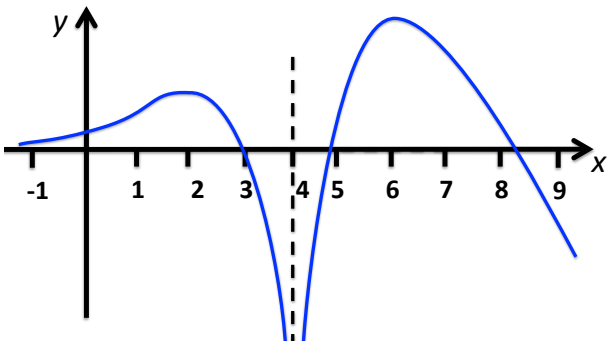
Assignment 3

Due Feb. 26, 2016

Problem 1.

The graph of function f is shown. Sketch the graphs of f' and f''.



Problem 2.

a. Sketch a possible graph of a function f that satisfies the following conditions and is differentiable for all x:

i. Domain of f is $0 < x < 12$

ii. f' is increasing on $(0, 3)$

iii. $f'' < 0$ on $(3, 6)$

iv. f is concave up on $(6, 9)$

v. f has an inflection point at $x = 9$

b. On your graph, find and label the local minimums and maximums.

c. Sketch a possible graph of a function f that satisfies the conditions given in part **a** and is differentiable for all x in its domain, except $x = 9$ where f is not continuous and $\lim_{x\to 9^+} f' = \lim_{x\to 9^-} f'$.

d. Sketch a possible graph of a function f that satisfies the conditions given in part **a** and is differentiable for all x in its domain, except $x = 9$ where f is continuous but not differentiable.

e. Sketch a possible graph of a function f that satisfies the conditions given in part **a** and is differentiable for all x in its domain, except $x = 9$ where f is not continuous but has a maximum and $\lim_{x\to 9^+} f' = \lim_{x\to 9^-} f'$.

f. Sketch a possible graph of a function f that satisfies the conditions given in part **a** and is differentiable for all x in its domain, except $x = 9$ where f is not continuous, $\lim_{x\to 9^+} f'$ is not defined, but $\lim_{x\to 9^-} f'$ is defined.

Problem 3.

Show that if $f(x)$ is increasing and concave up on $[a, b]$, then $f((a+b)/2) < \dfrac{f(a)+f(b)}{2}$. *Hint: Use the mean value theorem on the function f on the intervals $(a, (a+b)/2)$ and $((a+b)/2, b)$.*