A Guaranteed Cost LMI-Based Approach for Event-Triggered Average Consensus in Multi-Agent Networks

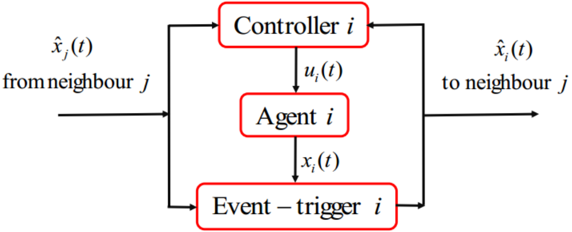
Amir Amini †, Arash Mohammadi ‡, Amir Asif †

† Electrical and Computer Engineering, Concordia University, Montreal, Canada.
‡Concordia Institute for Information System Engineering, Concordia University, Canada.

5^{th} IEEE Global Conference on Signal and Information Processing.

Outline

Event-triggered Average Consensus



1. $x_i(t)$: The state of agent i
2. $\hat{x}_i(t)$: The last transmitted state of agent i up to time t

Average Consensus Definition:

$$\lim_{t\to\infty} \left| x_i(t) - \frac{1}{N}\sum_{j=1}^{N} x_j(0) \right| = 0, \; 1 \le i \le N. \tag{1}$$

Motivation and Objective

Motivation

- Transmission saving for average consensus in multi-agent networks with bandwidth constrained environments.
- Adapting Guaranteed Cost approach to event-triggered average consensus.

Objective

- Achieving event-triggered average consensus with restricted guaranteed operational cost.
- Compute optimal parameters to achieve average consensus with small number of transmission.

Event-triggered Average Consensus

1. Agent model: $\dot{x}_i(t) = u_i(t), \quad 1 \le i \le N;$
2. Last transmitted state: $\hat{x}_i(t) = x_i(t^i_k), \; t \in [t^i_k, t^i_{k+1}).$
3. Controller: $u_i(t) = -\sum_{j \in \mathcal{N}_i} a_{ij}(\hat{x}_i(t) - \hat{x}_j(t)),$
4. Error: $e_i(t) = \hat{x}_i(t) - x_i(t)$

Closed-loop system:

$$\dot{\boldsymbol{x}}(t) = -L\Big(\boldsymbol{x}(t) + \boldsymbol{e}(t)\Big), \tag{2}$$

L : Laplacian Matrix;
$\boldsymbol{x}(t) = [\, x_1(t)\,, \ldots\,, x_N(t)\,]^T,$
$\hat{\boldsymbol{x}}(t) = [\, \hat{x}_1(t)\,, \ldots\,, \hat{x}_N(t)\,]^T,$
$\boldsymbol{e}(t) = \left[\, e_1(t)\,, \ldots\,, e_N(t)\,\right]^T$

Event-triggered Average Consensus

Event-triggering function:

Transmit new state if

$$e_i(t) \text{ exceeds the threshold } \phi|\hat{\mathbb{X}}_i(t)|$$

- $|\hat{\mathbb{X}}_i(t)|$: $|\mathcal{N}_i|\hat{x}_i(t) - \sum_{j=1}^{\mathcal{N}_i} \hat{x}_j(t)$,
- Positive scalar ϕ: The transmission threshold to be computed

How to design the optimal value for transmission threshold ϕ?

- If $\phi = 0 \longrightarrow$ Constant Transmission
- Inadequate small $\phi \longrightarrow$ Waste of communication resources
- Inadequate large $\phi \longrightarrow$ No consensus agreement

Cost function

The proposed cost function:

$$J = \int_0^\infty \left(\boldsymbol{x}^T(t) R \boldsymbol{x}(t) + \boldsymbol{u}^T(t) Q \boldsymbol{u}(t) \right) dt \tag{3}$$

- R and Q: given positive definite weighting matrices.
- Matrix R assigns desired penalty on deviation of the states $\boldsymbol{x}(t)$ from the target value.
- Matrix Q assigns desired penalty on control input $\boldsymbol{u}(t)$.

If there exists a positive scalar J^* such that the cost J associated with the event-triggered average consensus process satisfies $J \leq J^*$, then J^* is said to be a guaranteed cost.

How to restrict (minimize) J^* ?

Converting Consensus problem to Stability problem

In order to use the Lyapunov stability theorem and incorporate the cost function J in parameter design, Consensus in transformed to an equivalent stability problem

$$\underbrace{\dot{\boldsymbol{x}}(t) = -L\Big(\boldsymbol{x}(t) + \boldsymbol{e}(t)\Big)}_{\text{Consensus problem}} \Longleftrightarrow \underbrace{\dot{\boldsymbol{x}}_r(t) = -\mathbb{L}\Big(\boldsymbol{x}_r(t) + \boldsymbol{e}_r(t)\Big)}_{\text{Stability problem}}$$

$\boldsymbol{x}_r(t) = \hat{L}\boldsymbol{x}(t)$, $\quad \boldsymbol{e}_r(t) = \hat{L}\boldsymbol{e}(t)$, $\quad$ and $\quad \mathbb{L} = \hat{L}L\hat{L}^{\dagger}$.
$\hat{L}$: The reduced order Laplacian matrix.

The global Event-triggering condition:

$$\boldsymbol{e}_r^T(t)\boldsymbol{e}_r(t) \leq \Big(\boldsymbol{e}_r(t)+\boldsymbol{x}_r(t)\Big)^T M^T \phi^2 M(\boldsymbol{e}_r(t)+\boldsymbol{x}_r(t)). \qquad (4)$$

Computing Optimal Transmission Threshold

Lyapunov Candidate: $V(t) = \boldsymbol{x}_{\mathrm{r}}^T(t) P \boldsymbol{x}_{\mathrm{r}}(t)$
Incorporating the Lyapunov Stability theorem and proposed cost with this inequality:

$$\dot{V}(t) + \boldsymbol{x}_{\mathrm{r}}^T(t) R \boldsymbol{x}_{\mathrm{r}}(t) + \boldsymbol{u}^T(t) Q \boldsymbol{u}(t) < 0 \tag{5}$$

- If (5) is satisfied $\rightarrow$ $\dot{V}(t) < 0$ $\rightarrow$ The system is stable $(\lim_{t\to\infty} \boldsymbol{x}_{\mathrm{r}}(t) = 0)$ $\rightarrow$ Reaching average consensus
- Integrating (5) results in $V(\infty) - V(0) + \int_0^\infty \boldsymbol{x}_{\mathrm{r}}^T R \boldsymbol{x}_{\mathrm{r}}(t) + \boldsymbol{u}^T(t) Q \boldsymbol{u}(t)\, dt < 0$, which is equivalent to $J < \underbrace{[V(0) = \boldsymbol{x}_{\mathrm{r}}^T(0) P \boldsymbol{x}_{\mathrm{r}}(0)]}_{J^*}$.

Computing Optimal Transmission Threshold

Compute Transmission Threshold ϕ from:

$$\phi = \sqrt{\tau^{-1}\gamma^{-1}} \tag{6}$$

which is conditioned on the solvability of the following convex optimization problem

$$\min_{\gamma,\tau,P} \quad \overbrace{\tau + \gamma}^{\text{To enlarge } \phi} \quad + \overbrace{\text{trace}(P^2)}^{\text{To restrict } J^\star}$$

$$\text{S.t:} \begin{bmatrix} -P\mathbb{L}-\mathbb{L}^T P+R & -P\mathbb{L} & -(L\hat{L}^\dagger)^T & M^T \\ * & -\tau I & -(L\hat{L}^\dagger)^T & M^T \\ * & * & -Q^{-1} & 0 \\ * & * & * & -\gamma I \end{bmatrix} < 0 \tag{7}$$

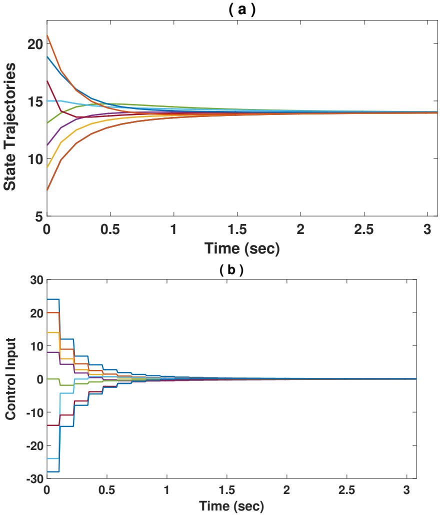
Experimental Results

- Laplacian Matrix:

$$L = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & -1 & 0 & -1 \\ 0 & 2 & 0 & 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 2 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 2 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ -1 & -1 & -1 & 0 & -1 & 5 & -1 & 0 \\ 0 & 0 & -1 & -1 & 0 & -1 & 3 & -1 \\ -1 & -1 & 0 & 0 & 0 & 0 & -1 & 3 \end{bmatrix}$$

- Given optimization values:
  $R = rI$, $Q = qI$, with $r = 10$, $q = 0.1$

- Computed Parameters:
  $\phi = 0.1393$,
  $J^* = 13327$,
  $J = 4010. \rightarrow J < J^*$



Experimental Results

How different choices for R and Q affect the average consensus process?

- $\bar{TI}$: Total iteration to reach average consensus
- $\bar{AT}$: Average number of state transmission instants

Table:

The effect of weighting matrices R, and Q on the event-triggered average consensus performance

r	q	ϕ	$\bar{TI}$	$\bar{AT}$	J	J^*
1	0.1	0.1156	310	24.875	404.27	1675.3
10	0.1	0.1393	307	20.875	2322.2	5993.5
20	0.1	0.1402	301	19.5	8026.3	25952.2
1	0.05	0.1307	308	21.5	399.47	1475.9
1	0.1	0.1156	310	24.875	404.27	1675.3
1	1	0.1102	311	24.125	1737.0	4681.2

Conclusion and Future work

Conclusion

1. The data transmission threshold ϕ is affected by a different selection of weighting matrices R and Q.
2. A larger ϕ causes a faster consensus convergence rate with fewer number of transmissions which is at the expense of more cost J.
3. For a fixed Q, increasing R results in obtaining a relatively larger value for ϕ.

Future Work

1. Guaranteed Cost average consensus in networks with random link failures.
2. Guaranteed Cost average consensus in networks with time-varying communication delay.

Question?

Thank You