Distributed Inference and Management of Future Cyber-Physical Networks

Georgios B. Giannakis

Digital Technology Center and Dept. of ECE
University of Minnesota

Acknowledgements: NREL, NSF CCF 1423316; CyberSEES 1442686; EPCN 1509040
Profs. N. Gatsis (UTSA), V. Kekatos (VaTech), H. Zhu (UIUC),
S. Dhople (UMN), Dr. E. Dall'Anese (NREL), and L. Zhang (UMN)
**SMART GRID**: Advanced infrastructure and information technologies (Cyber) to enhance the electrical power network (Physical)

- controllable
- resilient
- efficient
- participation
- sustainable
- self-restoring
- green
- situational awareness

Dept. of Energy, “The smart grid: an introduction”
Enabling technology advances for optimization, learning, and signal processing.

- Renewables
- Distributed generation
- Micro-grids
- Sensing/metering
- Demand response
- Communication networks
- Electric vehicles
- Power electronics
Outline

- Distributed and robust power system state estimation (PSSE)
- Distributed optimal power flow (OPF)
- Distributed demand response (DR)
- Distributed electric vehicle (EV) charging
Complex power

- Power injection to bus $m$
  \[ S_m = V_m I_m^* = P_m + jQ_m \]

- (Re) active power generated or consumed at a bus

- Power flow over line $(m, n)$
  \[ S_{mn} = V_m I_{mn}^* = P_{mn} + jQ_{mn} \]

- Multivariate nodal power model (*quadratic* in $v$)

\[ s = \text{diag}(v)i^* = \text{diag}(v)Y^*v^* \]

- Concatenating $\{ V_m \}$
- Bus admittance matrix

- Concatenating $\{ S_m \}$
- Concatenating $\{ I_m \}$
Power system state estimation
Motivation for PSSE

**Goal:** Given meter readings and grid parameters, find state vector $v$

- Quantities of interest expressible as functions of bus voltages in $v$
- PSSE is of paramount importance for
  - Situational awareness
  - Reliability analysis and planning
  - Load forecasting
  - Economic operations and billing
- Can be formulated as an estimation problem [Schweppe et al’70]

SCADA-based PSSE

- Supervisory control and data acquisition (SCADA) system
  - Terminals forward readings to control center (~4 secs)
  - Phases cannot be used due to timing mismatches

- Available measurements ($M$)
  \[
  \{V_m, P_m, Q_m, P_{mn}, Q_{mn}, I_{mn}\}
  \]
  \[
  z = h(v) + \epsilon
  \]

- Nonlinear (weighted) least-squares
  \[
  \hat{v} := \arg \min_v \|z - h(v)\|^2
  \]

- Constraints
  - Zero-injection buses  \[P_m = Q_m = 0\]
  - Feasible ranges  \[V_m^{\min} \leq V_m \leq V_m^{\max}\]
Popular solvers

(M1) Gauss-Newton iterations

- Approximate \( h(v) \approx h(v_k) + G_k^T(v - v_k) \), \( G_k \) : Jacobian at \( v_k \)
- Linear LS in closed form \( v_{k+1} = v_k + (G_k G_k^T)^{-1} G_k(z - h(v_k)) \)

- Cholesky factorization based remedies for numerical stability
- Sensitive to initialization; No convergence guarantee

(M2) Fast decoupled solver

- Active powers depend only on \( \{\theta_m\} \); reactive only on \( \{V_m\} \)
- Approximate \( (G_k G_k^T)^{-1} \) at flat voltage profile \( v = 1 + j0 \)

Semidefinite relaxation

- Rectangular coordinates: measurements are \textit{quadratic} in $\mathbf{v}$

$$P_m + jQ_m = \mathbf{v}_m^T \mathbf{I}_m^* = \mathbf{e}_m^T \mathbf{v} (\mathbf{Yv})^H \mathbf{e}_m = \text{Tr}(\mathbf{Y}^H \mathbf{e}_m \mathbf{e}_m^T \mathbf{vv}^H \mathbf{H}_m)$$

- Yet \textit{linear} in $\mathbf{V} = \mathbf{vv}^H$

$$\min_{\mathbf{v}} \sum_{m=1}^{M} (z_m - h_m(\mathbf{v}))^2$$

$$\min_{\mathbf{V}} \sum_{m=1}^{M} (z_m - \text{Tr}(\mathbf{H}_m \mathbf{V}))^2$$

\text{s.to } \mathbf{V} \succeq 0 \text{ and } \text{rank}((\mathbf{V}) = 1$$

- SDR popular in SP and communications [Goemans et al’95]
- SDR for SE [Zhu-GG’11], SDR for OPF [Bai etal’08, [Lavaei-Low’11]
  - Generalizations include PMU data, and robust SDR-based PSSE
  - (Near-)optimal regardless of initialization; polynomial complexity $O(N^{4.5} \log(1/\epsilon))$

Numerical tests

- IEEE 30, 57, and 118-bus benchmarks
- $V_m \sim \mathcal{N}(1, 0.01), \theta_m \sim \mathcal{U}[-\theta, \theta]$
- Closer to global optimum at higher complexity

Average running time in secs.

<table>
<thead>
<tr>
<th># of buses</th>
<th>WLS</th>
<th>SDR</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.216</td>
<td>1.62</td>
</tr>
<tr>
<td>57</td>
<td>0.558</td>
<td>4.32</td>
</tr>
<tr>
<td>118</td>
<td>2.87</td>
<td>21.6</td>
</tr>
</tbody>
</table>
Decentralized PSSE - motivation

- Scalable with control area size, and privacy preserving
- Area 2 buses (states): \{3,4,7,8\}
- Area 2 collects flow measurements \{(4,5), (4,9), (7,9)\}...

- **Option 1:** Ignore tie-line meters
  - statistically suboptimal
  - observability at risk (bus 11)
  - tie-line mismatches (trading)

- **Option 2:** Augment \(v_2\) to \{3,4,7,8,5,9\}
  - consent with neighbors on shared states

Cost decomposition

- Include tie-line buses to split local LS cost per $\mathcal{N}(k)$

$$f_k(V_{(k)}) := \sum_{\ell=1}^{M_k} \left[ z_k^\ell - \text{Tr}(H_{(k)}^\ell V_{(k)}) \right]^2$$

$$\hat{V} := \arg\min_{V} \sum_{k=1}^{K} \sum_{\ell=1}^{M_k} \left[ z_k^\ell - \text{Tr}(H_{(k)}^\ell V) \right]^2 \quad \text{s.to} \quad V \succeq 0$$

$$\hat{V} := \arg\min_{V} \sum_{k=1}^{K} f_k(V_{(k)}) \quad \text{s.to} \quad V \succeq 0$$

**Challenge:** as $\{\mathcal{N}_{(k)}\}$ overlap partially, PSD constraint couples $\{V_{(k)}\}$

**Blessing:** overlap $\rightarrow$ global; no overlap: $V \succeq 0 \iff V_{(k)} \succeq 0, \ \forall k$
Distributed SDP for PSSE

- If **graph** with **areas-as-nodes** and **overlaps-as-edges** is a **tree**, then

\[
\hat{\mathbf{V}} := \arg \min_{\mathbf{V}} \sum_{k=1}^{K} f_k(\mathbf{V}_{(k)}) \quad \text{(C-SDP)}
\]

\[
\{\hat{\mathbf{V}}_{(k)}\} := \arg \min_{\{\mathbf{V}_{(k)}\}} \sum_{k=1}^{K} f_k(\mathbf{V}_{(k)})
\]

\[
\text{s.to } \mathbf{V}_{(k)} \succeq 0, \quad \mathbf{V}_{(k)}^{[j]} = \mathbf{V}_{(k)}^{[i]}
\]

- ADMM [Glowinski-Marrocco’75]; for D-Estimation [Schizas-Giannakis’07]
  - Iterates between local variables and multipliers per equality constraint

- Converges \( \mathbf{V}_{(k)}(i) \to \hat{\mathbf{V}}_{(k)} \) even for noisy-async. links [Schizas-GG’08], [Zhu-GG’09]
ADMM convergence in action

- IEEE 14-bus grid with 4 areas; 5 meters on tie-lines

- Errors $\|V^{(k)}(i) - \hat{V}^{(k)}\|_F$ vanish asymptotically
118-bus test case

- Triangular configuration [Min-Abur’06]
- Power flow meters on all tie lines except for (23, 24)
  - graph of areas is a tree

Local norms

\[ \|v_k(i) - v_k\|_2 \]

converge in only 20 iterations!

Decentralized PSSE for linear models

- Local linear(ized) model: \( z_k = H_k x_k + n_k \)

- Regional PSSEs

- Coupled local problems

\[ \min_{x_k \in X_k} f_k(x_k) \]

\[ \min_{\{x_k\}} \sum_{k=1}^{K} f_k(x_k) \]

s.t. \( x_k[l] = x_l[k] \)

**S1.** \( x_{k}^{t+1} \leftarrow \arg \min_{x_k \in X_k} f_k(x_k) + \frac{c}{2} \sum_{i \in S_k} (x_k(i) - p_k^t(i))^2 \)

**S2.** \( p_{k}^{t+1}(i) \leftarrow p_k^t(i) + \left( x_{l}^{t+1}[i] - \frac{x_k^t(i) + x_l^t[i]}{2} \right) \)

- ADMM solver: convergent with minimal exchanges and privacy-preserving

**Simulated test**

\[ \mathbf{x}_{k}^{t+1} \leftarrow (\mathbf{H}_{k}^{T} \mathbf{H}_{k} + c \cdot \mathbf{D}_{k})^{-1} (\mathbf{H}_{k}^{T} \mathbf{z}_{k} + c \cdot \mathbf{D}_{k} \mathbf{p}_{k}^{t}) \quad [\mathbf{D}_{k}]_{ii} = |S_{k}^{i}| \]

\[ \mathbf{p}_{k}^{t+1}(i) \leftarrow \mathbf{p}_{k}^{t}(i) + \left( \mathbf{x}_{l}^{t+1}[i] - \frac{x_{k}^{t}(i) + x_{l}^{t}[i]}{2} \right) \]

---

Decentralized bad data cleansing

\[ z = Hx + n + o \]

- Reveal \textit{single} and \textit{block} outliers via

\[ f(x) := \min_o \frac{1}{2} \| z - Hx - o \|_2^2 + \lambda \| o \|_1 \]

\[ = \sum_{m=1}^{M} h(z_m - h_m^T x) \]

\textbf{S1.} \[ x_k^{t+1} \leftarrow \left( H_k^T H_k + c \cdot D_k \right)^{-1} \left( H_k^T (z_k - o_k^t) + c \cdot D_k p_k^t \right) \]

\textbf{S2.} \[ o_k^{t+1} \leftarrow [z_k - H_k x_k^{t+1}]^+ \]

\textbf{S3.} \[ p_k^{t+1}(i) \leftarrow p_k^t(i) + \left( x_i^{t+1}[i] - \frac{x_i^t(i) + x_i^t[i]}{2} \right) \]
D-PSSE on a 4,200-bus grid
Optimal Power Flow


Generation cost

- Thermal generators
- Power output $P_{G_i}$ (MW)
- Generation cost $C_i(P_{G_i})$ ($/h$ or €/h)

Economic dispatch (ED): Find most economically generated power output to serve given load

- ED typically solved every 5-10 minutes
AC optimal power flow

- **Motivation**: Minimize generation cost respecting physical constraints

\[
\min_{p_G, q_G, v} \sum_{m=1}^{N_b} C_m(P_{G_m}) \\
\text{subj. to } p_G - p_D + j(q_G - q_D) = \text{diag}(v)(Yv)^* \\
|\text{Re}\{S_{mn}\}| \leq f_{mn}^{\max}; |S_{mn}| \leq S_{mn}^{\max} \\
V_m^{\min} \leq |V_m| \leq V_m^{\max} \\
p_G^{\min} \leq p_G \leq p_G^{\max}; q_G^{\min} \leq q_G \leq q_G^{\max}
\]

- Quadratic equality constraints \(\rightarrow\) nonconvex problem
- Traditional approaches rely on KKT conditions
SDP relaxation

$$\mathcal{V}_m \mathcal{I}_m^* = e_m^H v (Y v)^H e_m = \text{tr}[e_m^H v v^H Y^H e_m] = \text{tr}[Y^H e_m e_m^H v v^H]$$

- Nodal balance constraint linear in $V := v v^H$

$$P_{G_m} - P_{D_m} + j(Q_{G_m} - Q_{D_m}) = \text{tr}[Y^H e_m e_m^H V]$$

- Line flow and bus voltage constraints also linear in $V$

- AC-OPF with variables $p_G, q_G, V$ and additional constraints
  $$V \succeq 0 \quad \text{rank}[V] = 1$$

  Nonconvex $\rightarrow$ Drop

- Works in many practical OPF instances and IEEE benchmarks
- Optimal in tree graphs [Lam et al.'12]


AC OPF for multi-phase

- Power and voltage magnitude as linear functions of \( V := \mathbf{v} \mathbf{v}^H \)
- Regulating constraints per node \textit{and} per phase (can be unbalanced)

\[
\begin{align*}
\min_{\mathbf{V}, \mathbf{p}_G, \mathbf{q}_G} & \quad C(\mathbf{p}_G, \mathbf{q}_G) \\
\text{sub. to} & \quad \{P_{G,n}^\phi, Q_{G,n}^\phi\} \in \mathcal{B}_n^\phi, \text{ and } \forall \phi \ n \\
\text{Tr} (\mathbf{\Phi}_{n}^\phi \mathbf{V}) = & \quad P_{G,n}^\phi - P_{\ell,n}^\phi \\
\text{Tr} (\mathbf{\Psi}_{n}^\phi \mathbf{V}) = & \quad Q_{G,n}^\phi - Q_{\ell,n}^\phi \\
(V_{\text{min}}^\phi)^2 \leq & \quad \text{Tr} (\mathbf{M}_n^\phi \mathbf{V}) \leq (V_{\text{max}}^\phi)^2 \\
\mathbf{V} \geq & \quad 0, \quad \text{rank}(\mathbf{V}) = 1
\end{align*}
\]

- \( \text{rank}(\mathbf{V}^\text{opt}) = 1 \) \( \rightarrow \) globally optimal AC OPF solution!

Distributed three-phase OPF

- Multi-area based on non-convex OPF [Kim-Baldick’97, Hug-Andersson’09, Erseghe’14]

- Node-to-node for single-phase systems [Zhang et al’12]

- Distributed SDP for three-phase systems

  \[
  \begin{align*}
  \min_{\mathbf{V}, \{s_n\}} & \quad \sum_a C_a(\mathbf{V}^{(a)}, s^{(a)}) \\
  \text{subject to} & \quad \{\mathbf{V}^{(a)}, s^{(a)}\} \in \mathcal{B}^{(a)} \quad \forall a \\
  & \quad \mathbf{V} \succeq 0
  \end{align*}
  \]

  \textbf{Challenge:} PSD constraint couples local quantities!

  \textbf{Q:} \quad \mathbf{V} \succeq 0 \iff \{\mathbf{V}^{(a)} \succeq 0\}
**Topology-based decoupling**

**Result:** If the graph of areas is a tree, without “loops” across areas, then the centralized PSD constraint *decouples*

\[
\{ \mathbf{V}^{(a)}_{\text{opt}}, s_{\text{opt}}^{(a)} \} = \arg \min_{\{ \mathbf{V}^{(a)}_{\text{a}}, \{s^{(a)}_{\text{a}}\} } \sum_a C_a(\mathbf{V}^{(a)}, s^{(a)}) \\
\text{subject to } \{ \mathbf{V}^{(a)}_{\text{a}}, s^{(a)}_{\text{a}} \} \in \mathcal{B}^{(a)} \ \forall a \\
\mathbf{V}^{(a)}_{\text{a}} \succeq 0 \ \forall a \\
\mathbf{V}^{(a)}_{j} = \mathbf{V}^{(j)}_{\text{a}} \ \forall (a, j) \in \mathcal{E}_\mathcal{A}
\]

- Distributed solution via alternating direction method of multipliers
Illustrative test case

- **Consensus error**

- **Comparison with sub-gradient** [Zhang et al’12]

- Convergence rate does not depend on area size
Demand Response
Motivation for DR

- Changes in electricity consumption by end-users in response to
  - Changes in electricity prices over time
  - Incentive payments at times of high wholesale prices

- Benefits of DR
  - Reduced demand reduces the potential of forced outages
  - Lower demand holds down electricity prices in spot markets
  - Can reduce the amount of generation and transmission assets

- DR programs
  - Incentive-based programs
  - Price-driven programs

Cooperative DR

- Set of users (residences) \{1, \ldots, R\} served by the same utility
- Set of smart appliances \(A_r\) per user \(r\)
- Power consumption \(p_{ra}^t\)
- End-user utility function \(U_{ra}(p_{ra})\)

- Cost of power procurement for utility company

\[
C^t \left( \sum_{r=1}^{R} \sum_{a \in A_r} p_{ra}^t \right)
\]
**Motivation:** Reduce peak demand respecting preferences of users

- Convexity depends on $P_{ra}$
- Challenges
  - Scalable scheduling over AMI; and privacy issues
Solution approaches

- Gradient projection, block coordinate descent, dual decomposition
  [Chen et al.’12], [Mohsenian-Rad et al.’10], [Papavasiliou et al.’10],
  [Samadi et al.’11], [Gatsis-GG’12]

- **Dual decomposition:** Introduce variable $s^t$ for total supplied power

\[
\sum_{r=1}^{R} \sum_{a \in A_r} p_{ra}^t \leq s^t
\]

  Demand-supply balance

  - Lagrange multiplier $\lambda^t$ for supply-demand balance

- **Upshot**
  - Sub-problems for utility and smart meters are separated
  - Privacy respected
Distributed DR algorithm

- **Schedule update:** At the utility company and smart meters

\[ \min_{0 \leq s^t \leq s_{\text{max}}} \{ C^t(s^t) - \lambda^t(\ell) s^t \} \]

prices

\[ \sum_{a \in A} p_{ra}^t(\ell) \]

total hourly consumption

- **Multiplier update:** At utility company

\[ \lambda^t(\ell + 1) = \left[ \lambda^t(\ell) + \beta \left( \sum_{r=1}^{R} \sum_{a \in A} p_{ra}^t(\ell) - s^t(\ell) \right) \right]^+ \]
Lost AMI messages

- Messages in both ways may be lost
  - Not transmitted, due to failure
  - Not received, due to noise
  - Cyber-attacks

- Use the latest message available
- Convergence established for different lost-message patterns
  - Asynchronous subgradient method

- **Benefit:** Resilience to communication network outages

Plug-in Electric Vehicles
Plug-in electric vehicles

- PEVs feature batteries that can be plugged in
  - At end-user premises
  - At charging stations

- Benefits of high PEV penetration
  - Environmental: reduce carbon emissions
  - Economic: reduce dependency on oil

- Charging coordination is well motivated to avoid
  - Overloading of distribution networks [Clement-Nyns et al’10]
  - Creating new peaks
Charging coordination

- Fleet of vehicles $n = 1, \ldots, N$ to charge on top of baseload $D^t$
- Fraction of charge (rate) $r^t_n$ per slot $t = 1, \ldots, T$
  - Vehicle plugged in at different slots $0 \leq r^t_n \leq r^{t,\text{max}}_n$

- Centralized charging coordination

\[
\begin{align*}
\min_{\{r_n\}} \quad & \sum_{t=1}^{T} C^t \left( D^t + \sum_{n=1}^{N} r^t_n \right) \\
\text{subj.to} \quad & 0 \leq r^t_n \leq r^{t,\text{max}}_n \\
& \sum_{t=1}^{T} r^t_n = R_n \\
& n = 1, \ldots, N
\end{align*}
\]

- Convex and differentiable $C^t(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$, e.g., $C^t(\cdot) := (\cdot)^2$

Distributed PEV scheduling

- Relies on Frank-Wolfe (FW) optimization method

  - Identical per-vehicle partial gradients of the costs
    \[
    \nabla r_n \left( \sum_{t=1}^{T} C^t \left( D^t + \sum_{n=1}^{N} r_n^t \right) \right) = g, \quad n = 1, \ldots, N
    \n    \]

  - At iteration \(i\), vehicle \(n\) solves a linear program
    \[
    \hat{r}_n(i) \in \arg \min_{r_n \in \mathcal{E}_n} \hat{r}_n^T g(i)
    \]
    Solution: charge first small entries of \(g(i)\)

  - Vehicle \(n\) updates
    \[
    r_n(i + 1) = (1 - \eta(i)) r_n(i) + \eta(i) \hat{r}_n(i)
    \]
    \[
    \eta(i) := \frac{2}{i+2} \quad \text{or chosen via line-search}
    \]

Asynchronous updates

- To deal with process delays of EV controllers and/or commutation failures

(as1) Lost updates occur independently at random
(as2) Probability of a successful update larger than $\alpha$

- Guaranteed convergence with

$$\eta(i) := \frac{2}{\alpha i + 2}$$

- $\mathcal{O}(1/i)$ convergence rate in expectation

Numerical tests

- 51 out of 52 EVs are updated in an asynchronous setting

- Projected gradient descent (GD) and ADMM must project (expensive!)

- *Speed-up advantage* of FW thanks to simple updates
Take-home messages

- Distributed and robust PSSE
  - Non-convexity tackled via semidefinite relaxation
  - Decentralized estimation via ADMM
  - Sparse outlier models for robustness to "bad data"

- Distributed OPF
  - Semidefinite relaxation is tight for radial microgrids
  - ADMM solver for decentralized multiphase OPF

- Distributed DR
  - Decentralized management through dual decomposition
  - Resilience to lost AMI messages

- Distributed EV charging
  - Scalable and decentralized scheduler via Frank-Wolfe iteration
  - Robust to random communication outages

Thank you!