Performance Analysis and Optimization of Multi-Selective Scheme for Cooperative Sensing in Fading Channels

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Abstract—We propose a multi-selective sensing scheme where the primary user activity is detected in cognitive radio through cooperation among different sensing nodes and the fusion center. The proposed cooperative sensing scheme is based on order statistics of the reporting links between the cooperative nodes and fusion center where the links with high signal-to-noise ratios (SNRs) are selected as reliable reporting links. The performance of the proposed scheme is compared to other existing schemes in terms of the probability of detection and probability of false alarm over independent identical distributed (i.i.d.) and independent non-identical distributed (i.n.d.) Rayleigh fading channels. Both simulations and analytical results show that the proposed scheme outperforms conventional sensing schemes under different system parameters. Furthermore, we examine the optimum N-out-of-K rule of our scheme under different detection threshold and SNR. Our results show that the proposed multi-selective scheme offers improvement in terms of the probability of detection when compared with other existing schemes such as selection combining (SC), square-law selection (SLS) and general N-out-of-K rule.

Index Terms—Cooperative spectrum sensing, fusion center, selection combining.

I. INTRODUCTION

Cognitive radio technology provides secondary user (SU) with temporary access to under-utilized licensed bands originally assigned to primary user (PU) (licensed user). Recently many works have been proposed to leverage the large potential gain of cognitive radios in wireless networks (e.g., [1]-[5]). There has been One key aspect to enable spectrum sharing is based on spectrum sensing strategies where SU detects the PU activity through spectrum sensing and monitoring techniques [6]-[8]. Once unoccupied, the SU utilizes the spectrum hole as long as the PU is absent. Thus spectrum sensing techniques are proposed to detect unoccupied licensed bands and to avoid interference to the PU. Energy detection, which is based on the received signal power [9], is preferred due to its low implementation complexity and simple structure compared with other current spectrum sensing techniques, e.g., matched filter detection based on coherent detection through maximization of the signal-to-noise ratio (SNR) [2],[10], the cyclostationary feature detection realized by exploitation of the inherent periodicity of PU signal [2],[11]. However, energy detection is not reliable in scenarios where multipath fading and shadowing exist. To overcome these problems, cooperative energy detection schemes are proposed to improve the detection probability through SUs’ cooperation [12], [13]. In a cognitive radio network, cognitive users/nodes report to a fusion center their observations on PU activity. In such a network setting, two techniques are used; i) decision fusion, where cognitive users observe the PU activity and send their hard decisions (1 bit) to the fusion center which in turn employs an N-out-of-K rule to decide on the PU activity [14], ii) data fusion, where cognitive users send their soft observations to the fusion center for combining followed by PU detection. In this case, combining techniques such as, square-law combining (SLC) [9], square-law selection (SLS) [9], selection combining (SC) [15] and maximal ratio combining (MRC) [16] can be used at the fusion center.

With the advantages offered by cooperative/relay communications in providing reliable information and high throughput in wireless environments, many works have investigated the performance of such networks over different system configurations (e.g., [17]-[19] and references therein). Considering cooperative spectrum sensing, square-law selection and square-law combining are known to offer high detection performance while requiring low bandwidth usage among the different data fusion schemes [20]. Although the N-out-of-K rule is more bandwidth efficient, its performance is not superior to the performance of square-law combining [14], [21]. In the literature, researchers have focused on improvements of square-law combining and MRC and only few works have considered the performance of selection combining. Also most of the existing works have only considered a homogenous case where the channels are modeled as i.i.d. and few works considered heterogeneous case where the channels are modeled as i.n.d. (e.g., [22] and [23]). For instance, the performance of spectrum sensing over i.i.d. fading channels based on detection threshold has been studied in [4], [24]-[26]. In more realistic environments, the signals from the different relays travel through different propagation paths to the destination, which results in non-identical fading statistics and signal strengths (i.e., i.n.d.). Thus, [22] and [23] considered an i.n.d. fading channel model for cooperative spectrum sensing by involving local results from all relay links, regardless of the reliability of these links. The general optimization of the N-out-of-K rule in terms of...
general error rate is also investigated in [13].

In this paper, and different from previous works, we propose a selection combining scheme named multi-selective cooperation which incorporates the low bandwidth advantage of the $N$-out-of-$K$ rule in addition to superior performance compared with selection combining. Our multi-selective scheme considers the condition of all relay links where the concept of order statistics is employed to select $K$ links with the highest SNR for cooperative spectrum sensing over i.i.d. and i.n.d. Rayleigh fading channels. The probabilities of detection and false alarm of the proposed scheme are derived for both channel models. Our results show significant performance improvement relative to general $N$-out-of-$K$ scheme, square-law selection and selection combining schemes. Furthermore, the performance of our scheme is investigated through optimization of the $N$-out-of-$K$ rule in terms of general error rate when considering different system parameters. We also assume that the PU activity is modeled using the ON/OFF activity model (i.e., ON and OFF represent respectively the active and absence states of the PU). This model is commonly employed as it represents many realistic PU scenarios [27]. In addition, the ON/OFF model is commonly used by other models such as the two-state Markov model where the BUSY/IDLE states correspond to the ON/OFF states. Therefore, we will further analyze the performance of the proposed scheme under different ON/OFF model parameters and their effect on the total error rate.

The paper is organized as follows. In Section II, we first introduce the energy detection model in each relay over Rayleigh fading channels. Then, the multi-selective scheme is introduced and the probabilities of detection and false alarm are derived over i.i.d. and i.n.d. channels. Section III compares the performance of the multi-selective scheme with other current schemes. In section IV, the optimization of the $N$-out-of-$K$ rule used in the proposed scheme is introduced and examined under different system parameters. Also we consider the effect of the ON/OFF model parameters on the performance of the proposed scheme. Finally, conclusions are drawn in Section V.

II. Multi-Selective Cooperative Sensing System Model

We consider a cooperative cognitive network where at any time a SU can be a source, relay, or destination. The number of links including the P-R (primary to relay) and P-D (primary to destination) is denoted by $L$. Also to simplify the analysis, we first consider the R-D (relay to destination) links are error-free. Then we relax this assumption by modeling the R-D links as AWGN channels. This assumption can be justified in scenarios where the destination and relays are in the vicinity of each other forming a cluster. In the sequel, without loss of generality, in some instances the fusion center is referred to as the destination where the cognitive network is of ad-hoc structure. On the other hand, in a centralized cognitive network the base station can be considered as the fusion center with some powerful processing. In either network settings, our proposed scheme applies equally with no loss in performance.

Before adopting multi-selective cooperative scheme in relays, let us first introduce the operation of energy detection in each relay. In each relay, detection problem of unknown signal is a binary hypothesis test [28]. PU signal $s(t)$ is transmitted via fading channel with channel gain $h$. The signal $\beta(t)$ received at the detector follows a binary hypothesis: $H_0$ and $H_1$ representing the absence and presence of PU activity respectively, expressed as

$$\beta(t) = \begin{cases} a(t), & H_0, \\ hs(t) + a(t), & H_1. \end{cases}$$

where $a(t)$ is the zero-mean additive white Gaussian noise (AWGN) with variance $\sigma^2$.

Over Rayleigh fading channel, the probability of detection is expressed according to [9], as

$$P_d = Q_q\left(\sqrt{2\gamma}, \sqrt{\lambda}\right)$$

where $Q_q(\cdot)$ is the gamma function, $\Gamma(q, \frac{x}{q})$ is the upper incomplete gamma function given by $\Gamma(q, \frac{x}{q}) = \int_0^\infty e^{-x/t - t} dt$ with $Q_q(\cdot)$ representing the $q$th order generalized Marcum-Q function [30], $\lambda$ is the energy threshold of the detector, $q = TW$ is the time bandwidth product, and $\gamma$ represents the SNR defined as $\gamma = \frac{\left|h_0\right|^2E_s}{N_{01}}$, where $E_s$ is the energy of the PU signal.

Considering a block-fading channel, the SNR remains constant for the duration of one observation time (i.e., block) and varies independently from one block to another. Given this, the destination receives local SNR information on the P-R and local decision on PU activity from each relay link. The destination then selects $K$ links with the highest SNRs as final report links. The destination makes a final decision on the state of PU based on these reporting links’ local decisions through an $N$-out-of-$K$ rule. In this case, at least $N$ report links out of $K$ selected ones are considered for final decision at the destination. For instance, the OR rule corresponds to the case of $N = 1$; AND rule corresponds to the case of $N = K$; and Majority rule corresponds to the case of $N = \lceil N/2 \rceil$ [14]. It is worth mentioning that the difference between the proposed scheme and the general $N$-out-of-$K$ scheme is that our scheme considers only $K$ links with the highest SNRs out of the $L$ available links while the general $N$-out-of-$K$ scheme considers all $L$ links (i.e., $K = L$) in forming the final decision about the PU activity.
To assess the performance of our detection scheme, in what follows, we derive the receiver operating characteristics (ROC) and analyze the overall probabilities of false alarm and detection.

**Probability of False Alarm over both i.i.d. and i.n.d. Rayleigh Fading Channels:** Under the multi-selective cooperation scheme, the destination selects $K$ links with the highest SNRs for final decision. However, in the case of $H_0$ in (1), the detector receives noise alone. Therefore, under $H_0$, the destination picks $K$ links randomly irrespective of the SNRs. The overall probability of false alarm incorporating the N-out-of-K rule over both i.i.d. and i.n.d. Rayleigh fading channels is given by

$$Q_f = \sum_{j=0}^{K} \binom{K}{j} P_f^j (1 - P_f)^{K-j}, \quad (5)$$

where $P_f$ represents the local probability of false alarm of each link over i.i.d. Rayleigh fading channels and $P_f = P_{f,i}$ ($i = 1, 2, L$). Here, $P_{f,i}$ represents the local probability of false alarm of the $i$th link over i.n.d. Rayleigh fading channels.

**A. Probability of Detection over i.i.d. Rayleigh Fading Channels**

Over i.i.d. Rayleigh fading channels, the probability density function (PDF) and cumulative distribution function (CDF) of the SNR $\gamma$, are given by [31, Table 9.5]

$$f(\gamma) = \frac{1}{\gamma} e^{-\gamma/\tau} \quad \text{(6)}$$

$$F(\gamma) = 1 - e^{-\gamma/\tau}. \quad \text{(7)}$$

The destination first orders the R-D and P-D links in terms of their SNRs to form the set $U = \{\gamma^{(1)}, \gamma^{(2)}, \ldots, \gamma^{(L)}\}$, considered strictly decreasing set. The destination then selects the $K$ ($1 \leq K \leq L$) links corresponding to the $K$ highest SNRs as reporting links from set $U$ to form the sub-set $S_K = \{\gamma^{(1)}, \gamma^{(2)}, \ldots, \gamma^{(K)}\}$ as the one that represents these report links. In what follows, the PDF of the $\gamma^{(k)}$ ($1 \leq k \leq K$) in the set $U$ and the joint PDF of the elements in the set $S_K$ are first evaluated and then used to derive the probability of detection.

Since the multi-selective scheme selects $K$ links from the set $U$ as report links, we need to derive the PDF of $\gamma^{(k)}$ ($1 \leq k \leq K$) in the set $U$. As the SNRs of the $L$ links are i.i.d., the probability of any link being in the $\gamma^{(k)}$ in the set $U$ is equal to $(1/L)$ and the corresponding PDF of the SNR is given by

$$f^{(k)}(\gamma^{(k)}) = \binom{L-1}{k-1} F^{L-k}(\gamma^{(k)}) [1 - F(\gamma^{(k)})]^{k-1} f(\gamma^{(k)}) \quad \text{(8)}$$

where $F(\gamma^{(k)})$ and $f(\gamma^{(k)})$ are given respectively by (6) and (7) with $F(\gamma^{(k)})$ representing the CDF of any link’s SNR smaller than $\gamma^{(k)}$. Substituting (6) and (7) in (8), this PDF can be rewritten as

$$f^{(k)}(\gamma^{(k)}) = \frac{L!}{(k-1)!(L-k)!} \left(1 - e^{-\gamma^{(k)}/\tau}\right)^{L-k} \left(e^{-\gamma^{(k)}/\tau}\right)^{k}$$

$$= \frac{L!}{(k-1)!(L-k)!} \sum_{i=0}^{L-k} \frac{(-1)^i}{i!} \binom{L-k}{i} e^{-\frac{\gamma^{(k)}}{\tau}}$$

(9)

Since the multi-selective scheme adopts the N-out-of-K rule, we derive the joint PDF of any $N$ elements, regarded as $z_1, z_2, \ldots, z_N$, in the set $S_K$. Accordingly to (8), this joint PDF, represented by $\{\gamma^{(z_1)}, \gamma^{(z_2)}, \ldots, \gamma^{(z_N)}\}$ ($1 \leq z_1 < z_2, \ldots, < z_N \leq K$; $I < N \leq K$) for $\gamma^{(z_1)} > \ldots > \gamma^{(z_N)}$, is given as [32, Section 2.2]

$$f^{(z_1), \ldots, (z_N)}(\gamma^{(z_1)}, \ldots, \gamma^{(z_N)}) = \frac{L!}{(z_1 - 1)!(z_2 - z_1 - 1)! \cdots (L - z_N)!} \left[1 - F(\gamma^{(z_1)})\right]^{z_1-1} \times F(\gamma^{(z_1)}) \times F^{z_2-z_1-1}(\gamma^{(z_2)}) \times F^{z_3-z_2}(\gamma^{(z_3)}) \cdots (10)$$

Considering the probability of detection of $\gamma^{(k)}$, from (3) and (9), and by making the change of variable $x = \sqrt{2} \gamma$ in [30, Eq.(30)], one can show that

$$\mathcal{P}_d^{(k)} = \int f_k(\gamma^{(k)}) Q_d(\sqrt{2} \gamma^{(k)}, \sqrt{\lambda}) d\gamma^{(k)} \quad \text{(11)}$$

where $\mathcal{P}_d^{(k)}$ is given by (4) with replacing $\tau$ by $\frac{\tau}{\tau+\tau}$. Now, the joint probability of detection corresponding to $f^{(z_1), \ldots, (z_N)}$ in the set $U$ is given by (3) and (10) as

$$\mathcal{P}_d^{(z_1), (z_2), \ldots, (z_N)} = \int_{\gamma^{(z_1)}}^{\gamma^{(z_N)}} \cdots \int_{\gamma^{(z_1)}}^{\gamma^{(z_N)}} f^{(z_1), (z_2), \ldots, (z_N)}(\gamma^{(z_1)}, \ldots, \gamma^{(z_N)}) \times Q_d(\sqrt{2} \gamma^{(z_1)}, \sqrt{\lambda}) \cdots Q_d(\sqrt{2} \gamma^{(z_N)}, \sqrt{\lambda}) d\gamma^{(z_1)} \cdots d\gamma^{(z_N)} \quad \text{(12)}$$

Finally, the total probability of detection is evaluated using (11) and (12). Since the multi-selective scheme selects $K$ links with highest SNRs as final report links from the set $U$, the probability of detection is equal to the overall probability of detection in the set $S_K$. Now adopting the N-out-of-K rule for the overall probability of detection in the set $S_K$ (i.e., signal is present when at least any $N$ of the reporting links from $\gamma^{(1)}$ to $\gamma^{(K)}$ having detected the PU signal), the joint probability of any $N$ elements in the set $S_K$ has been derived in (11). Note that after ordering, the elements in $S_K$ are no longer independent. Therefore, the overall probability of detection of $N$ elements
in $S_K$ is given by

$$Q_d = \begin{cases} 
\frac{1}{\mathcal{P}_d(z_1, z_2, \ldots, z_N)} - P_N^c & \text{if } N < K \\
0 & \text{if } N = K
\end{cases}$$

(13)

where $S_K$ is the set of all combinations of $N$ indices chosen from the set $S_K$. For example, when $K = 3$ and $N = 2$, $S_{K,N} = \{\{1,2\}, \{1,3\}, \{2,3\}\}$. $\mathcal{P}_d(z_1, z_2, \ldots, z_N)$ represents the joint probability of any $N$ elements, regarded as $\{\gamma(z_1), \gamma(z_2), \ldots, \gamma(z_N)\}$, in the $S_{K,N}$ set. Also $\mathcal{P}_d(z_1, z_2, \ldots, z_N)$ is calculated using (11) when $N > 1$.

Given that the SNR of the PU signal follows exponential distribution, $P_N$ is written as

$$P_N^c = \sum_{y=0}^{K-N} (-1)^{y+1} N_{K-N,y}$$

(14)

where $N_{K,N+g}$ is the set of all combinations of $N+g$ indices chosen from the set $S_K$, and $\mathcal{P}_d(z_1, z_2, \ldots, z_N)$ represents the joint probability of any $N+g$ elements, regarded as $\{\gamma(z_1), \gamma(z_2), \ldots, \gamma(z_{N+g})\}$, in the set $N_{K,N+g}$ is also evaluated using (11).

\section{B. Probability of Detection over i.i.d. Rayleigh Fading Channels}

Given that the SNR of the PU signal follows exponential distribution for all P-R and P-D links, for any $i$ link ($1 \leq i \leq L$), the corresponding PDF and CDF are given by [31, Table 9.5, Section 9.8]

$$f_i(\gamma) = \frac{1}{\bar{\gamma}_i} e^{-\gamma/\bar{\gamma}_i}$$

(15)

$$F_i(\gamma) = 1 - e^{-\gamma/\bar{\gamma}_i}$$

(16)

where $\bar{\gamma}_i$ represents the average SNR of the $i$th link. Similar to the i.i.d. case, the destination first orders the R-D and P-D links in terms of their SNRs to form the set $U = \{\gamma^{(1)}, \gamma^{(2)}, \ldots, \gamma^{(L)}\}$ in a descending manner. By combining (15) and (16), the PDF of $\gamma^{(k)}$ ($1 \leq k \leq K$) in the $U$ set is expressed as

$$f(k)(\gamma^{(k)}) = \sum_{x=1}^{L} f_{x_1}(\gamma^{(x)}) \sum_{R_{L,k,x_1}} \left( \prod_{m=2}^{L-k+1} F_x(\gamma^{(x)}) \right) \times \prod_{x=L-k+2}^{L} \left( 1 - F_x(\gamma^{(x)}) \right)$$

where $R_{L,k,x_1}$ is the set of all combinations of $L-k$ indices chosen from the difference set $\{1, 2, \ldots, L\} - \{x_1\}$ with $1 \leq x_1 \leq L$. For instance, when $L = 4$, $k = 2$ and $x_1 = 1$, $R_{L,k,x_1} = \{(2, 3), (2, 4), (3, 4)\}$. $f_{x_1}$ is given by (15), $F_{x_1}$ and $F_{x_2}$ are given by (16). It is to be noted that the term $\prod_{m=2}^{L-k+1} F_x(\gamma^{(x)})$ in (17) can be expressed as [33, Eq.(11)]

$$\prod_{m=2}^{L-k+1} F_x(\gamma^{(x)}) = \prod_{m=2}^{L-k+1} \left( 1 - \frac{\gamma^{(x)}}{\sigma^2} \right)$$

(18)

Therefore, the PDF $f(k)(\gamma^{(k)})$ in (17) is simplified as

$$f(k)(\gamma^{(k)}) = \sum_{x=1}^{L} \prod_{x=1}^{L-k+1} \left( 1 - \frac{\gamma^{(x)}}{\sigma^2} \right)$$

(19)

with

$$\sigma^2 = \sum_{m=1}^{L} \frac{1}{\sigma^2_m} - 1 / \sum_{m=L-k+2}^{L} \frac{1}{\sigma^2_m}$$

(20)

The joint PDF of any $N$ elements in the set $S_K$, regarded as $\{\gamma^{(1)}, \gamma^{(2)}, \ldots, \gamma^{(N)}\}$ ($1 \leq 1 < 2 < \ldots < N \leq K; 1 < N \leq K$) for $\gamma^{(1)} > \ldots > \gamma^{(N)}$, is given by

$$f(k)(\gamma^{(k)}) = \frac{1}{(z_1-1)! \cdot \ldots \cdot (z_N-z_{N-1})! \cdot (L-z_N)!} \prod_{x=1}^{L-k} \left( 1 - \frac{\gamma^{(x)}}{\sigma^2} \right)$$

(21)

where the set $P$ includes all $L!$ permutations $(r_1, r_2, \ldots, r_L)$ of $(1, 2, \ldots, L)$. For example, when $L = 3$, $P = \{1, 2, 3\}, \{1, 2, 3\}, \{1, 2, 3\}, \{1, 2, 3\}, \{1, 3, 2\}$ and $(r_1, r_2, r_3)$ can be any element in $P$.

Now, the probability of detection of $\gamma^{(k)}$ in the set $U$ can be obtained by averaging (3) over (19) while performing the change of variable $x = \sqrt{2y}$ and making use of [30, Eq.(30)] to yield

$$\mathcal{P}_d(k) = \frac{1}{\sigma^2} \exp \left( - \frac{\lambda}{2} \right) \left( \frac{\sigma^2}{\lambda} \right)^{k-1} \sum_{x=1}^{L-k+2} \frac{1}{\sigma^2} \exp \left( - \frac{\lambda}{\sigma^2} \right)$$

(22)
Finally, the overall probability of detection using the joint PDF $f(z_1, z_2, \ldots, z_N)$ can be obtained by averaging (3) over (21). The structure of the overall probability of detection over i.n.d. channels is the same as (12) and (13) while replacing the probability of detection of $\gamma^{(k)}$ in the set $U$ by (21) and the joint probability of detection of any $N$ elements in the set $S_K$ by the expression averaged (3) over (20).

III. PERFORMANCE RESULTS

In this section, we compare the proposed multi-selective scheme with some current schemes (SC [15], SLS [9] and general $N$-out-of-$K$ rule [14]) via receiver operating characteristics (ROC) over i.i.d. and i.n.d. Rayleigh fading channels scenarios. Without loss of generality, for simulation convenient, we assume that the one-sided bandwidth $W$ is 1000Hz, the observation time $T_{ob}$ is 0.005s, the time bandwidth product $q$ is 5, the number of SUs $L = 6$, multi-selective cooperation scheme selects $K=3$ reporting links for cooperation where it adopts an OR rule for final decision on PU activity. For the homogenous (i.i.d.) case, the average SNR $\bar{\gamma}$ is set to 5dB while the average SNR $\bar{\gamma}$ in all links ranges from 3dB to 8dB for the heterogeneous (i.n.d.) case. We first consider the case of error-free R-D links and then present results for the case with channel errors.

In Fig. 1, the ROC of the multi-selective cooperation scheme is compared to other combining schemes where it is shown that it outperforms all schemes. It is known that the SLS scheme only selects the link with the highest signal power to make final decision on the PU activity, while the SC scheme only selects the link with the highest instantaneous SNR for detection. On the other hand, our proposed scheme selects several links with high instantaneous SNR to make final decision on the PU activity. Compared with SLS and SC, our proposed scheme not only guarantees that selected links have high detection performance but also has higher spatial diversity. The same remarks can be drawn for the case of i.n.d. Rayleigh fading channels as evident from Fig. 2.

To relax the error-free assumption on the R-D links, we consider the case where the links between the relays and destination suffer from AWGN. Since the R-D links are not error-free any more, selection combining and the proposed multi-selective cooperation scheme consider each relay channel condition separately while selection combining picks out the link with the largest SNR among $SNR_{pr-rd}$ and $SNR_{rd}$ without loss of generality, for simulation convenient, we assume that $L = 6$, $P_f = 0.3$, the average SNR ($\tau_{PR} = \tau_{PD} = 5$dB) for the i.i.d. scenario. These results are shown in Fig. 3 where the proposed scheme still offers the best performance compared to other detection schemes. As seen from Fig. 3, we notice that both SC and our proposed scheme perform better than SLS when $\tau_{PR}$ is low. This is mainly due to the fact that SC selects the link with the highest instantaneous SNR to make final decision and our proposed scheme selects several links with high instantaneous SNR for detection. However, as $\tau_{PR}$ increases, the performance of SLS approaches the SC whereas our scheme offers better detection performance due to the higher spatial diversity.

IV. OPTIMIZATION OF MULTI-SELECTIVE SCHEME OVER I.I.D. AND I.N.D. CHANNELS

Since the proposed scheme selects $K$ reporting links to form the final decision on the PU activity by using $N$-out-of-$K$ rule, there exists optimal values for $N$ and $K$ that allow for optimum performance results. In this section, we consider such an investigation and find the optimal $N$ and $K$.

When the threshold $\lambda$ and $N$ from the $N$-out-of-$K$ rule are given, the probability of detection $Q_d$ and false alarm $Q_f$ increase with the number of selected links $K$ according to (5) and (12). Thus, the total probability of miss detection $Q_m = 1 - Q_d$ decreases as $K$ increases. In this case, a performance metric that can be used to assess the performance of the receiver is the total error rate $Q_f + Q_m$ proposed in [13]. In what follows, we use the total error rate as the performance metric.
metric to find the optimal $K$ that achieves target error bound $Q_f + Q_m \leq \rho$ for the multi-selective scheme.

Given the detection threshold $\lambda$, according to (5) and (12), the number of selected links $K$ directly affects the total error rate. However when $K$ is fixed, the performance of the multi-selective scheme is dominated by the $N$-out-of-$K$ rule chosen according to (12). Therefore, we first solve for the optimal $N$ for a given $K$. Then we evaluate the optimal number of selected links $K$ that achieves a target error bound $Q_f + Q_m \leq \rho$ for a given optimal $N$.

A. Decision Rule

We first examine the performance of the multi-selective cooperation scheme under different $N$-out-of-$K$ rules, i.e., OR, majority, and AND rules. The results of this investigation are presented in Fig. 4 showing the total error rate $Q_f + Q_m$ versus threshold with different $N$-out-of-$K$ rules varying from $N = 1$ to $N = 6$ when the multi-selective scheme selects $K = 6$ reporting links among $L = 10$ SU links. As Fig. 4 shows, when the detection threshold is small, i.e., $\lambda = 6$, the OR rule $N = 1$ outperforms all other rules. Meanwhile, the AND rule $N = 6$ offers the best performance when the threshold is large, i.e., $\lambda = 21$. Therefore, the optimal $N$ in $N$-out-of-$K$ rule, $N_{\text{opt}}$, is different under different thresholds $\lambda$ when $K$ is fixed.

We express the total error rate using a function $H(N)$ as follows:

$$Q_f + Q_m = Q_f + 1 - Q_d = 1 + H(N)$$

$$= 1 + \sum_{j=N}^{K} \binom{K}{j} P_f^j (1 - P_f)^{K-j}$$

$$= \sum_{S_k,N} T_d^{(2z_1, z_2, \ldots, z_N)} - P_{\text{com}}$$

(23)

when $N < K$

After some simplifications, the optimal $N$ is reached when

$$\frac{\partial H(N)}{\partial N} = 0.$$ 

$$\frac{\partial H(N)}{\partial N} \approx H(N + 1) - H(N)$$

$$= \sum \frac{H(z_1, z_2, \ldots, z_N)}{S_k,N} + 2 \sum_{j=1}^{K-N} (-1)^j T_d^{(2z_1, z_2, \ldots, z_N+j)}$$

(24)

$$(K\bigg) P_f^N (1 - P_f)^{K-N} = 0$$

when $N < K$.

Since the number of selected links $K$ is fixed, $T_d^{(2z_1, z_2, \ldots, z_N)}$ varies with $N$. Now let us define a function $G(N) = \sum_{S_k,N} \frac{T_d^{(2z_1, z_2, \ldots, z_N)}}{S_k,N} + 2 \sum_{j=1}^{K-N} (-1)^j T_d^{(2z_1, z_2, \ldots, z_N+j)}$. Thus, $N_{\text{opt}}$ is obtained when it satisfies the following function

$$G(N_{\text{opt}}) = \left(\frac{K}{N}\right) P_f^{N_{\text{opt}}} (1 - P_f)^{K-N_{\text{opt}}}.$$ 

(25)

According to (2),(3) and (11), $T_d^{(2z_1, z_2, \ldots, z_N)}$ and $P_f$ decrease with the threshold $\lambda$ (this was also proved in [15]). Therefore, when the threshold is small enough, the right hand side of (24) increases with $N$. From (12), since the left hand side of (24) decreases with $N$, $N_{\text{opt}} = 1$ when the threshold is small and $N_{\text{opt}} = K$ when the threshold is large.

B. Optimal Number of Selected Links ($K$)

In the multi-selective scheme, the local decisions of $K$ selected links are used for final decision at the destination. Our objective here is to find the optimal $K_{\text{opt}} (1 \leq K_{\text{opt}} \leq L)$ that achieves a total error rate target bound, $Q_f + Q_m \leq \rho$ for a given detection threshold $\lambda$.

We first evaluate the corresponding optimal $N$-out-of-$K$ rule for each $K$, optimal voting rule ($N_{\text{opt}}^K$), by using (24). Then we define a new function $T(\ldots)$ in terms of the variable $K$ as

$$T(K, N_{\text{opt}}^K) = Q_f + Q_m - \rho = 1 + Q_f - Q_d - \rho$$

(26)

where $K$ is the number of selected links used for final decision. Note that the probabilities of false alarm $Q_f$ and detection $Q_d$.
are functions of \( K \) and \( N_{K_{opt}}^m \) is given by (24). The optimal \( K \) is the one that satisfies the following conditions

\[
T(K_{opt}, N_{K_{opt}}^m) \leq 0 \tag{27}
\]

\[
T(K_{opt} - 1, N_{K_{opt} - 1}^m) > 0. \tag{28}
\]

From (27) and (28), \( K_{opt} \) is the first crossing zero of the function \( T(K, N_{K_{opt}}^m) \) plotted against \( K \). For example, consider \( L = 20 \) available links to the destination with average SNR \( \gamma = 5 \text{dB} \) over all links, and a target error rate bound \( Q_J + Q_m \leq 0.1 \) with a given detection threshold \( \lambda = 18 \). From Figs. 7, 8, two important remarks are noted, (i) Increasing the number of selected links \( K \) results in lower probability of miss detection \( Q_m \), while increasing the number of nodes \( N \) in the \( N\text{-out-of-}K \) rule results in higher \( Q_m \). (ii) As the number of selected links \( K \) goes high, the probability of false alarm \( Q_J \) also goes high with \( K \) and while as \( N \) goes high \( Q_J \) decreases. From these remarks, one can achieve a target error rate bound \( Q_J + Q_m \leq 0.1 \) for a given detection threshold through (27) and (28). Fig. 5 shows \( Q_J(K, N) + Q_m(K, N) \) versus \( K \) applied with optimal \( N\text{-out-of-}K \) rule, \( N = N_{K_{opt}}^m \). As the results in Fig. 5 show, \( Q_J + Q_m \leq 0.1 \) is achieved when \( K_{opt} = 4 \) under a corresponding optimal voting rule \( N_{K_{opt}}^m = 2 \) which is shown in Fig. 6. It is to be noted that the due to the inverse dependence of \( K \) and \( N \) with \( Q_J \) and \( Q_m \) as given by from (5) and (12), the total error rate \( Q_J + Q_m \) performance in Fig. 5 is justified.

As a final investigation, we consider the case of i.n.d. Rayleigh fading channels. The optimization analysis is similar to the i.i.d. case where the probability of detection is given by (21) for the i.n.d. case. Let us consider the performance of the multi-selective scheme over i.n.d. Rayleigh fading channels with \( L = 14 \) where the SNR of the different P-R links ranges from 2dB to 15dB, and the target error rate \( Q_J + Q_m \leq 0.02 \) with a given threshold \( \lambda = 18 \). As Figs. 9, 10 show, the required total error rate is achieved when \( K_{opt} = 2 \) with a corresponding optimal voting rule \( N_{K_{opt}}^m = 2 \).
C. PU Activity model

Now we will apply one typical PU activity model, ON/OFF model, to the proposed multi-selective scheme for the i.i.d. Rayleigh fading case. The aim here is to study the effect of the ON/OFF model parameters on the total error rate performance. In ON/OFF model, the ON and OFF states correspond to $H_1$ and $H_0$, respectively. Without loss of generality, we assume that the probability of PU presence (ON state) denoted by $\alpha$ [27] to vary from 0.1 to 0.9. The overall PU operation time, including the ON and OFF states, is represented by $T$. Other parameters are as the same as the i.i.d. case discussed in Sec. III. Hence, the OFF state relates to $Q_f$ and ON state relates to $Q_d$. In this case, the total error rate corresponding to $T$ can be expressed as:

$$T\alpha(1-Q_d) + T(1-\alpha)Q_f = Q_f + \alpha(1-Q_f-Q_d).$$  \hspace{1cm} (29)

Fig. 11 shows the performance of the proposed multi-selective scheme as a function of $Q_f$, $\alpha$, and the achieved total error rate. The same simulation parameters as before are considered, the average SNR=5dB for a system with $L=6$ SUs, $K=3$ selected/reporting links for cooperation, and adopting the OR rule (i.e., $N=1$) for the final decision on PU activity. The effect of the primary user activity on the total error rate and probability of false alarm is evident from these results. As shown, when the probability of the primary user being active goes high, the total error rate decreases for a given probability of false alarm. Also comparing (23) and (29), we can notice that the ON/OFF model does not affect the general optimization method of the proposed multi-selective scheme in terms of the total error rate.

V. CONCLUSIONS

We proposed a new cooperative sensing scheme which incorporates the low bandwidth advantage of the $N$-out-of-$K$ rule with superior performance compared with the selection combining scheme. The probabilities of detection and false alarm are derived for both i.i.d. and i.n.d. Rayleigh fading channels. Through optimization, we have evaluated the optimal decision rule $N$ and the number of selected links $K$ under a total error rate requirement. Simulation results have shown that the multi-selective scheme can offer a significant performance improvement in detecting the PU activity compared with other existing detection schemes.

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