative systems. It has been shown that cooperative diversity can offer these gains by introducing both temporal and spatial correlation into the transmitted signals from different relays without increasing the total transmitted power. In [2, 3], a two-user cooperative scheme using orthogonal code-division multiple-access (CDMA) codes was developed. The cooperation among users can, in general, be categorised into two methods; amplify-and-forward (AAF) and decode-and-forward (DAF) [2]. The DAF method is based mainly on the fact that each relay (cooperative user) decodes the partner’s data followed by retransmission of the estimated data. On the other hand, the relay, using the AAF method, transmits an amplified version of the received partner’s signal (corrupted by noise).

In CDMA systems, the effect of multi-user interference arising from non-orthogonal spreading codes can be mitigated using multi-user detection. The optimal maximum-likelihood multi-user detector although provides a performance close to the single-user bound, it requires large computational complexity which is known to grow exponentially with the number of users. Therefore other suboptimal detectors such as the decorrelator and minimum mean-square-error (MMSE) detectors have been introduced to reduce this complexity. Employing non-orthogonal spreading codes, Venturino et al. [4] investigated the performance of cooperative networks based on synchronous direct-sequence (DS)CDMA. In their work, they considered both DAF and AAF in cases of perfectly and partially known channel conditions. In [5], a modified MMSE detector was introduced for CDMA cooperative networks in synchronous flat-fading channels. Furthermore, the impact of asynchronous communication on cooperative networks in frequency non-selective fading channels was studied in [6]. Along the same lines, the authors in Vardhe et al. [7] studied the impact of non-orthogonality and asynchronous communications in flat fading channels for a CDMA system that employs DAF cooperation. Considering single relay, the performance of uncoded a cooperative network has been analysed over asynchronous CDMA systems in [8].

For a synchronous cooperative CDMA network, Huang et al. [9] studied the multi-user cooperation technique where each relay can cooperate with multiple active users over flat fading channels. The opportunistic relaying and selection cooperation are two relay selection methods that have been studied in [10], where closed-form expressions for the outage probability and probability of error have been provided for the uncoded single-relay cooperation case. Other cooperative techniques including coded cooperation, where cooperative diversity integrates with channel coding, have been considered in [11, 12]. A scenario where multiple transmitting sources and destinations cooperate simultaneously using the MMSE method through a number of relays was proposed in [13]. More recently, the performance of multi-relay cooperative networks was studied using AAF in flat fading channels with orthogonal frequency division multiplexing [14].

Coded cooperation was introduced in [11], where the codewords for the active transmitting users are split and sent over a number of independent fading channels. In [15],
serial concatenated convolutional codes have been introduced to a half duplex time-division multiple-access single-link cooperative scheme where the performance of the system was studied over flat fading channels for both AAF and DAF.

It is noted that in previous works related to CDMA systems, the performance of the cooperative network was either accompanied with the assumption of orthogonal subchannels (e.g. [2, 3, 16]) or considering synchronous communications (e.g. [4, 9]) all for the single-relay case. The impact of codes’ non-orthogonality was studied in [7] for CDMA cooperative network but over flat-fading channels with synchronous transmission. Since we consider an uplink channel (user to base station), the assumption of synchronous transmission is no longer valid. Therefore in this paper we consider asynchronous transmission between users and the base station. Also since CDMA is based on wideband transmission, the channel is modelled as being frequency-selective. Given this model, we study the impact of inter-user channel reliability on the performance of asynchronous DS-CDMA multi-relay-coded cooperative networks over frequency-selective fading channels. Our results show that the performance of the cooperative network is greatly affected not only by the quality of inter-user channels but also by the level of multi-user interference. To this end, we analyse the performance of a system that employs convolutional coding at both the source and the cooperative relay. In that, we consider two scenarios: (i) perfect inter-user channels (source-relays) where the relay (also referred as partner) correctly decodes the received user signal. This case is important since it serves as the optimal performance achieved using cooperation, where the distributed multiple-input multiple-output system reaches the performance of the centralised one and (ii) imperfect inter-user channels where we assume that each relay has an error detection capability (e.g. cyclic redundancy check) to decide whether cooperation can take place or not. In case of error, the cooperating user keeps silent (no cooperation), preventing the system from error propagation. The underlying receiver employs RAKE combining after interference suppression. The RAKE receiver exploits the path diversity inherent in multipath propagation where the decorrelator detector is used at both the relay side and the base station to mitigate the effect of multiple-access interference (MAI) and the known near–far problem.

The rest of this paper is organised as follows. The following section describes the asynchronous DS-CDMA system model used in this paper. In Section 3, the system performance for a DAF scheme is examined for the case of multi-user system where we obtain the probability density function (pdf) of the signal-to-noise ratio (SNR) at the receiver output. Then we derive the pairwise probability of error, which will be used to obtain an upper bound on the bit-error rate (BER) performance. The performance of DAF with imperfect inter-user channels is also analysed in this section. In Section 4, both analytical and simulation results are compared and discussed. Finally, conclusions are given in Section 5.

2 System model

Consider an uplink transmission for an asynchronous K-user DS-CDMA system. Each active transmitting user in the network can cooperate with V relays. The system employs a single antenna at both sides of the link. All users and relays are embedded with convolutional encoders of rate $R_e = n_c / n_e$ and a Viterbi decoder. In what follows, to simplify the notation, we refer to the base station with subscript b and the vth cooperating relay with subscript v.

The transmission scheme considered is described in Figs. 1 and 2, for a V-relay system. As shown, during the first transmission phase each active transmitting user sends its own data to the base station and its V partners (Fig. 1), whereas during the second transmission phase (relaying phase), each cooperating relay transmits the decoded version of its partner’s data to the base station over orthogonal time slots (Fig. 2). The relaying phase is done in V time slots where each cooperating relay will start transmitting at a different time slot. Hence, the data transmission rate of the system will decline as the number of cooperating relays per transmitting user increases.

Consider a multipath channel with P resolvable paths during each transmission period. For the DAF, the low-pass equivalent of the received signal at the base station and relay (v) during the first transmission phase can be expressed, respectively, as

$$r_{b1}(t) = \sum_{n=0}^{L-1} \sum_{k=1}^{K} \sum_{p=1}^{P} \sqrt{R_e U_k S_k(m)}$$

$$\times C_k(t - \tau_k - \tau_{kp} - mT_b) h_{eb}(m) + n_{b1}(t)$$

(1)

$$r_{v1}(t) = \sum_{n=0}^{L-1} \sum_{k=1}^{K} \sum_{p=1}^{P} \sqrt{R_e U_k S_k(m)}$$

$$\times C_k(t - \tau_k - \tau_{kp} - mT_b) h_{e1}(m) + n_{v1}(t)$$

(2)

Fig. 1 First transmission phase

Fig. 2 Second transmission phase (relaying phase)
\[
R_v(t) = \sum_{m=0}^{L-1} \sum_{k=0}^{K-1} \sum_{p=1}^{P} \sqrt{R_v E_{U_k}} S_k(m) \times C_v(t - \tau_k - \tau_{kp} - \frac{mT_b}{T}) h_{vb}^m(m) + n_v(t) \tag{2}
\]

where \( L \) is the size of the transmitted frame, \( S_k(m) \) is the \( m \)th transmitted bit of user \( k \), \( E_{U_k} \) and \( E_{L_k} \) are the received signal energies per path of user \( k \) at the base station (uplink channel) and the relay \( v \) (inter-user channel) respectively, \( C_v(t) \) is the spreading code assigned to the \( k \)th user with processing gain \((T_b/T_v)\), where \( T_b \) is the bit period, \( T_v \) is the chip period and \( \tau_v \) is the random transmit delay of the \( k \)th user which is assumed uniformly distributed along the symbol period. The parameter \( \tau_{kp} \) represents the delay of the \( p \)th path of user \( k \) during one transmission period. The channel coefficient \( h_{vb}^m \) models the fading of the inter-user channel between user \( k \) and \( v \) over the \( p \)th path, whereas \( h_{vb}^m \) represents the fading coefficient for the uplink channel between user \( k \) and the base station \( b \) over the path \( p \). These fading coefficients are modelled as independent Gaussian random variables with zero mean and unit variance. The noise \( n_v(t) \) and \( n_v(t) \) are complex Gaussian, each with zero mean and variance \( \sigma_v^2 = N_v/2 \).

During the second transmission phase (relaying phase), \( V \) set of relays \( \{f_1(k) \cdots f_V(k)\} \) cooperate with user \( k \) in which each cooperating relay retransmits the received signal. Each cooperating relay first decodes the partner’s received signal, then using error checking techniques it decides whether or not to forward the estimated partner’s data to the base station in the relaying phase. The relaying phase is done in \( V \) time slots where each cooperating relay will start transmitting at different time slots. In the proposed system model, each cooperative user can cooperate with only a single user at a time and instead of being idle (empty time slot), users can cooperate interchangeably. The low-pass equivalent of the received signal at the base station during the relaying phase can then be expressed as

\[
Y_{vb}(m)b_1 = \sum_{m=0}^{L-1} \sum_{k=0}^{K-1} \sum_{p=1}^{P} \sqrt{R_v E_{U_k}} R_{vb}^{(p,s)}(m, n) h_{vb}^m(n) S_k(n) + h_{vb}^m(m) \tag{4}
\]

where \( R_{vb}^{(p,s)}(m, n) \) is the cross-correlation between bit \( m \) of user \( k \) transmitted over path \( p \) and \( n \) bit of user \( w \) transmitted over path \( s \) and it is equivalent to

\[
R_{vb}^{(p,s)}(m, n) = \int C_v(t - \tau_k - \tau_{kp} - \frac{nT_b}{T}) x_c(t) dt \tag{5}
\]

The output of the bank of matched filters at both the base station and relay \( v \) can be expressed in vector format as

\[
Y_{hv} = R_{hv} H_{hv} X_{hv} + N_{hv} \tag{6}
\]

where \( X_{hv} \) is the \((KLP \times 1)\) data vector of the \( K \) users,
expressed for the first transmission phase as
\[X_b = \left[\sqrt{R_c E_{U_1} S_1(1)} \cdots \sqrt{R_c E_{U_1} S_1(L)} \cdots \sqrt{R_c E_{U_K} S_K(1)} \cdots \sqrt{R_c E_{U_K} S_K(L)}\right]^T\]

T represents transpose, \(N_b\) is \((KLP \times 1)\) noise vector with Gaussian elements each of zero mean and variance \(N_o/2\), \(H_b\) is \((KLP \times KLP)\) uplink channel matrix defined as
\[H_b = \begin{bmatrix}
h_{b,1}(1) & 0 & \cdots & 0 \\
0 & \cdots & \cdots & 0 \\
\vdots & \cdots & \cdots & \vdots \\
0 & \cdots & \cdots & h_{b,K}(L) \\
\end{bmatrix} \quad (7)
\]

assuming the channels to be fixed over the whole frame \(L\), \(h_{b,1}^p(1) = \cdots = h_{b,K}(L)\). Similarly, at the relay side
\[X_r = \left[\sqrt{R_c E_{U_1} S_1(1)} \cdots \sqrt{R_c E_{U_1} S_1(L)} \cdots \sqrt{R_c E_{U_K} S_K(1)} \cdots \sqrt{R_c E_{U_K} S_K(L)}\right]^T\]

\(N_r\) is \(((K-1)LP \times 1)\) Gaussian noise vector and \(H_r\) is \(((K-1)LP \times (K-1)LP)\) inter-user channel matrix. The matrices \(R_b\) and \(R_{b,r}\) are the \((KLP \times KLP)\) base stations and \(((K-1)LP \times (K-1)LP)\) relay cross-correlation, given respectively by
\[R_{b,r} = \begin{bmatrix}
R_{b,1,1}^{(1,1)}(1,1) & \cdots & R_{b,1,1}^{(1,1)}(1,L) & \cdots & R_{b,1,K}^{(1,1)}(1,L) \\
\vdots & \cdots & \vdots & \cdots & \vdots \\
R_{b,1,1}^{(1,L)}(1,1) & \cdots & R_{b,1,1}^{(1,L)}(1,L) & \cdots & R_{b,1,K}^{(1,L)}(1,L) \\
\vdots & \cdots & \vdots & \cdots & \vdots \\
R_{b,K,1}^{(1,1)}(1,1) & \cdots & R_{b,K,1}^{(1,1)}(1,L) & \cdots & R_{b,K,K}^{(1,1)}(1,L) \\
\end{bmatrix}
\quad (8)
\]

where \(R_{b,r}^{(m,n)}(m,n)\) is defined in (5). Note that the outputs of the matched filter bank in (6) suffer from MAI which can be eliminated using decorrelator detector at both relay and base station receivers. In this case, the output of the matched filter banks \(Y_b\), \(Y_r\), are applied to linear mappers as follows
\[Z_{b,r} = (R_{b,1})^{-1}Y_{b,r}\]

where \((R_{b,1})^{-1}\) and \((R_{b,r})^{-1}\) are the inverses of the cross-correlation matrices. The \((KLP \times 1)\) and \(((K-1)LP \times 1)\) vectors represent the \(Z_b\) and \(Z_r\), outputs of the decorrelator at the base station and relay \(r\), respectively, defined by
\[Z_{b,r} = H_{b,r}X_{b,r} + (R_{b,r})^{-1}N_{b,r}\quad (9)\]

### 3 BER analysis

Here an upper bound on the average BER at the base station decorrelator output using DAF cooperative method and considering both cases of perfect and imperfect inter-user channels is derived. For the sake of simplicity, we consider binary phase-shift-keying (BPSK) transmission.

#### 3.1 Perfect inter-user channels

We consider perfect inter-user channels between the transmitting user and relays. This assumption is too optimistic, however, it serves as a benchmark for optimum performance (i.e. lower bound). Later we will consider the more realistic case where relays are subject to decoding errors.

The decorrelator output in the first transmission phase at the base station can be expressed as
\[Z_1 = H_b[X_1]_1 + (R_b)^{-1}[N_1]_1\]

where \([X_1]_1\) represents the data vector transmitted during the first transmission phase
\[X_1 = \left[\sqrt{R_c E_{U_1} S_1(1)} \cdots \sqrt{R_c E_{U_1} S_1(L)} \cdots \sqrt{R_c E_{U_K} S_K(1)} \cdots \sqrt{R_c E_{U_K} S_K(L)}\right]^T\]

During the relaying phase, the decorrelator output at the base station for time slot \(v\) can be expressed as
\[Z_1 = H_b[X_1]_v + (R_b)^{-1}[N_1]_v\]

with \([X_1]_v\) being the data vector transmitted during the \(v\)th time slot of the relaying phase
\[X_1 = \left[\sqrt{R_c E_{U_1} S_1(1)} \cdots \sqrt{R_c E_{U_1} S_1(L)} \cdots \sqrt{R_c E_{U_K} S_K(1)} \cdots \sqrt{R_c E_{U_K} S_K(L)}\right]^T\]

where \(S_1(v)\) is the estimate of the \(v\)th bit of the active user cooperating with user \(k\) during the \(v\)th time slot of the relaying phase. The \(P\) elements of \([Z_1]_v\) correspond to the decision statistics of the \(m\)th bit for user one before RAKE combining, during the first transmission phase, are given by
\[Z_1 = \sqrt{R_c E_{U_1} h_{b,1}(m)S_1(m)} + [(R_b)^{-1}N_{b,1}]^1\]

\[\vdots\]

\[Z_1 = \sqrt{R_c E_{U_K} h_{b,K}(m)S_K(m)} + [(R_b)^{-1}N_{b,1}]^K\]

where \([Z_1]_v^m\) represents the \(m\)th element of \([Z_1]_v\) that corresponds to the decision statistics of the \(m\)th bit for user one before RAKE combining. \([(R_b)^{-1}N_{b,1}]^p\) is the \(p\)th entry of the vector, while the \(VP\) elements of \([Z_1]_v\), that represent the contribution of the relaying phase are given by
\[Z_1 = \sqrt{R_c E_{U_1} h_{b,1}(m)S_1(m)} + [(R_b)^{-1}N_{b,2,1}]^1\]

\[\vdots\]
\[ [Z_1(m)]_{b_2,V}^p = \sqrt{R_e \frac{E_{U_{1(i)}}}{V}} h_{f_{1(i)b}}(m) S_1(m) + [(R_b)^{-1} N_{b_2,V}]^p \] 

(11)

Under the assumption of perfect inter-user channels, \( S_1(m) = \hat{S}_1(m) \) and by combining the \( P(V+1) \) elements, the decision statistic of the desired user signal is

\[
\hat{S}_1(m) = h_{1b}^1(m)[Z_1(m)]_{b_1}^1 + \cdots + h_{1b}^p(m)[Z_1(m)]_{b_1}^p + h_{f_{1(i)b}}(m) \hat{Z}_1(m) \] 

(12)

Assuming all relays transmit with the same energy \( E_{U_1} = E_{U_{1(i)}} = \cdots = E_{U_{b_1(i)}} \), one can show that

\[
\hat{S}_1(m) = \left[ \sqrt{V} |h_{1b}^1(m)|^2 + \cdots + \sqrt{V} |h_{1b}^p(m)|^2 + |h_{f_{1(i)b}}(m)|^2 \right] \sqrt{\frac{E_{U_1}}{V}} S_1(m) \\
+ \Re \left[ h_{1b}^1(m)(R_b)^{-1} N_{b_1} \right] \\
+ \cdots + h_{1b}^p(m)(R_b)^{-1} N_{b_1} \] 

(13)

where \( (\cdot)^* \) denotes complex conjugate operation and \( \Re \) designates the real part operator.

The pairwise error probability can be defined as the number of errors between the received codeword \( \hat{S}_1(m) = [\hat{S}_1(m), \hat{S}_2(m), \ldots, \hat{S}_{b_1}(m)] \) and all-zero transmitted codeword. From (13), the pairwise error probability at the base station conditioned on the uplink channel coefficients can be expressed as (see (14))

\[
Q(x) = \int_{-\infty}^{\infty} \left( \frac{1}{\sqrt{2\pi}} \right) \exp \left( -\frac{x^2}{2} \right) dx 
\] 

(14)

where \( \phi(jw) \) is a function of the cross-correlation among users [17]. The joint pdf, \( f(A, B) \), is then given by

\[
f(A, B) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(jw, w_2) \exp(-jw_1A) \times \exp(-jw_2B) dw_1 dw_2 
\]

(15)

By solving the double integral and letting \( B = W \), we have

\[
f(\gamma, W) = \frac{\Gamma_{b_2}^p}{4\pi^2(2\sigma^2)^{p(V+1)}} \sum_{U=1}^{P(V+1)} \Psi_U W^{1/2} \\
\times \left( \frac{\gamma \sqrt{W} - W}{\kappa_U} \right)^{P(V+1)-2} \exp \left( -\frac{\gamma \sqrt{W}}{2\sigma^2} \right) 
\]

(16)

where \( \Psi_U = 4\pi^2 K_U/(P(V+1) - 1) \). Hence, one can determine the pdf of the SNR as

\[
f(\gamma) = \frac{\Gamma_{b_2}^p}{4\pi^2(2\sigma^2)^{p(V+1)}} \sum_{U=1}^{P(V+1)} \Psi_U R_U(\gamma) 
\]

(17)

Define the variable \( \gamma \) as

\[
\gamma = \frac{A}{\sqrt{B}} 
\]

(18)

where \( A = \sum_{p=1}^{p(V+1)} \sqrt{V} \alpha_{1b} + \sum_{v=1}^{V} \alpha_{f_{1(i)b}} \) and \( B = \sum_{p=1}^{p(V+1)} \alpha_{1b} + \sum_{v=1}^{V} \alpha_{f_{1(i)b}} \). By generalising (14), the pairwise conditional probability can be expressed as

\[
P_{b_1}(d) = Q \left( \frac{\sqrt{V} \alpha_{1b} \kappa_1 + \left( \sum_{v=1}^{V} \alpha_{f_{1(i)b}} \kappa_{f_{1(i)}} \right)}{\sqrt{\sigma_a^2}} \right) 
\]

(19)

The noise variance \( \sigma_a^2 \) in (15) can be written as

\[
\sigma_a^2 = \sigma_a^2 \sum_{p=1}^{p(V+1)} \alpha_{1b} \kappa_1 + \left( \sum_{v=1}^{V} \alpha_{f_{1(i)b}} \kappa_{f_{1(i)}} \right) 
\]

(20)
where

\[ R_U(\gamma) = \int_0^{2\gamma} W^{1/2} \left( \gamma W - \frac{W^{(V+1)-2}}{\kappa_U} \right) \exp \left( -\frac{\gamma W}{2\alpha^2} \right) dW \]  

(22)

Using the binomial series expansion, and by setting \( t = \sqrt{W} \) and expressing (22) in terms of confluent hypergeometric function [17], we obtain

\[ R_U(\gamma) = 2 \gamma^{P(V+1)+1} \kappa_U^{P(V+1)+1} \exp \left( -\frac{\kappa_U \gamma^2}{2\alpha^2} \right) \times \sum_{m=0}^{P(V+1)-2} \left( \frac{P(V+1)-2}{m} \right) \left( \frac{-1}{2P(V+1) - m - 1} \right) I_1 \]  

\[ \times \left( 1; 2P(V+1) - m - m; \frac{\kappa_U \gamma^2}{2\alpha^2} \right) \]  

(23)

The pdf of the SNR can be obtained by substituting (23) into (21) and hence, the pairwise probability of error can be expressed as

\[ P_{BE}(d) = \frac{1}{\pi} \int_0^{\infty} \exp \left( -\frac{R_c d \gamma^2}{2 \sin^2 \theta} \right) \gamma d\gamma d\theta \]  

\[ = \frac{1}{\pi} \sum_{U=1}^{P(V+1)+1} \Psi_U R_U(\gamma) d\gamma d\theta \]  

(24)

where \( \delta = E_{\gamma_U}/V \sigma_a^2 \)

\[ F_U(\delta) = \frac{1}{(2\alpha^2)^{P(V+1)+1}} \int_0^{\infty} \exp \left( -\frac{R_c d \gamma^2}{2 \sin^2 \theta} \right) R_U(\gamma) d\gamma d\theta \]  

\[ = 2 \kappa_U^{P(V+1)+1} \sum_{m=0}^{P(V+1)-2} \left( \frac{P(V+1)-2}{m} \right) \]  

\[ \times \left( \frac{-1}{2P(V+1) - 1 - m} \right) G_m(\delta) \]  

(25)

and

\[ G_m(\delta) = \frac{1}{(2\alpha^2)^{P(V+1)+1}} \int_0^{\infty} \gamma^{P(V+1)-1} \]  

\[ \times \exp \left[ -\gamma \left( \frac{R_c d \gamma^2}{2 \sin^2 \theta} + \frac{\kappa_U}{2\alpha^2} \right) \right] \]  

\[ \times I_1 \left( 1; 2P(V+1) - m; \frac{\kappa_U \gamma^2}{2\alpha^2} \right) d\gamma d\theta \]  

(26)

Using [18]

\[ \int_0^\infty \exp(-st) b^{-1} F_1(a; c; \kappa) d\tau \]  

\[ = \Gamma(b)(s - k)^{-b} F_1 \left( c - a, b; c; \frac{k}{k - s} \right) \]  

(27)

one can show that

\[ G_m(\delta) = \frac{\Gamma(1/2)\Gamma((2P(V+1)+1)/2)}{4P(V+1)(R_c d \delta) \Gamma(P(V+1)+1)} \]  

\[ \times \frac{1}{3} \int_0^{(2P(V+1)+1)/2} \]  

\[ \times \frac{2P(V+1)+1}{2P(V+1) - 1 - m, -\kappa_U}{R_c d \delta} \]  

(28)

where \( \delta = E[|h_{LU}^0(m)|^2]E_{\gamma_U}/V \sigma_a^2 \) is the normalised average SNR per relay.

Finally, using (28) and (25), the average pairwise probability for \( V \) cooperating relays in (24) can be evaluated as

\[ P_{BE}(d) = \frac{1}{\pi} \sum_{U=1}^{P(V+1)+1} 2 \Psi_U R_U(\gamma) d\gamma d\theta \]  

\[ = \frac{1}{\pi} \sum_{m=0}^{P(V+1)-2} \left( \frac{P(V+1)-2}{m} \right) \left( \frac{-1}{2P(V+1) - 1 - m} \right) \]  

\[ \times \left( \Gamma(1/2)\Gamma((2P(V+1)+1)/2) \right) \]  

\[ \times \frac{1}{4P(V+1)(R_c d \delta)} \]  

(29)

From (29), we can examine the asymptotic BER performance of the cooperative system as the SNR gets large. Noting that \( 3 \sqrt{F_1((2P(V+1)+1)/2, 2P(V+1) - 1 - m, -\kappa_U}{R_c d \delta} \rightarrow 1 \) as \( \delta \rightarrow \infty \), one can see that \( P_{BE} = 1/(c d^{P(V+1)+1}) \) where the achieved diversity order \( \text{div} = P(V+1) \).

By obtaining the average pairwise error probability, the upper bound on the probability of bit error can be expressed as [19]

\[ P_{BE} < \frac{1}{m_c \sum_{d \neq d_c} c(d)P_{BE}(d)} \]  

(30)

where \( d_c \) is the minimum free distance of the code, \( m_c \) is the number of information bits shifted to the encoder at the same time instant and \( c(d) \) is the sum of errors for error events of distance \( d \).

### 3.2 Imperfect inter-user channels

As mentioned earlier, the performance of the cooperative system with perfect inter-user channels is not realistic in some cases and can only serve as a lower bound indicating optimal performance. Here we consider the effect of errors in the inter-user channels where cooperation among users can only take place if the relay detection is error-free. If errors exist, the corresponding relay (cooperating user) stays idle during the second transmission phase. First we derive the pairwise error probability in the inter-user channel (source-relay channels), from which we can express the pairwise probability of error of the overall network.

Without loss of generality, consider the nth bit of user one as the desired bit, the decorrelator outputs at the cooperative
user side $f_s(1)$ is given by

$$[Z_1]_v = H_v[X_1]_v + (R_v)^{-1}[N_1]_v,$$

where the $P$ elements of $Z_1$ are

$$[Z_1(m)]_v = \sqrt{R_vE_{t_v}h_{f_{v(t)}}^1(m)S_{1}(m)} + [(R_v)^{-1}N_1]_v^1$$

$$\vdots$$

$$[Z_1(m)]_v = \sqrt{R_vE_{t_v}h_{f_{v(t)}}^1(m)S_{1}(m)} + [(R_v)^{-1}N_1]_v^P$$

At the output of the RAKE combiner, we have the user data decision statistic

$$\bar{S}_v(m) = \left[|h_{f_{v(t)}}^1(m)|^2 + \cdots + |h_{f_{v(t)}}^P(m)|^2\right]\sqrt{R_vE_{t_v}S_{1}(m)}$$

$$+ \text{Re}\{h_{f_{v(t)}}^1(m)(R_v)^{-1}N_1\} + \cdots +$$

$$+ h_{f_{v(t)}}^P(m)(R_v)^{-1}N_1)^P$$

(33)

and the pairwise probability of error over the inter-user channel

$$P_{be}(d) = Q\left(\frac{\sum_{p=1}^P \sqrt{dR_vE_{t_v}(h_{f_{v(t)}}^p(m))^2})}{\sqrt{\sigma_v^2}}\right)$$

(34)

where

$$\sigma_v^2 = \sigma_n^2 \sum_{p=1}^P |h_{f_{v(t)}}^p(m)|^2 (R_v)^{-2}$$

(35)

and $(R_v)^{-2}$ is the square of the sum of the $k$th row in the inverse of the cross-correlation matrix. Similar to the case of perfect inter-user channel, we define $d_{f_{v(t)}}^p = |h_{f_{v(t)}}^p(m)|^2$ with characteristic function given by (16). Also, letting $(R_v)^{-1} = \kappa_1$, the noise variance $\sigma_n^2$ can be expressed as

$$\sigma_n^2 = \sigma_n^2 \sum_{p=1}^P d_{f_{v(t)}}^p \kappa_1$$

(36)

Following the same analysis as in the perfect inter-user channel, we define the variable $\gamma_v$ as

$$\gamma_v = \frac{A_v}{\sqrt{B_v}}$$

(37)

where $A_v = \sum_{p=1}^P d_{f_{v(t)}}^p$ and $B_v = \sum_{p=1}^P d_{f_{v(t)}}^p \kappa_1$. Then, it is easy to show that

$$\phi_{d_{f_{v(t)}}}(w_1, w_2) = \frac{\Gamma(1/\kappa_U)}{(2\pi\kappa_U)^{P+1}} \left(\sum_{p=1}^P \frac{\kappa_U}{(w_2 - y_j/\kappa_U)}\right)$$

(38)

correlation among users. The pdf, $f(A_v, B_v)$, is then given by

$$f(A_v, B_v) = \frac{\Gamma(1/\kappa_U)}{4\pi^2(2\pi)^P} \sum_{p=1}^P \Psi_{U_v} \left(\frac{B_v - \frac{B_v}{\kappa_U}}{\kappa_U} \right)^P \exp\left(-\frac{A_v}{2\pi}\right)$$

with $\Psi_{U_v} = 4\pi^2 K_{U_v} / \Gamma(P)$ and

$$f(\gamma_v) = \frac{\Gamma(1/\kappa_U)}{4\pi^2(2\pi)^P} \sum_{U_v=1}^P \Psi_{U_v} R_{U_v}(\gamma_v)$$

(39)

where

$$R_{U_v}(\gamma_v) = 2\gamma_v^{2P-1} \sum_{m=0}^{P-2} \left(-\frac{\gamma_v}{2\pi}\right)^m \frac{\Gamma(1/\kappa_U)}{\Gamma(2P-1)} \left(\frac{\Gamma(1/\kappa_U)}{\Gamma(2P-1)}\right)^P$$

(40)

The pairwise probability of error at the relay can then be obtained using (34) and (39)

$$P_{be}(d) = \frac{1}{\pi} \int_{\gamma_v=0}^{\gamma_v=\infty} \exp\left(-\frac{R_v d_{\delta_v} \gamma_v^2}{2\pi^2}\right) f(\gamma_v) d\gamma_v d\theta$$

(41)

with

$$F_{U_v}(\delta_v) = 2\kappa_U^{P-1} \sum_{m=0}^{P-2} \left(\frac{P-2-m}{2P-m-1}\right) G_{U_v}(\delta_v)$$

(42)

$$G_{U_v}(\delta_v) = \frac{\Gamma(P)}{2(R_v d_{\delta_v})^P} \int_{\theta=0}^{\theta=\pi/2} \sin^2(\theta)^P \left(2P-m-1, P; 2P-m, -\frac{\kappa_U \sin^2(\theta)}{R_v d_{\delta_v}}\right) d\theta$$

(43)

Given the pairwise error probability at the relay, one can find the average pairwise probability at the base station for the

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imperfect inter-user channel case as

\[ P_b(d) = \frac{(1 - P_{b_E}(d))^{V} (P_{b_P}(d))^{V}}{1} \]

\[ + \sum_{n=1}^{V-1} \binom{V}{n} (P_{b_E}(d))^{n} (1 - P_{b_E}(d))^{V-n} (P_{b_P}(d))^{V-n} \]  

\[ + (P_{b_E}(d))^{V} (P_{b_P}(d))_{0} \]  

(44)

In (44), the first term represents the system pairwise probability in case of error-free inter-user channels (all \( V \) relays are cooperating) with \( P_{b_E}(d) \) representing the pairwise probability per inter-user channel given by (43), and \( (P_{b_P}(d))_{V} \) is the overall pairwise probability with \( V \) cooperating relays in (29). The second term expresses the combination of erroneous and error-free inter-user channels with \( (P_{b_P}(d))_{V-n} \) as the overall pairwise probability with \( (V - n) \) relays, whereas the third term is the pairwise probability in case of all inter-user channels are in error, with \( (P_{b_E}(d))_{0} \) as the pairwise probability for the case of no cooperation.

**Fig. 4** BER performance for an eight-user asynchronous uncoded DS-CDMA system over two-path frequency-selective slow fading channels (one relay)

**Fig. 5** BER performance for an eight-user asynchronous DS-CDMA network over two-path frequency-selective slow fading channel (one relay)
4 Simulation results

Here we present simulation results to assess the performance of a multi-relay coded cooperative system when considering different scenarios. Also, we examine the accuracy of our analytical results obtained in Section 3. We consider the cooperative diversity spreading scheme described before with one transmit and one receive antennas. A multi-user asynchronous DS-CDMA system with BPSK transmission is considered where every user data are spread using non-orthogonal gold codes of length 31 chips. Since the focus of our work is not on the relay selection problem, to simplify our simulations, relays are selected at random where any number of users (relays) can cooperate with each other. Convolutional coding of rate 1/2 with constraint length \([7, 5]\) is used at both active user and relay sides. For the asynchronous channel, the transmitted frames are 100 bits each, the fading coefficients are fixed for a number of frames and the delay between users, \(\tau_k\), is uniformly distributed along the symbol period. We also assume perfect knowledge of the channel coefficients at the base station and the relay. In order to maintain a constant data

![Fig. 6 BER performance for an eight-user asynchronous DS-CDMA with DAF cooperation as a function of the number of paths over frequency-selective slow fading channels for perfect inter-user channels (two relays)](image1)

![Fig. 7 BER performance for an eight-user asynchronous DS-CDMA with DAF cooperation as a function of the number of paths over frequency-selective slow fading channels for imperfect inter-user channels (two relays)](image2)
rate for all cases, the transmitting users in the non-cooperation case will transmit each data bit $V$ times, so that the data rate is the same as the cooperating case [4].

Before proceeding, first we examine the effect of multi-user interference at the cooperative user. In Fig. 4, the performance of an uncoded single-relay cooperative system is examined over a frequency-selective slow fading channel. In these results, we consider the case of error propagation because of decoding errors at the relay. The performance of the system with conventional matched filter receiver at the relay is compared with a system that employs a decorrelator multi-user detector. As shown when the decorrelator receiver is used at the relay, the system is immune to multiple-access interference whereas the conventional receiver becomes almost ineffective resulting in an error floor because of the high levels of interference. As a result, the performance of the cooperative system can in some cases be worse than the non-cooperative system. It is clear that the system can achieve the full diversity gain when considering perfect inter-user channel. The results also suggest that, in the presence of MAI at the relay, the overall system performance degrades significantly and no benefit for user cooperation can take place.

By fixing the SNR at the inter-user channels, Fig. 5 compares the performance of the coded cooperative network for eight users over two-path frequency-selective fading channel. With the overall system performance for the non-cooperative case acting as an upper bound and the perfect inter-user channels as lower bound, it can be concluded that as the quality of the inter-user channel improves, the system achieves large diversity gains. It is shown that the system can achieve the full diversity gain when considering perfect inter-user channels.

For the multi-relay case shown in Figs. 6 and 7, the overall diversity achieved is $(V + 1)^P$ when the inter-user channels are error-free. That is, the overall diversity order of the system is a function of the number of cooperating relays $V$ and the number of resolvable paths $P$. In all the above results, the accuracy of our analytical results is clear.

5 Conclusions

We examined the performance of a multi-relay coded cooperative diversity using DAF method in asynchronous DS-CDMA systems over frequency-selective slow fading channels. In order to mitigate the multi-access interference, decorrelator multi-user detectors have been employed at both the relays and the base station. The performance of the system has been studied where an exact expression for the pairwise probability of error is obtained for the case of imperfect and perfect inter-user channels.

The impact of erroneous inter-user channels on the overall system performance was also studied, where we have shown that as the quality of the channel gets poorer, the system starts losing diversity. The effect of multi-user interference on the system performance was examined and it was shown that the benefits of cooperative diversity cannot be achieved without interference suppression at the relay side. We have compared the performance of the system for different number of cooperating relays. A trade-off between the system performance and transmission rate showed that as the number of relays increases, the system gains higher diversity order, but on the other side the transmission rate starts to decline.

6 References