Feb 1–5, 201

Week 5

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Reminder: Quiz on Fri. on MVT, critical points

This week: 1st derivative test (Ref book: sec. 3.3)

Note: lecture notes are available on section website

Recap: f differentiable ~~on~~ for $a<x<b$

and $\begin{cases} \text{increasing} \Rightarrow f' \geq 0 \\ \text{decreasing} \Rightarrow f' \leq 0 \\ \text{constant} \Rightarrow f' = 0 \end{cases}$

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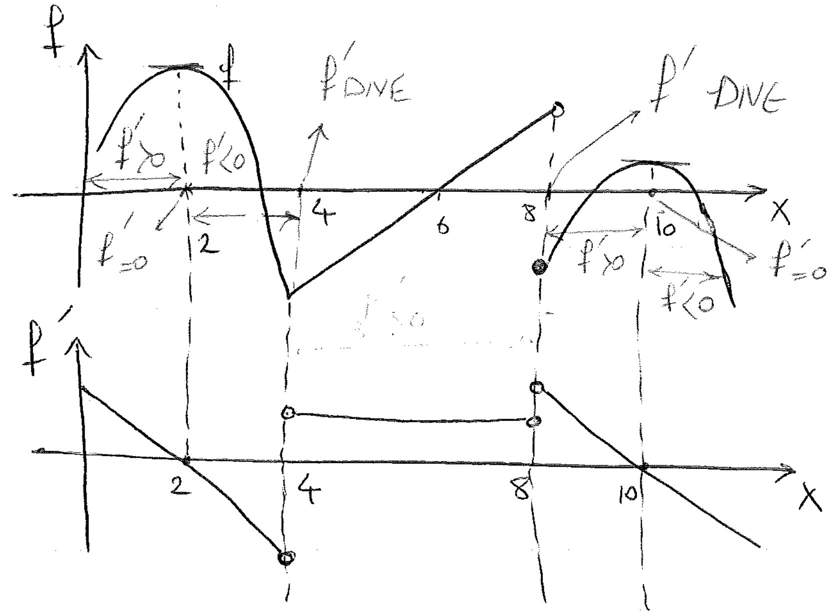
Now can you sketch the graph of f' given the graph of f



Critical numbers:
2, 4, 8, 10

How about sketching f given the graph of f'?

$\boxed{\text{Theorem}}$ 2^{nd} shape theorem

For a function f which is differentiable on an interval I

i) if $f'(x) > 0$ for all x in the interval I, then f is increasing on I

ii) if $f'(x) < 0$ for all x in the interval I, then f is decreasing on I

iii) if $f'(x) = 0$ for all x in the interval I, then f is constant on I.

Proof on page 12

Example: Use information about f' to help graph $f(x) = \frac{x^3}{3} - 3x^2 + 5x - 1$

$$\frac{df}{dx} = x^2 - 6x + 5 \qquad \frac{df}{dx} = 0 \Rightarrow x = \frac{6 \mp \sqrt{36 - 20}}{2} = \frac{6 \mp \sqrt{16}}{2}$$

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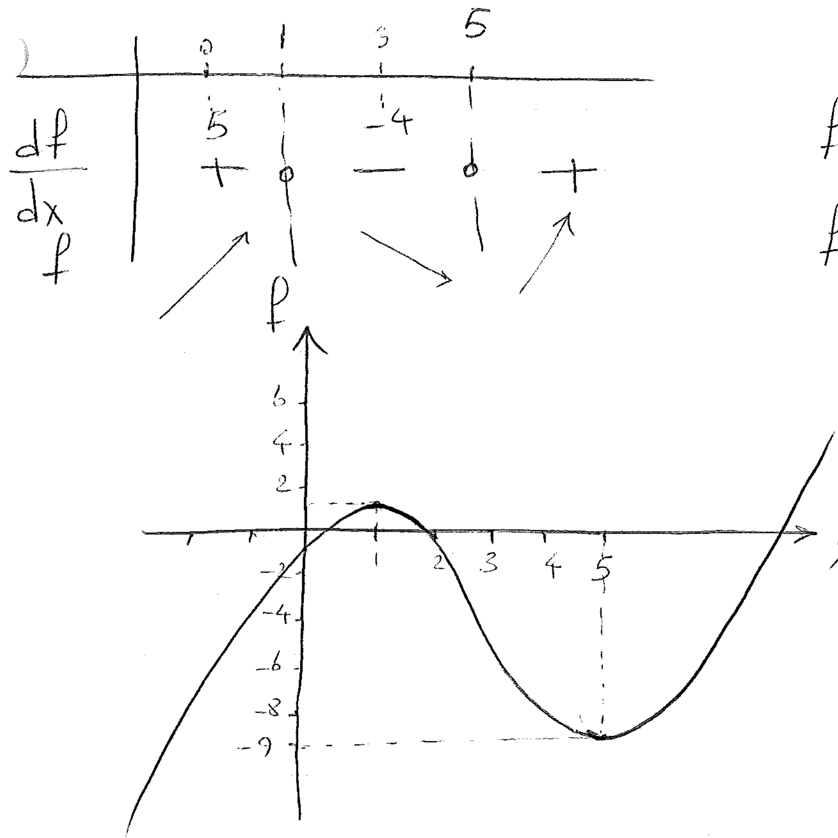


$$f(1) = \frac{1}{3} - 3 + 5 - 1 = \frac{4}{3}$$

$$f(5) = \frac{5^3}{3} - 3*25 + 5*5 - 1$$

$$= 5^2\left(\frac{5}{3} - 3 + 1\right) - 1$$

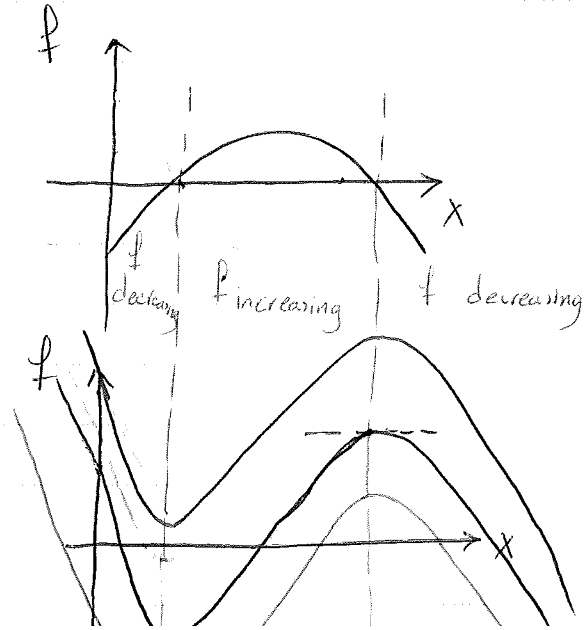
$$= 5^2\left(\frac{5-}{3}\right) - 1 = \frac{-25}{3} - 1$$

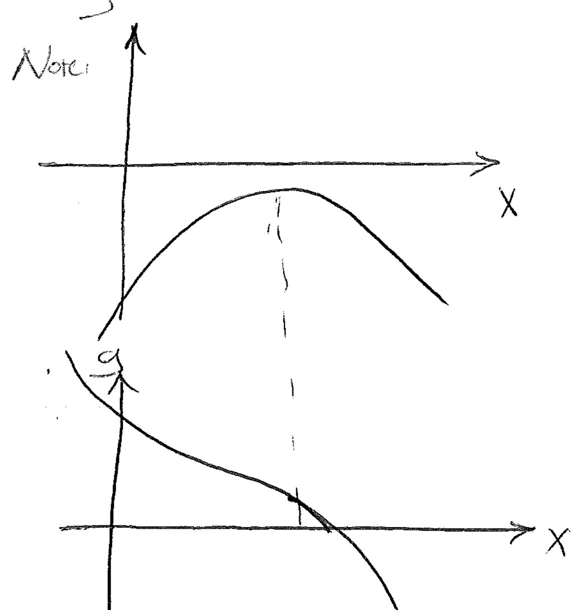
$$= -\frac{28}{3} \approx -9.$$

Example:

Does a graph of f' completely determine the graph of f?

i.e is your answer, f, unique? a) yes (b) no





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Proof of i: (Contradiction proof)

Let's assume that f is not increasing on I, then there exist $a < b$ in I so that $f(b) < f(a)$.

Since f' is defined for all x in I, f must be continuous for $a \leq x \leq b$ and differentiable for $a < x < b$.

So MTV states that there is at least one number c so that

$$f'(c) = \frac{f(b) - f(a)}{b - a}, \quad a < c < b$$

However, since $a < b$, we have $b - a > 0$. Also, $f(b) < f(a)$; therefore $f(b) - f(a) < 0$. So we have

$$f'(c) = \frac{f(b) - f(a)}{b - a} < 0$$

This contradicts the statement that $f' > 0$ for all x in I.

So the assumption that f is not increasing on I must be false.

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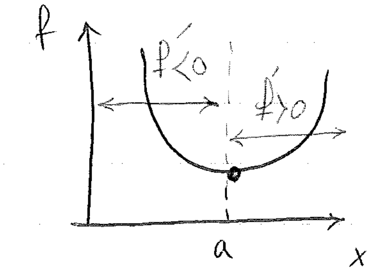
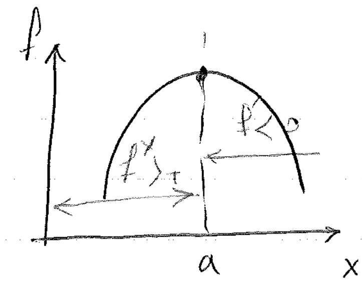
Wed. Feb 3,

Last time: if f diff on I then

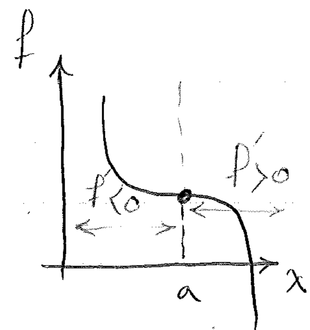
(f increasing on I $\Rightarrow$ $f' \geq 0$ on I)
(f decreasing on I $\Rightarrow$ $f' \leq 0$ on I)
(f is constant on I $\Rightarrow$ $f' = 0$ on I)



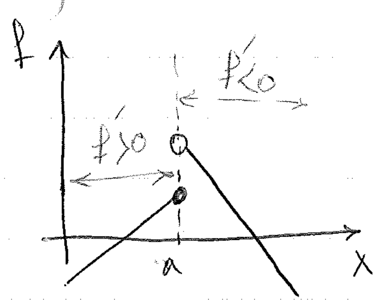
Min



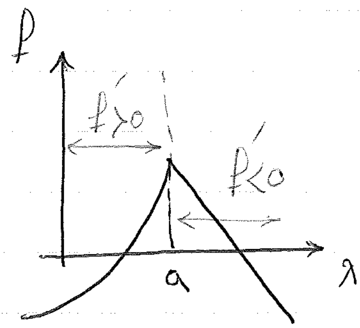
Max



neither.



neither



Max

Don't forget to email me the questions you want me to solve in the review. Wed. Feb 10th.

a) I never have had a question in class that I didn't ask
b) Sometimes I don't ask my questions
c) In most classes I have at least one question that I didn't ask

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Question : f is continuous on I and $f'(a)=0$, $a \in I$.

Assuming that $f'(x) < 0$ for $x < a$ and $f'(x) > 0$ for $x > a$,

Which statement is true ? ~~abou~~

a) f has a local min at a
b) f has a local max at a
c) f does not have a local min or max at a Mon. Class

What if $f'(a)$ ~~is~~ does not exist? a) b) c) Graded her

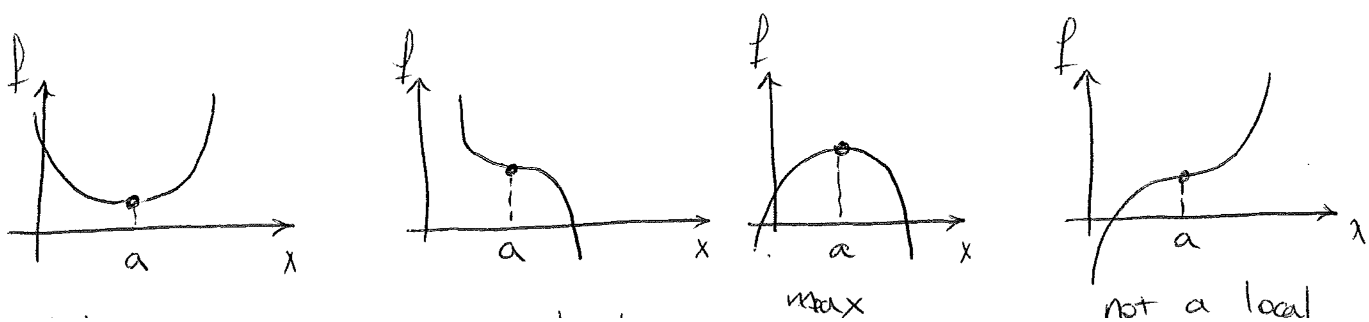
$\boxed{\text{first derivative test for local extrema}}$

Let f be a continuous function with $f'(a)=0$ or $f'(a)$ is undefined

i) if $f'(x \to a^-) < 0$ and $f'(x \to a^+) > 0$, then $(a, f(a))$ is a local minimum

ii) if $f'(x \to a^-) > 0$ and $f'(x \to a^+) < 0$, then $(a, f(a))$ is a local maximum

iii) if $f'(x)$ does not change sign at $x=a$, then a is not a local extremum.



Example: find all the local extrema of $f(x) = x^4 + x^3 - \frac{x^2}{2} + 1$ and use the 1^{st} derivative test to determine if they are maximum minimums or neither.

looking for local extrema $\rightarrow$ have to find critical points
$\rightarrow$ find numbers where $f' = 0$ or f' DNE

$$f'(x) = \frac{df}{dx} = 4x^3 + 3x^2 - \frac{2x}{2} = 4x^3 + 3x^2 - x$$

f' is a polynomial $\Rightarrow$ f' exists/is defined everywhere. So let's fin x so that $f'(x) = 0$

$$f'(x) = 0 \Rightarrow 4x^3 + 3x^2 - x = 0 \Rightarrow x(4x^2 + 3x - 1) = 0$$

$$\Rightarrow \begin{cases} x = 0 \Rightarrow x = 0 \text{ is a critical number} \\ 4x^2 + 3x - 1 = 0 \Rightarrow x = \frac{-3 \mp \sqrt{9+16}}{8} = \frac{-3 \mp \sqrt{25}}{8} = \frac{-3 \mp 5}{8} \end{cases}$$

$$\Rightarrow x = \begin{cases} \frac{-8}{8} = -1 \Rightarrow x = -1 \text{ is a critical number} \\ \frac{2}{8} = \frac{1}{4} \Rightarrow x = \frac{1}{4} \text{ is a critical number} \end{cases}$$

Easiest (not fastest) approach

		-1		0		$1/4$	
x	$-$		$-$		$+$		$+$
$4x^2+3x-1$	$+$	0	$-$		$-$	0	$+$

$$4x^2 + 3x - 1 = (x+1)\left(x - \frac{1}{4}\right)4$$

f has local min at $x = -1, 1/4$

Example:

Find all the local min and max of $f(x) = \dfrac{|x^2-4|}{x^3}$

Reminder: $|a| = a$ if $a \geqslant 0$

$|a| = -a$ if $a < 0$

before I can easily diff., I need to deal with the abs.

$x^2 - 4 = 0 \Rightarrow x = \mp 2$

	-2	0	2	
x^2-4	$+$	-4 $-$		$+$

OR

	$x<-2$	$-2<x<2$	$x>2$
$x-2$	$-$	$-$	$+$
$x+2$	$-$	$+$	$+$
x^2-4	$+$	$-$	$+$

So...

$$f(x) = \begin{cases} \dfrac{x^2-4}{x^3} & x \leqslant -2 \text{ or } x \geqslant 2 \\ \dfrac{-x^2+4}{x^3} & -2 < x < 2 \end{cases}$$

for $(x \leqslant -2$ or $x \geqslant 2)$

could also use quotient rule

$$f(x) = \frac{x^2-4}{x^3} \Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(\frac{x^2-4}{x^3}\right) = \frac{d}{dx}\left(\frac{1}{x} - \frac{4}{x^3}\right) = \frac{-1}{x^2} - \frac{4*(-3)}{x^4}$$

$$\Rightarrow \frac{df}{dx} = \frac{-x^2+12}{x^4}$$

$\frac{df}{dx} = 0 \Rightarrow x = \mp\sqrt{12}$

$\frac{df}{dx}$ DNE $\Rightarrow x = 0$

for $-2 < x < 2$

$$f(x) = \frac{-x^2+4}{x^3} \Rightarrow \frac{df}{dx} = \frac{d}{dx}\left(-\frac{x^2-4}{x^3}\right) = -\frac{d}{dx}\left(\frac{x^2-4}{x^3}\right) = -\frac{-x^2+12}{x^4} = \frac{x^2-12}{x^4}$$

Fri. class started here

What numbers should I be careful about? $x = 0, \mp 2, \mp\sqrt{12}$

Note $x^4 \geq 0$

		$-\sqrt{12}$		-2		0		2		$\sqrt{12}$	
x^2-12	$+$		$-$		$-$		$-$		$-$		$+$
f'	$-$		$+$		$-$		$-$		$+$		$-$
f	$\searrow$		$\nearrow$		$\searrow$		$\searrow$		$\nearrow$		$\searrow$

local min: $x = -\sqrt{12}, 2$

local max: $x = -2, \sqrt{12}$

Is $x=0$ a critical point of f? no. $x=0$ is not in the domain

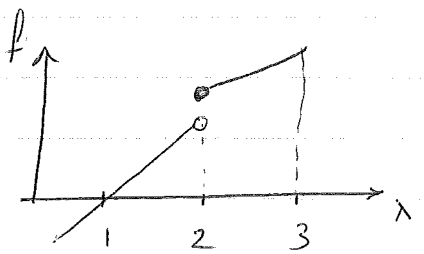
a) yes

b) no

Question : a) True OR b) False

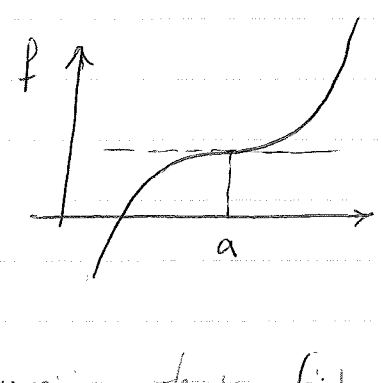
1). if f is increasing on an interval then $f'(x) \geq 0$ for all x in I

Nope. f may not be diff. for all x in I False



2) if f is increasing and differentiable on I, then $f'(x) > 0$ for all x in I. False

It is possible that $f' = 0$ for some x in I



Assuming that fig1 shows the graph of derivative of a function f, which statement is true?

a) f has a local min at 2 and a local max at 5

(b) f has a local min at 3 and two local max

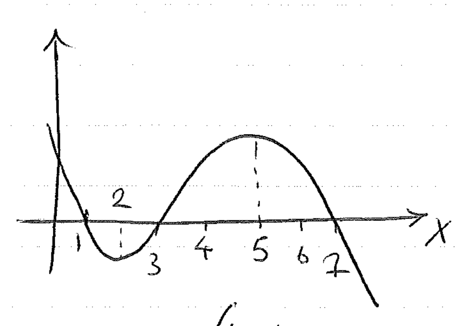


fig. 1

c) f has two local min at $x=1$, $x=7$ and a local max at $x=$

d) One can't find the local extrema of f using this fig. as a gra of f'

e) None of the above