A Robust Event-Triggered Consensus Strategy for Linear Multi-Agent Systems with Uncertain Network Topology

Amir Amini [†], Amir Asif [†], Arash Mohammadi [‡]

[†] Electrical and Computer Engineering, Concordia University, Montreal, Canada.
[‡]Concordia Institute for Information System Engineering, Concordia University, Canada.

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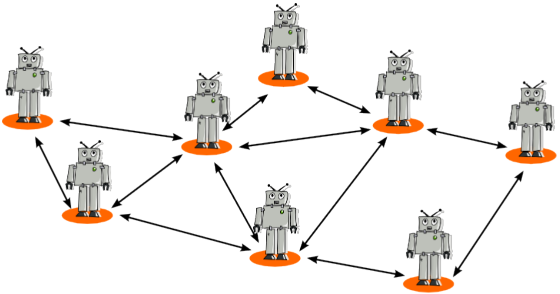
Multi-agent Systems



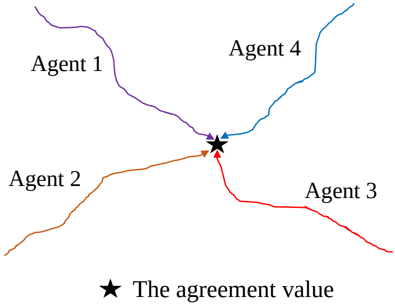
A multi-agent system consists of multiple agents that interact to achieve a cooperative objective.

An agent can represent a moving vehicle, a sensor node, an electric bus, etc.

Cooperative Objectives: Formation, Consensus, Containment, Rendezvous, ...

Consensus

Consensus Control:



Consensus: To reach an agreement upon a common value.

Agent Dynamics

General Linear Agent Model :

$$\dot{\boldsymbol{x}}_i(t) = \boldsymbol{A}\boldsymbol{x}_i(t) + \boldsymbol{B}\boldsymbol{u}_i(t), \quad 1 \leq i \leq N \tag{1}$$

- $\boldsymbol{x}_i(t) \in \mathbb{R}^n$: The state of agent i at time instant t;
- $\boldsymbol{A} \in \mathbb{R}^{n \times n}$: System matrix (known and constant);
- $\boldsymbol{B} \in \mathbb{R}^{n \times m}$: Input matrix (known and constant);
- $\boldsymbol{u}_i(t) \in \mathbb{R}^{m \times n}$: A proposed distributed control input;
- N : Number of agents in the network.

Consensus

Consensus Definition:

For any initial condition $\boldsymbol{x}_i(0)$, the consensus problem for (1) is said to be solved iff **:**

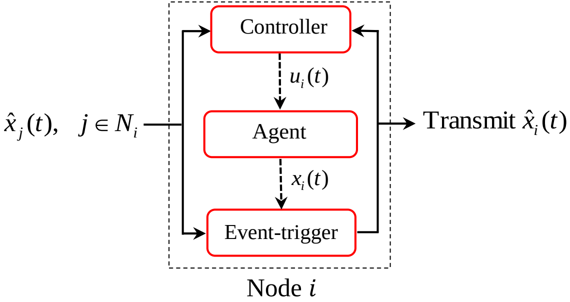
- Global sense **:** $\lim_{t\to\infty} \| \boldsymbol{x}_i(t) - \boldsymbol{x}_j(t) \| = 0,\ (1 \leq i, j \leq N),$
- Average sense **:** $\lim_{t\to\infty} \|\boldsymbol{x}_i(t) - \frac{1}{N}\sum_{j=1}^{N}\boldsymbol{x}_j(0)\| = 0,\ (1 \leq i \leq N),$

Average consensus is usually considered for first-order agents defined by $\dot{x}_i(t) = u_i(t)$, with $x_i(0)$ as initial local observation.

Key components in reaching consensus:

- Distributed control input $\boldsymbol{u}_i(t)$,
- Information exchange between the neighbouring agents.

Event-triggered Consensus



1. $x_i(t)$: The state of agent i
2. $\hat{x}_i(t)$: The last transmitted state of agent i up to time t

The received information is subject to uncertainty due to existence of communication unreliabilities $\rightarrow$ robustness is required

Motivation and Objective

Motivation:

- Transmission saving for consensus in multi-agent systems with bandwidth constrained environments and unreliable channel.

Objective:

- Achieve event-triggered consensus with a desired exponential rate of convergence (as opposed to asymptotic rate);
- Compute optimal consensus parameters to achieve consensus in presence of network uncertainties.

Main Features

- Event-based disagreement vector :

$$\boldsymbol{q}_i(t) = \sum_{j\in\mathcal{N}_i} \bar{a}_{ij}\,(e^{\boldsymbol{A}(t-t^i_{k_i})}\boldsymbol{x}_i(t^i_{k_i}) - e^{\boldsymbol{A}(t-t^j_{k_j})}\boldsymbol{x}_j(t^j_{k_j}))$$

where $\bar{a}_{ij}$ is the uncertain (but norm-bounded) weight for channel link between agent i and j.

- Measurement error : $\boldsymbol{e}_i(t) = e^{\boldsymbol{A}(t-t^i_{k_i})}\boldsymbol{x}_i(t^i_{k_i}) - \boldsymbol{x}_i(t)$.

  - **Event-triggering function :** given an event time $t^i_{k_i}$, the next event for agent i is triggered at $t = t^i_{k_i+1}$, where

$$t^i_{k_i+1} = \inf\{t > t^i_{k_i} \mid \|\boldsymbol{e}_i(t)\| - \phi\|\boldsymbol{q}_i(t)\| \geq 0\}, \qquad (2)$$

$\phi > 0$: Transmission threshold to be designed.

Design unknown parameters

The proposed control law :

$$\boldsymbol{u}_i(t) = \boldsymbol{K}_i \boldsymbol{q}_i(t), \tag{3}$$

$\boldsymbol{K}_i$: Control gain to be designed.

Question: How to design optimal[1] values for transmission threshold ϕ and control gain $\boldsymbol{K}_i$ that guarantee an exponential rate of consensus in *norm-bounded* uncertain network channel?

[1] maximize ϕ to minimize events, and minimize $\boldsymbol{K}_i$ to minimize control force

Preliminary steps prior to optimization

- Consider the augmented closed-loop system;
- Convert the consensus problem into an equivalent stability problem $\rightarrow$ Lyapunov stability method
- Obtain sufficient conditions and inequalities for uncertain connectivity links.

Compute optimal consensus parameters

Solve the following convex optimization problem with desired convergence rate ζ

$$\min_{\boldsymbol{\Theta}_i,\mu,\epsilon,\tau_j,\boldsymbol{P},\omega_1,\omega_2,\omega_3,\omega_4} f = \overbrace{\omega_1+\omega_2}^{\text{To maximize } \phi} + \overbrace{\omega_3+\omega_4}^{\text{To minimize } \boldsymbol{K}_i} \tag{4}$$

$$\text{S.t:} \quad \boldsymbol{\Pi} = \begin{bmatrix} \boldsymbol{\Pi}_1 & \boldsymbol{\Pi}_2 \\ * & \boldsymbol{\Pi}_3 \end{bmatrix} < 0, \begin{bmatrix} -\omega_1 & \tau_1 \\ * & -1 \end{bmatrix} < 0, \begin{bmatrix} -\omega_2 & \mu \\ * & -1 \end{bmatrix} < 0,$$

$$\begin{bmatrix} \omega_3 \boldsymbol{I} & \boldsymbol{I} \\ * & \boldsymbol{P} \end{bmatrix} > 0, \begin{bmatrix} -\omega_4 \boldsymbol{I} & \boldsymbol{\Theta}^T \\ * & -\boldsymbol{I} \end{bmatrix} < 0,$$

- $\boldsymbol{\Theta}_i$ $(1 \le i \le N)$, μ, ϵ, τ_j $(1 \le j \le 3)$, $\boldsymbol{P}$, ω_c $(1 \le c \le 4)$ are decision variables;
- Block Matrix $\boldsymbol{\Pi}$ contains information about agent models, network connectivity, exponential convergence criterion, uncertainty upper bound, control gain $\boldsymbol{K}_i$, and transmission threshold ϕ.

Compute optimal consensus parameters

Once the optimization problem (4) is solved, compute consensus parameters

$$\phi = \sqrt{{\tau_1}^{-1}\mu^{-1}}, \text{ and } \boldsymbol{K}_i = \boldsymbol{B}_i^{\dagger}\boldsymbol{P}^{-1}\boldsymbol{\Theta}_i, \quad (1 \leq i \leq N) \tag{5}$$

Consensus parameters are bounded for the minimized objective function $f = \omega_1 + \omega_2 + \omega_3 + \omega_4$

$$\phi \geq (\omega_1\omega_2)^{\frac{-1}{4}}, \quad {\boldsymbol{K}_i}^T\boldsymbol{K}_i \leq \omega_3\,\omega_4^2\boldsymbol{B}_i^{\dagger}{\boldsymbol{B}_i^{\dagger}}^T, \ (1 \leq i \leq N). \tag{6}$$

Experimental Results

- A network of six second-order heterogeneous agents

$$\begin{aligned} \dot{r}_i(t) &= v_i(t), \\ m_i \dot{v}_i(t) &= u_i(t), \quad (1 \le i \le 6), \end{aligned} \tag{7}$$

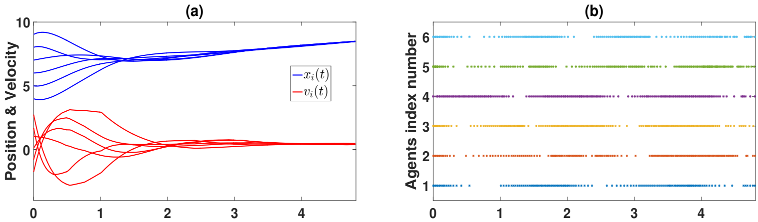
$r_i(t) \in \mathbb{R}$: Position, $v_i(t) \in \mathbb{R}$: Velocity, m_i: Inertia

- Consensus in this problem is to, distributively, reach a common position and velocity
- Laplacian Matrix (two unreliable links)

$$L = \begin{bmatrix} 2.5 & 0 & 0 & -0.5 & -1 & -1 \\ 0 & 2 & 0 & 0 & -1 & -1 \\ 0 & -1 & 3 & -1 & 0 & -1 \\ 0 & -1 & 0 & 2 & 0 & -1 \\ -1 & -1 & 0 & -1 & 3 & 0 \\ -1 & -1 & -0.5 & 0 & 0 & 2.5 \end{bmatrix} \longleftrightarrow \bar{L} = \begin{bmatrix} 2 & 0 & 0 & 0 & -1 & -1 \\ 0 & 2 & 0 & 0 & -1 & -1 \\ 0 & -1 & 3 & -1 & 0 & -1 \\ 0 & -1 & 0 & 2 & 0 & -1 \\ -1 & -1 & 0 & -1 & 3 & 0 \\ -1 & -1 & 0 & 0 & 0 & 2 \end{bmatrix} \tag{8}$$

- Solve the optimization problem (4) to compute $\boldsymbol{K}_i$ and ϕ

Experimental Results



How different values for decay rate ζ affect the consensus process?

Table 1: Consensus performance for varying ζ.

decay rate ζ	Number of transmissions per agent						Consensus time (sec)	Objective function f
	1	2	3	4	5	6		
0.2	262	295	333	318	369	321	10.57	401.19
0.3	133	154	175	180	164	142	4.81	406.84
0.4	68	58	95	194	50	68	3.51	411.27

Conclusion

1. For a desired rate of convergence, robust event-triggered consensus is reached for norm-bounded uncertain networks;
2. Using convex optimization, the transmission threshold ϕ is maximized (to trigger minimum number of events) and control gain $\boldsymbol{K}_i$ is minimized (to minimize the control force);
3. As convergence rate ζ is increased, the consensus time constantly gets reduced until the optimization problem becomes infeasible

Question?

Thank You