# Event-Based Control and Estimation

**Tongwen Chen**

Department of Electrical and Computer Engineering

University of Alberta, Canada

www.ece.ualberta.ca/~tchen

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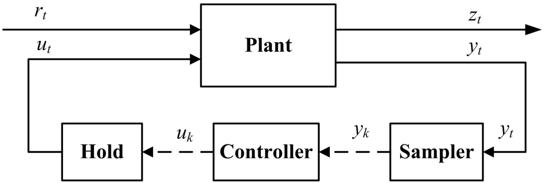

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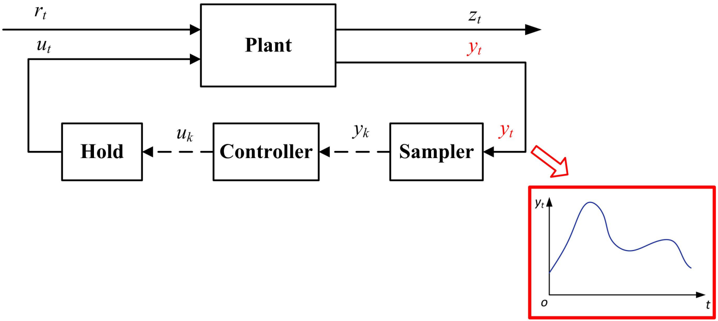

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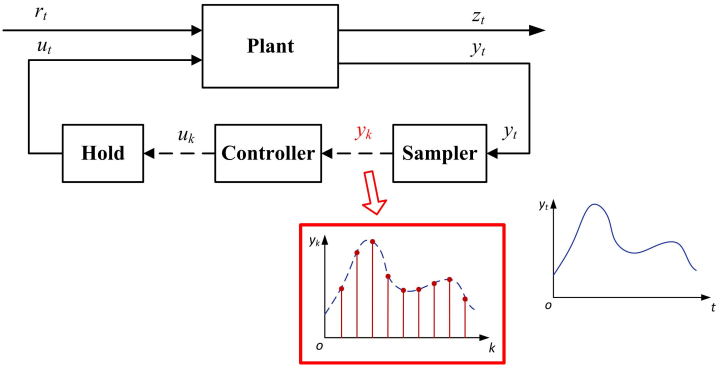

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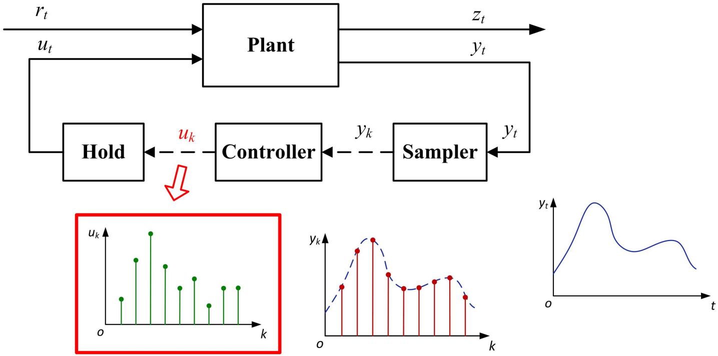

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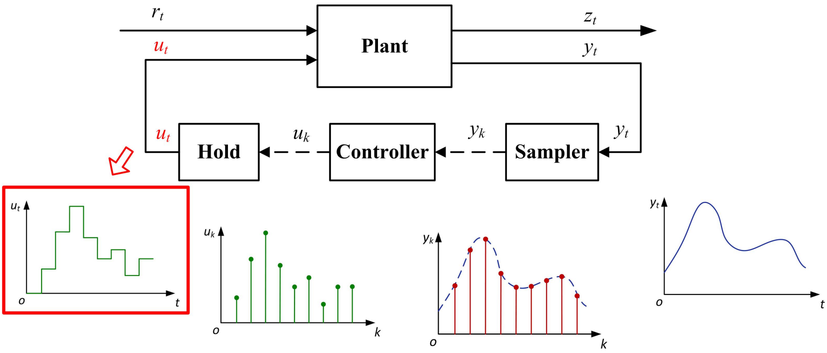

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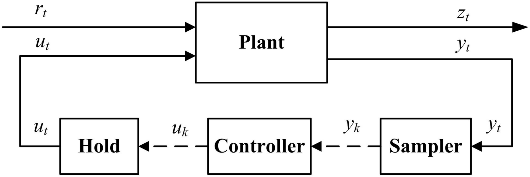

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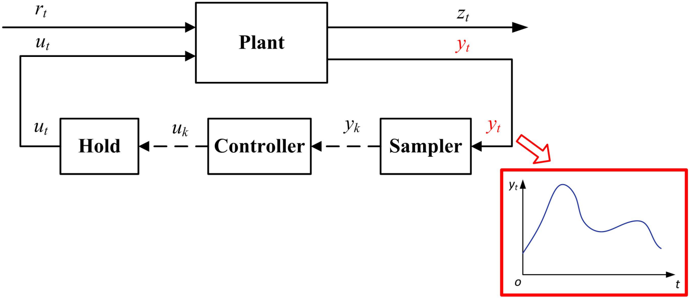

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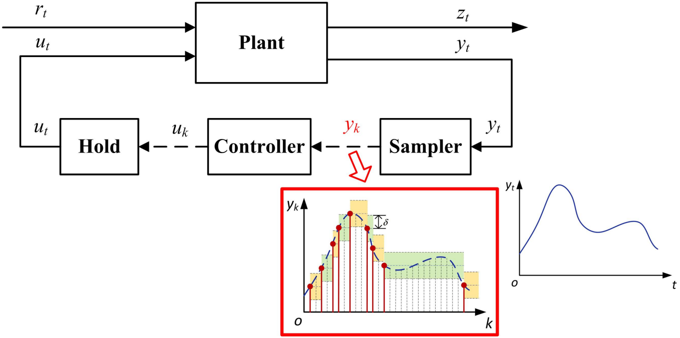

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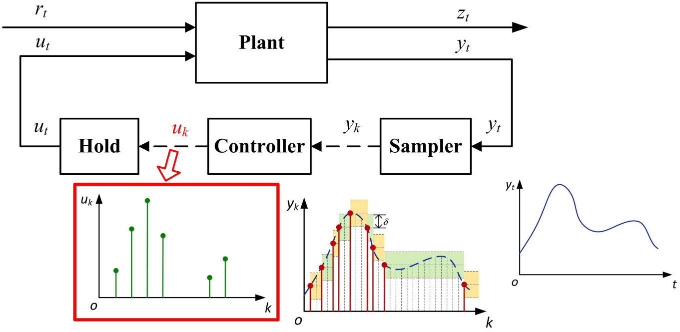

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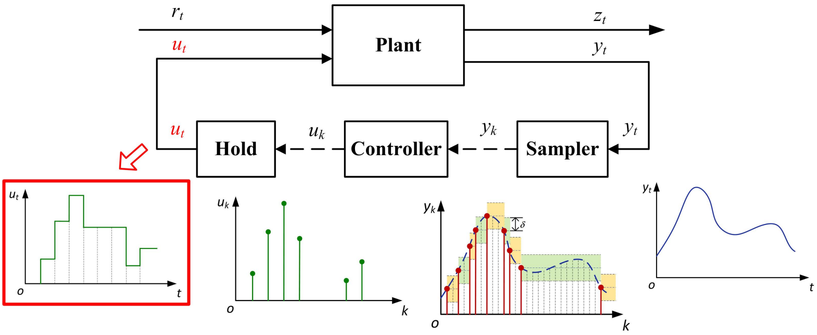

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**Periodic sampled-data design** **Event-based sampled-data design**

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- Good performance can be guaranteed for high average sampling rates
- Performance can be maintained at low sampling rates by appropriately designing the event-triggering schemes
- Relatively new

# Pioneering Work by Åström and Bernhardsson (CDC'02)

Scalar first-order process:

$$\mathrm{d}x = ax\mathrm{d}t + u\mathrm{d}t + \mathrm{d}w.$$

Impulsive control is updated whenever

$$|x(t)| \geq d$$

Asymptotic average variance:

$$V = \lim_{T\to\infty} \frac{1}{T} \int_0^T \mathbb{E}\left\{x^2(t)\right\} \mathrm{d}t$$

Comparison for $a = 0$ with the same average sampling period:

$$\frac{V_P}{V_E} = 3$$

# Event-triggering schemes

- Deterministic event-triggering conditions:

$$\gamma_k^i = \begin{cases} 0, & \text{if } y_k^i \in \Xi_k^i \\ 1, & \text{otherwise} \end{cases}$$

where $\Xi_k^i$ denotes the event-triggering set of sensor $i$ at time $k$.

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- Stochastic event-triggering conditions:

$$\gamma_k^i = \begin{cases} 0, & \text{if } \zeta_k^i \le \phi(y_k^i); \\ 1, & \text{otherwise,} \end{cases}$$

where $\zeta_k^i$ is a random variable with a uniform distribution over $[0, 1]$.

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"Send on delta" (Miskowicz, Sensors'06):

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Adaptive (Tabuada, TAC'07), (Mazo Jr. & Tabuada, TAC'11), (Wang & Lemmon, TAC'11), (Wang & Lemmon, Auto'11), (Dimarogonas, Frazzoli & Johansson, TAC'12):

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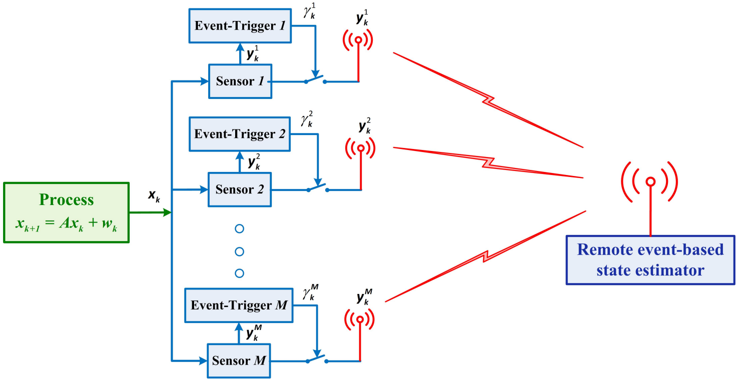
Model based (Lunze and Lehmann, Auto'10):

$$\|x(t_{k+1}) - x_m(t_{k+1})\| \leq e$$

# Event-based state estimation



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- Requirements on maintaining system performance at reduced communication cost

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  - § Communication rate analysis
  - § Tradeoff between performance and communication cost

# Event-based MMSE state estimation

- Linear Gaussian system:

$$x_{k+1} = Ax_k + w_k,$$
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- $w_k \sim \mathcal{N}(0, Q)$, $v_k^i \sim \mathcal{N}(0, R^i)$, $x_0 \sim \mathcal{N}(0, P_0)$. $x_0$, $w$ and $v^i$ are mutually uncorrelated.

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\gamma_k^i = \begin{cases} 0, & \text{if } y_k^i \in \Xi_k^i \\ 1, & \text{if } y_k^i \notin \Xi_k^i \end{cases} \tag{1}
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# Event-based MMSE state estimation

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$$s = [s_1, s_2, \ldots, s_M],$$

where $s_i \in \mathbb{N}_{1:M}$, $s_i \neq s_j$, unless $i = j$.

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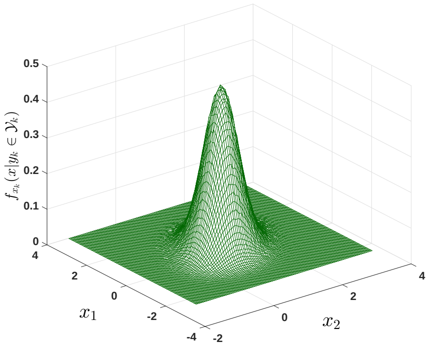
- For $i \in \mathbb{N}_{1:M}$, define

$$\mathcal{I}_k^i := \{\mathcal{Y}_1, \mathcal{Y}_2, ..., \mathcal{Y}_{k-1}, \{\mathcal{Y}_k^1, \mathcal{Y}_k^2, ..., \mathcal{Y}_k^i\}\} \tag{3}$$

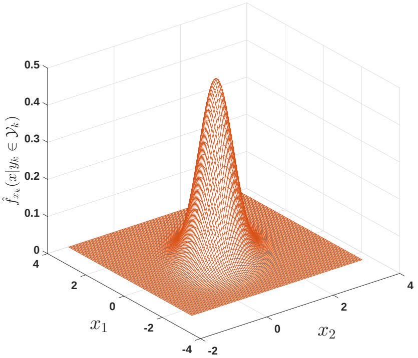
where $\mathcal{Y}_k := \{\mathcal{Y}_k^1, \mathcal{Y}_k^2, ..., \mathcal{Y}_k^M\}$.

# Gaussian Approximation

## Motivating observations:



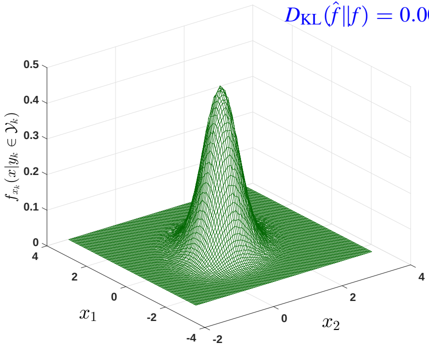

(a) Exact conditional distribution




(b) Approximate conditional distribution

## Motivating observations:

$D_{\mathrm{KL}}(\hat{f}\|f) = 0.0032$, $D_{\mathrm{KL}}(f\|\hat{f}) = 0.0033$.

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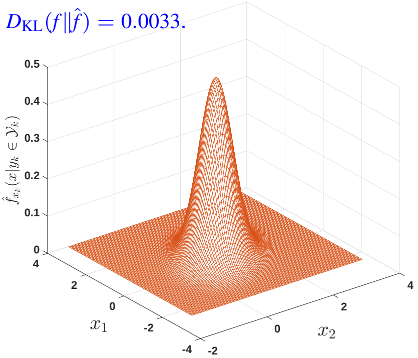


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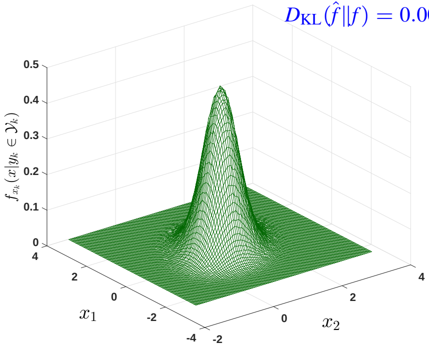


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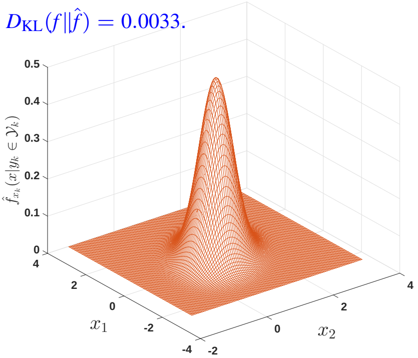


(f) Approximate conditional distribution

### Assumption

*The conditional distribution of $x_k$ on the event-triggered measurement information $\mathcal{I}_k^i$ is approximately Gaussian.*

# Results for general event-triggering conditions

## Theorem

1. The optimal prediction $\hat{x}_k^0$ of the state $x_k$ and the corresponding covariance $P_k^0$ are given by

$$\hat{x}_k^0 = A\hat{x}_{k-1}^M, \; P_k^0 = h(P_{k-1}^M).$$

2. For $i \in \mathbb{N}_{0:M-1}$, the fusion of information from the $(i+1)$th sensor leads to the following recursive state estimation equations:

If $\gamma_k^{i+1} = 1$,

$$\hat{x}_k^{i+1} = \hat{x}_k^i + L_k^{i+1}(z_k^{i+1} - \bar{z}_k^{i+1|i}), \; P_k^{i+1} = \tilde{g}_{i+1}(P_k^i);$$

If $\gamma_k^{i+1} = 0$,

$$\hat{x}_k^{i+1} = \hat{x}_k^i + L_k^{i+1}(\bar{z}_k^{i+1|i+1} - \bar{z}_k^{i+1|i}),$$
$$P_k^{i+1} = \tilde{g}_{i+1}(P_k^i) + L_k^{i+1}\mathrm{Cov}(z_k^{i+1}|\mathcal{I}_k^{i+1})(L_k^{i+1})^\top,$$

where $h(\cdot)$ and $\tilde{g}_{i+1}(\cdot)$ are the Lyapunov and Riccati operators, $\bar{z}_k^{i+1|i} := C^{i+1}(\hat{x}_k^i - \hat{x}_k^0)$, $\bar{z}_k^{i+1|i+1} := \mathrm{E}(z_k^{i+1}|\mathcal{I}_k^{i+1})$, and $L_k^{i+1} := P_k^i(C^{i+1})^\top[C^{i+1}P_k^i(C^{i+1})^\top + R^{i+1}]^{-1}$.

# Special case for single-channel sensors

- If $\gamma_k^{i+1} = 1$,
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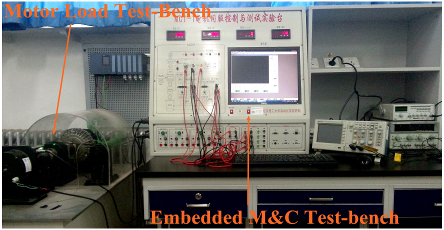
- If $\gamma_k^{i+1} = 0$,
$$\hat{x}_k^{i+1} = \hat{x}_k^i + L_k^{i+1}\hat{z}_k^{i+1},\quad P_k^{i+1} = \tilde{g}_{s_{i+1}}(P_k^i, \vartheta_k^{i+1}),$$

$$\hat{z}_k^{i+1} = \left[\phi\left(\frac{a_k^{i+1} - \bar{z}_k^{i+1|i}}{\Psi_{z_k^{i+1}}^{1/2}}\right) - \phi\left(\frac{b_k^{i+1} - \bar{z}_k^{i+1|i}}{\Psi_{z_k^{i+1}}^{1/2}}\right)\right]\Psi_{z_k^{i+1}}^{1/2} \Bigg/ \left[\mathcal{Q}\left(\frac{a_k^{i+1} - \bar{z}_k^{i+1|i}}{\Psi_{z_k^{i+1}}^{1/2}}\right) - \mathcal{Q}\left(\frac{b_k^{i+1} - \bar{z}_k^{i+1|i}}{\Psi_{z_k^{i+1}}^{1/2}}\right)\right],$$

$$\vartheta_k^{i+1} = \frac{\left(\hat{z}_k^{i+1}\right)^2}{\Psi_{z_k^{i+1}}} - \frac{\frac{a_k^{i+1} - \bar{z}_k^{i+1|i}}{\Psi_{z_k^{i+1}}^{1/2}}\phi\left(\frac{a_k^{i+1} - \bar{z}_k^{i+1|i}}{\Psi_{z_k^{i+1}}^{1/2}}\right) - \frac{b_k^{i+1} - \bar{z}_k^{i+1|i}}{\Psi_{z_k^{i+1}}^{1/2}}\phi\left(\frac{b_k^{i+1} - \bar{z}_k^{i+1|i}}{\Psi_{z_k^{i+1}}^{1/2}}\right)}{\mathcal{Q}\left(\frac{a_k^{i+1} - \bar{z}_k^{i+1|i}}{\Psi_{z_k^{i+1}}^{1/2}}\right) - \mathcal{Q}\left(\frac{b_k^{i+1} - \bar{z}_k^{i+1|i}}{\Psi_{z_k^{i+1}}^{1/2}}\right)},$$

with event-triggering sets $\Xi_k^{i+1} := [a_k^{i+1},\ b_k^{i+1}]$, $\Psi_{z_k^{i+1}} := C^{i+1}P_k^i(C^{i+1})^\top + R^{i+1}$, $\phi(z) := \frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}z^2)$ and $\mathcal{Q}(\cdot)$ being the standard Q-function.

# Event-based state estimation: An experimental case study

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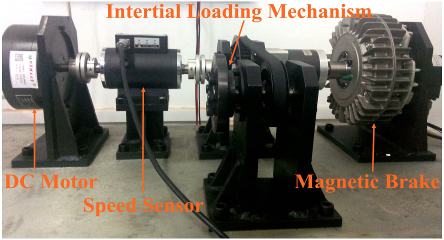


(b) DC motor system architecture

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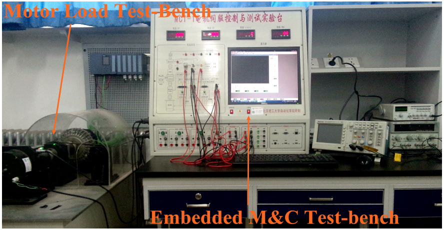


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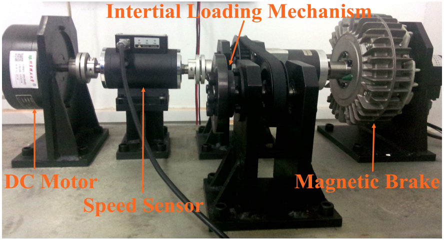


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- Event-triggered data transmission scheme (send-on-delta):

$$\gamma_k = \begin{cases} 0, & \text{if } \|y_k - y_{\tau_k}\|_2 \leq \delta; \\ 1, & \text{otherwise.} \end{cases}$$

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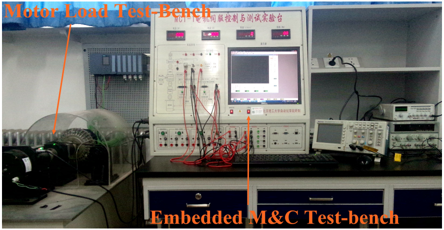


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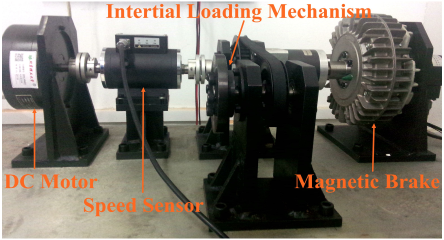


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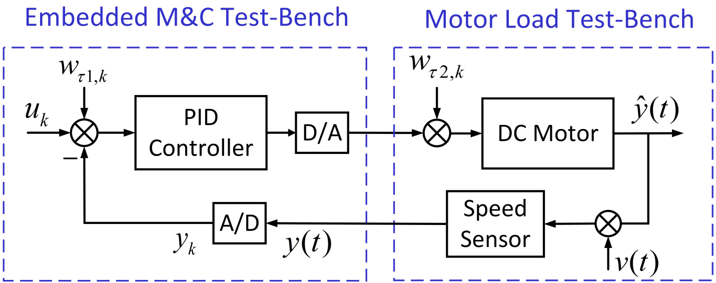
- Goal: To estimate the actual speed of the motor using event-triggered and noisy speed measurement information $\mathcal{I}_k$:

$$\mathcal{I}_k := \{(\gamma_0, \gamma_0 \mathbf{y}_0), (\gamma_1, \gamma_1 \mathbf{y}_1), \ldots, (\gamma_k, \gamma_k \mathbf{y}_k)\}.$$

## Modeling: Identifying a continuous-time model




- Model structure:

$$G(s) = \left( \frac{k_1}{T_1 s + 1} + \frac{k_2}{s^2/w_n^2 + 2\zeta s/w_n + 1} \right) \times e^{-ls};$$

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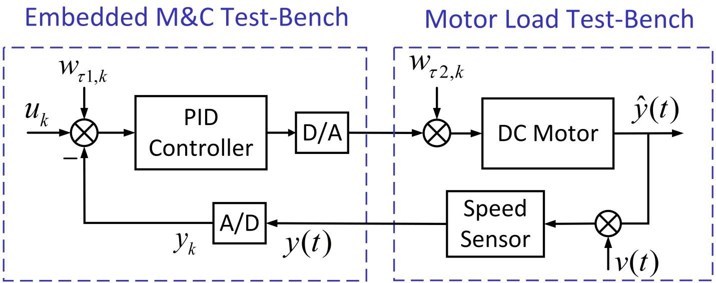


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- Identified model:

$$G(s) = \left( \frac{0.415}{1.937s + 1} + \frac{0.565}{s^2/12 + 0.15s + 1} \right) \times e^{-0.28s}.$$

## Modeling: Discretization and state-space realization

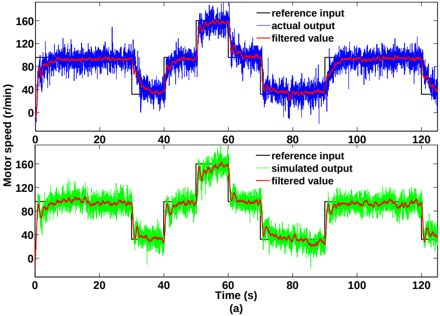
- Sampling time $h = 0.02\text{s}$

## Modeling: Discretization and state-space realization

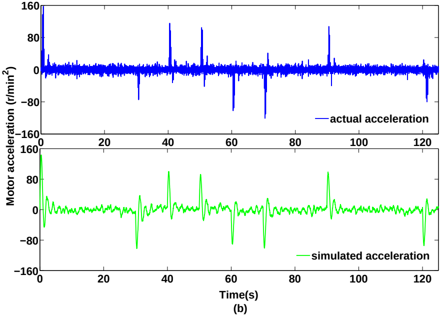
- Sampling time $h = 0.02\text{s}$
- Discrete-time state-space model:

$$
\begin{aligned}
x_{k+1} &= \begin{bmatrix} 1 & 0.01998 & 0.0002 \\ -0.0012 & 0.9974 & 0.0195 \\ -0.121 & -0.2537 & 0.9522 \end{bmatrix} x_k + \begin{bmatrix} 0.001374 \\ 0.13668 \\ -0.213 \end{bmatrix} u_k + w_k; \\
y_k &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x_k + v_k; \\
\text{Cov}(w_k) &= \begin{bmatrix} 1.5 \times 10^{-4} & 0 & 0 \\ 0 & 2 \times 10^{-4} & 0 \\ 0 & 0 & 2.4 \times 10^{-4} \end{bmatrix}, \\
\text{Cov}(v_k) &= 0.004.
\end{aligned}
$$

## Model validation




(a) Speed validation




(b) Acceleration validation

## Event-based MMSE estimator:

$$\hat{x}_k^- = A\hat{x}_{k-1},$$
$$P_k^- = AP_{k-1}A^\top + Q.$$
$$\hat{x}_k = \hat{x}_k^- + P_k^- C^\top (CP_k^- C^\top + R)^{-1} CP_k^- \left[\gamma_k (y_k - C\hat{x}_k^-) + (1-\gamma_k)\hat{z}_k\right],$$
$$P_k = P_k^- - \left[\gamma_k + (1-\gamma_k)\vartheta_k\right] P_k^- C^\top (CP_k^- C^\top + R)^{-1} CP_k^-$$

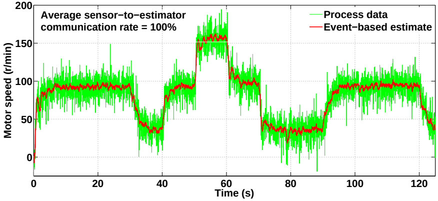
where $\hat{z}_k$ and $\vartheta_k$ are defined as

$$\hat{z}_k = \frac{\phi\left(\frac{a_k}{\Psi_{z_k}^{1/2}}\right) - \phi\left(\frac{b_k}{\Psi_{z_k}^{1/2}}\right)}{\mathcal{Q}\left(\frac{a_k}{\Psi_{z_k}^{1/2}}\right) - \mathcal{Q}\left(\frac{b_k}{\Psi_{z_k}^{1/2}}\right)} \Psi_{z_k}^{1/2}, \quad \vartheta_k = \left[\frac{\phi\left(\frac{a_k}{\Psi_{z_k}^{1/2}}\right) - \phi\left(\frac{b_k}{\Psi_{z_k}^{1/2}}\right)}{\mathcal{Q}\left(\frac{a_k}{\Psi_{z_k}^{1/2}}\right) - \mathcal{Q}\left(\frac{b_k}{\Psi_{z_k}^{1/2}}\right)}\right]^2 - \frac{\frac{a_k}{\Psi_{z_k}^{1/2}}\phi\left(\frac{a_k}{\Psi_{z_k}^{1/2}}\right) - \frac{b_k}{\Psi_{z_k}^{1/2}}\phi\left(\frac{b_k}{\Psi_{z_k}^{1/2}}\right)}{\mathcal{Q}\left(\frac{a_k}{\Psi_{z_k}^{1/2}}\right) - \mathcal{Q}\left(\frac{b_k}{\Psi_{z_k}^{1/2}}\right)},$$

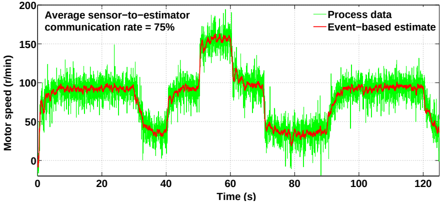
with $a_k$ and $b_k$ denoting the upper and lower limits of the event-triggering set, $\Psi_{z_k} := CP_k^- C^\top + R$, $\phi(z) := \frac{1}{\sqrt{2\pi}}\exp(-\frac{1}{2}z^2)$ and $\mathcal{Q}(\cdot)$ being the standard Q-function.

[1] D. Shi, T. Chen, and L. Shi, "An event-triggered approach to state estimation with multiple point-and set-valued measurements," Automatica, 50(6), pp. 1641-1648, 2014.

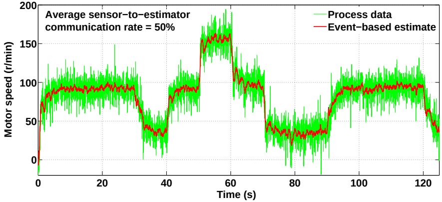
## Estimation performance for different measurement transmission rates



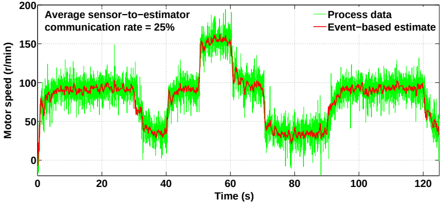

(a) Average communication rate $= 100\%$




(b) Average communication rate $= 75\%$

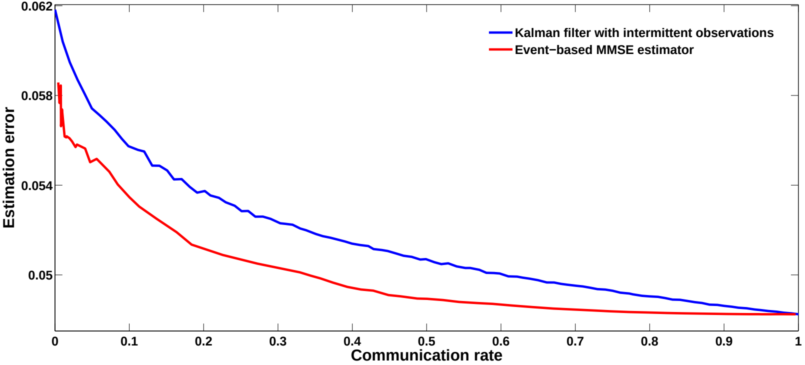



(c) Average communication rate $= 50\%$




(d) Average communication rate $= 25\%$

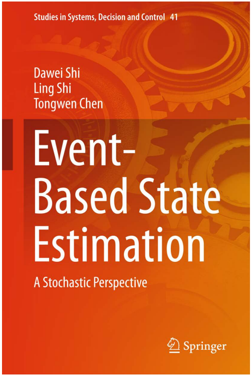
# Event-based state estimation: An experimental case study




Tradeoff between estimation error and average transmission rate

# New book on event-based state estimation

Dawei Shi, Ling Shi and Tongwen Chen, "Event-Based State Estimation: A Stochastic Perspective," Springer, 2016.



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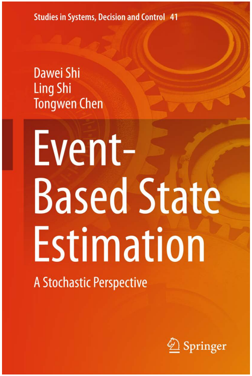


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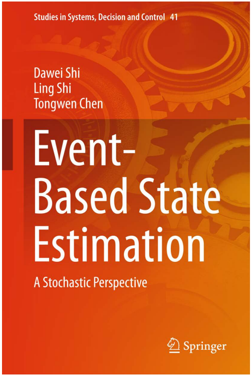


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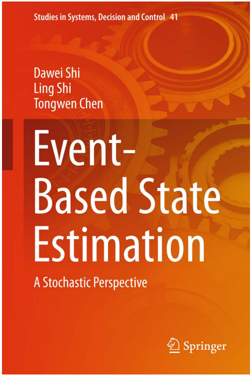


- The first book covering recent research developments on event-based state estimation and related methodologies.
- An up-to-date literature review is included in the emerging new area of event-triggered systems and state estimation.
- Extensive illustrative examples are provided to help understand and master the new concepts and techniques.

# Acknowledgment

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Thank you!

NSERC CRSNG

People. Discovery. Innovation.