Jan 18, 16

1 MATH110. Week 3

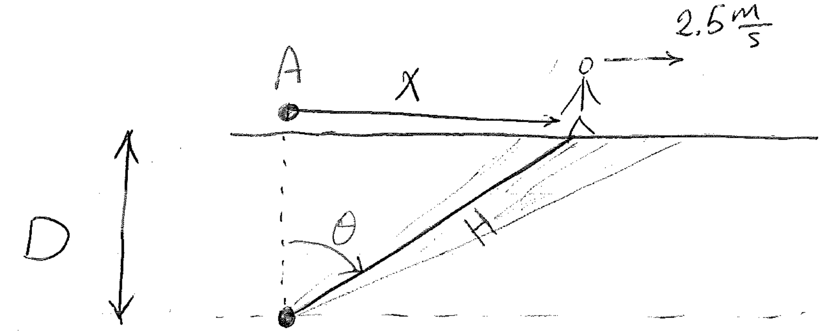
We will be having a quiz on Fri (15 min. at the end of the class)

This week: finish related rates
talk about critical points/min/max

Midterm on Feb 10, 6pm

Example: A man walks along a straight path at a speed of $2.5\ \frac{m}{s}$. A searchlight is located on the ground 15m from the path, and is kept focused on the man. At what rate is the searchlight rotating when the man is 20m from the point on the path closest to the searchlight?

Step 0:



A is the point on the path closest to the searchlight

Step 1: $D = 15\ m.$ $\frac{dx}{dt} = 2.5\ \frac{m}{s}$

Step 2: At what rate is the searchlight <u>rotating</u>?

rotating $\Rightarrow$ think of the rate of change of an angle w/ time.

$$\Rightarrow \frac{d\theta}{dt}$$

Step 3: Can we write an eq. w/ x and θ in there?

a) Similar triangles $\Rightarrow$ I only see 1 triangle here.
Similar triangles give relations between leng

b) Pythagorean theorem $\Rightarrow$ $D^2 + x^2 = H^2$ no θ!

c) Trig. functions. $\Rightarrow$ $\tan\theta = \frac{x}{D}$

Step 4: x and θ are changing w/ time $\Rightarrow$ $x(t)$, $\theta(t)$
D doesn't change w/ time $\Rightarrow$ $\frac{dD}{dt} = 0$

Step 5: $\frac{d}{dt}\left(\tan(\theta)\right) = \frac{d}{dt}\left(\frac{x}{D}\right)$

Q: diff and Simplify the above

a) $\frac{1}{\cos^2(\theta)} = \frac{x'}{D'}$ b) $\frac{1}{\cos^2(\theta)} = \frac{x'}{D}$

(c) $\frac{\theta'}{\ldots} = \frac{x'}{\ldots}$ d) $\frac{\theta'}{\ldots} = \frac{x' * D - x * D'}{\ldots}$

MATH 110
Jan 18, 16

$$\frac{d}{dt}\left(\tan(\theta)\right) = \frac{d}{dt}\left(\frac{x(t)}{D}\right)$$

$$\frac{d\tan(\theta)}{d\theta} * \frac{d\theta}{dt} = \frac{1}{D}\frac{dx}{dt}$$

$$\frac{1}{\cos^2(\theta)}\frac{d\theta}{dt} = \frac{1}{D}\frac{dx}{dt} \quad \text{OR} \quad \frac{\theta'}{\cos^2(\theta)} = \frac{x'}{D}$$

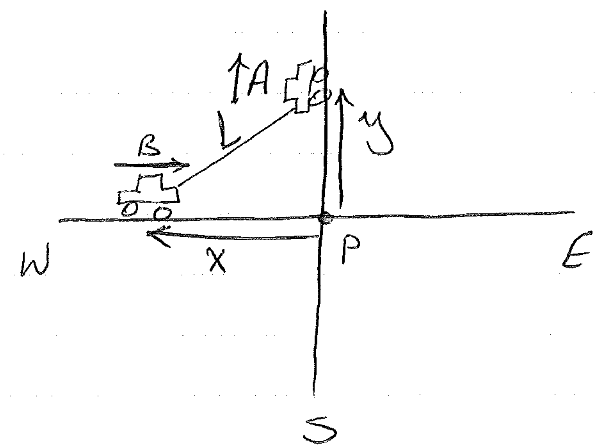
$$\Longrightarrow \quad \frac{d\theta}{dt} = \frac{\cos^2(\theta)}{D} * \frac{dx}{dt}$$

Step 6:

$$\frac{d\theta}{dt} = \frac{\frac{D}{\sqrt{D^2+x^2}}}{\frac{D}{1}} * \frac{dx}{dt} = \frac{\cancel{D}}{\cancel{D} * \sqrt{D^2+x^2}}\frac{dx}{dt}$$

$$= \frac{1}{\sqrt{\underbrace{15^2}_{(3*5)^2} + \underbrace{20^2}_{(4*5)^2}}} * 2.5 = \frac{1}{5*5} * 2.5 = 0.1 \ \frac{\text{Rad}}{s}$$

Example: A road running north to south crosses a road going east to west at the point p. Car A is driving north w/ constant speed of $60\frac{km}{h}$ and passed point p 1 hour ago. Car B is driving east at a constant speed of $80\frac{km}{h}$ and is 40 km to the west of p. How fast is the distance between the cars increasing an hour after car B has passed point p?



Step 0.

Step 1:

$$\frac{dy}{dt} = 60\frac{km}{h} \text{ : constant}$$

Now I have to pick the ref. time.
I pick the ref. time to be when A passed p. $y(0)=0$
~~...~~ t: time in hours since A passed p

$\frac{dx}{dt} = ?$
a) $80\frac{km}{h}$
b) $-80\frac{km}{h}$
c) none of the above
d) I don't know

$$\frac{dx}{dt} = -80\frac{km}{h} \text{ : constant}$$

$$X(1) = 40\,km$$

Step 2: $\frac{dL}{dt} = ?$

Step 3: $x^2 + y^2 = \ell^2$

Step 4: x, y and ℓ are changing w/ time

$\Rightarrow x(t), y(t), \ell(t)$

Step 5:

$$\frac{d}{dt}\left((x(t))^2 + (y(t))^2\right) = \frac{d}{dt}\left((\ell(t))^2\right)$$

$$\cancel{2}x(t)\frac{dx}{dt} + \cancel{2}y(t)\frac{dy}{dt} = \cancel{2}\ell(t)\frac{d\ell}{dt}$$

$$\frac{d\ell}{dt} = \frac{x}{\ell}\frac{dx}{dt} + \frac{y}{\ell}\frac{dy}{dt}$$

I should diff. with respect to ... ?

a) x
b) y
c) ℓ
d) t
e) I don't know

We need to find $\frac{d\ell}{dt}$ An HOUR after B has passed p...

$x(1) = 40$ km

$\frac{dx}{dt} = -80 \frac{km}{h}$ $\Rightarrow$

t	x
0	120
1	40
2	-40
3	-120

$x(t) = 40 - 80(t-1)$

$x(t^*) = 0 \Rightarrow$

$40 - 80(t^*-1) = 0$

$40 = 80(t^*-1)$

$\frac{1}{2} = t^* - 1$

$\frac{3}{2} = t^*$

Q) $x(t) = ?$

a) $40 - 80(t-1)$
b) $40 + 80(t-1)$
c) $40 - 80t$
d) $40 + 80t$
e) $80t$

t^*: When B passes point p

Now I know that I need to find $\frac{dl}{dt}$ at $t = 1.5 + 1 \mid t = 2.5h$

$$x(t) = 40 - 80(t-1)$$

$$x(2.5) = 40 - 80(2.5-1) = 40 - 80 * 1.5 = 40 - 120 = -80 \text{ k}$$

$y(0) = 0$

$\frac{dy}{dt} = 60 \frac{km}{h}$ $\Rightarrow$

t	y
0	0
1	60
2	120
3	180

$y(t) = 60t$

$y(2.5) = 60 * 2.5 = 150 \text{ km}$

$$\cancel{l} \ (l(2.5))^2 = (x(2.5))^2 + (y(2.5))^2$$

$$l(2.5) = \sqrt{(-80)^2 + (150)^2} \text{ km}$$

$$\frac{dl}{dt} = \frac{-80 \text{ km}}{\sqrt{80^2 + 150^2} \text{ km}} * \left(-80 \frac{km}{h}\right) + \frac{150 \text{ km}}{\sqrt{80^2 + 150^2} \text{ km}} * 60 \frac{km}{h}$$

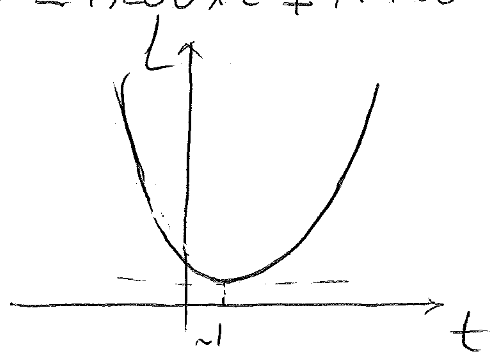
(7) Jan 18-22, 16
Week 3

Ok .... We've found that $x^2+y^2=L^2$. You can imagine at some point in time the distance betwee A and B is minimized / minimum. right? How can I figure out when that happens? or what's the min value of L?

We can evaluate L at many many points in time ...
or we can try to graph L ...
Calculus provides ways of significantly narrowing the number of points we need to examine

$$L^2 = x^2+y^2 = (40-80(t-1))^2 + (60t)^2$$

$$= 10000\,t^2 - 19200 * t + 14400 \qquad \frac{dL}{dt}=0 \Rightarrow t=0.96$$

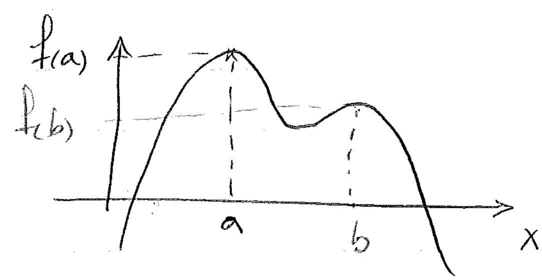


let's say I'm looking for all min/max of a function. How can I use calc. to do this?

Jan 18-12, 2016
Week 3

Now before we think more about this let's define min / max

$\boxed{\text{Definition}}$ f has a maximum or global maximum at a if $f(a) \geq f(x)$ for all x in the domain of f. The maximum of f is $f(a)$ and this maximum of f occurs at a.



what is b?

$\boxed{\text{Definition}}$ f has a local or relative maximum at a if $f(a) \geq f(x)$ for all x near a or in some open interval which contains a.

$\boxed{\text{Definition}}$ f has a global extreme at a if $f(a)$ is a global min or max.
f has a local extreme at a if $f(a)$ is a local min or max.

USE THE SLIDE with Min/Max

Wed. class ended here

Fri. Jan 22, 2016

New written assignment is available on section website

- due on Fri. Jan 29, 2016 at the beginning of the class
- Make sure to staple ur solution
- Staple them together in order

No calculator or phone on ur desk during quiz

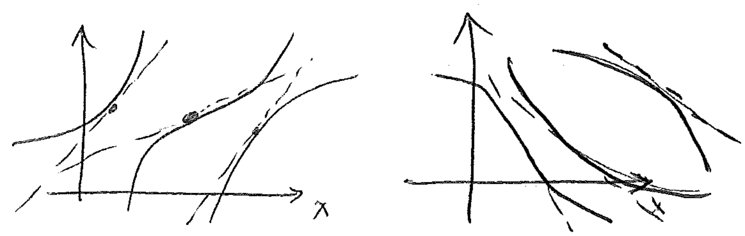
Last time we defined local and global extrema.

USE THE SLIDE

- Note that end points are not local extrema
- Don't forget that I have not labeled one of the extrema
- Notice that where $\frac{dy}{dx}$ DNE we may or may not have an extrema

MATH 110
Jan 18-22, 2016

Theorem if $f'(a) > 0$ or $f'(a) < 0$ the $f(a)$ is not a local min or max.



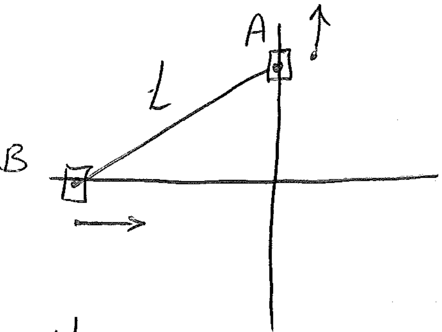
Theorem: If f is defined on an open interval and $f(a)$ is a local extreme of f, then either $f'(a) = 0$ or f is not differentiable at a.

So when we are looking for Min/Max we should find the points where f is not diff. or $f'(a) = 0$

Definition: A Critical number for a function f is a value $x = a$ in the domain of f such that

i) $f'(a) = 0$
or
ii) f is not differentiable
or
iii) a is an endpoint

~~Example~~ let's go back to our last problem on related rates:



$$L(t) = \sqrt{10000t^2 - 19200t + 14400}$$

let's find the critical numbers of L

$\Rightarrow$ find points where $\frac{dL}{dt} = 0$ or $\frac{dL}{dt}$ DNE

$\frac{dL}{dt} = 2 * 10000t - 19200$ this is a linear function and it is defined everywhere in $\mathbb{R}$

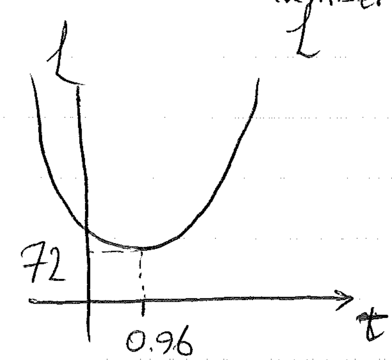
Reminder: polynomials are differentiable everywhere on $\mathbb{R}$

$\frac{dL}{dt} = 0 \Rightarrow 20000t - 19200 = 0$

$\Rightarrow 20000t = 19200$

$t = \frac{19200}{20000} = \frac{192}{200} = \frac{96}{100} = 0.96$ s : the only critical number of L

$L(0.96) = 72$ km



Example: Find the critical numbers of $y = e^{4x-2} - x$

chain rule $\quad u = 4x-2$

$$\frac{d}{dx}\left(e^{4x-2} - x\right) = \frac{d}{dx}\left(e^{4x-2}\right) - \frac{d}{dx}(x) = \frac{de^u}{du} * \frac{du}{dx} - 1$$

$$= e^u * \frac{d(4x-2)}{dx} - 1 = 4 * e^{4x-2} - 1$$

$\frac{dy}{dx}$ exists every where

$$\frac{dy}{dx} = 0 \Rightarrow 4e^{4x-2} - 1 = 0 \Rightarrow 4e^{4x-2} = 1$$

$$\Rightarrow e^{4x-2} = \frac{1}{4} \Rightarrow \ln\left(e^{4x-2}\right) = \ln\left(\frac{1}{4}\right)$$

$$\Rightarrow 4x - 2 = -\ln 4 \Rightarrow 4x = 2 - \ln 4$$

$$\Rightarrow x = \frac{2}{4} - \frac{\ln 4}{4} \quad : \text{critical point of } y$$

Fri class ended here