MATH110-001, L'Hopital's rule

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limit as $x \to 0$

$$\lim_{x \to 0} \frac{\sin(x)}{x} = ?$$

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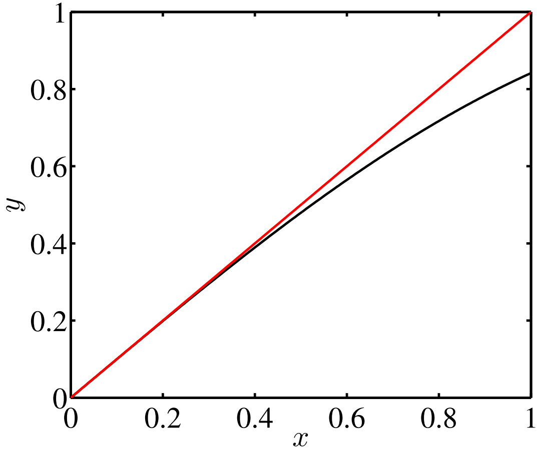
$f(x) = \sin(x)$

$g(x) = x$



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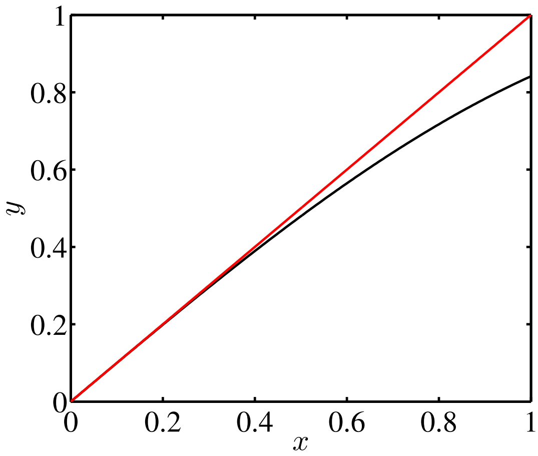
$$\lim_{x\to 0} \frac{\sin(x)}{x} = ?$$

$f(x) = \sin(x)$

$g(x) = x$

$$\lim_{x\to 0} \frac{\sin(x)}{x} = 1$$



limit as $x \to 0$

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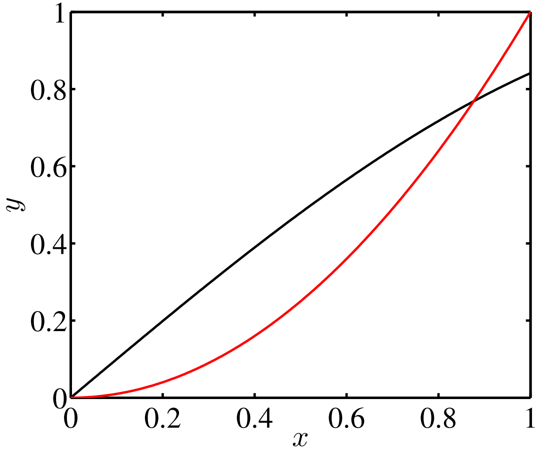


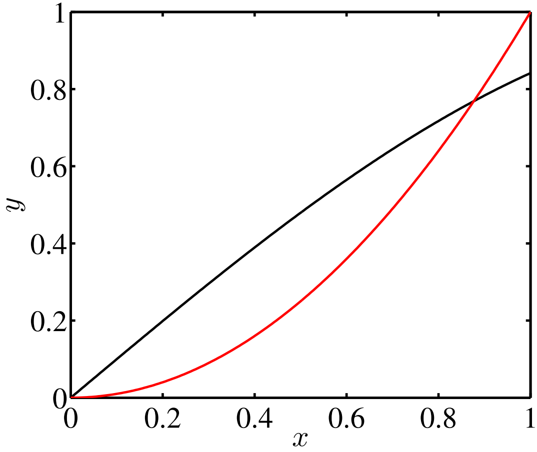
limit as $x \to 0$

$$\lim_{x \to 0} \frac{\sin(x)}{x^2} = ?$$

$f(x) = \sin(x)$

$g(x) = x^2$

$$\lim_{x \to 0} \frac{\sin(x)}{x^2} = \pm\infty$$



limit as $x \to 0$

Question: $\lim_{x \to 0^+} \frac{\sin(x)}{x^2}$ is ___ and $\lim_{x \to 0^-} \frac{\sin(x)}{x^2}$ is ___.

- $+\infty, +\infty$
- $+\infty, -\infty$
- $-\infty, -\infty$
- $-\infty, +\infty$

limit as $x \to 0$

$$\lim_{x \to 0} \frac{x^3}{1 - \cos(x)} = ?$$

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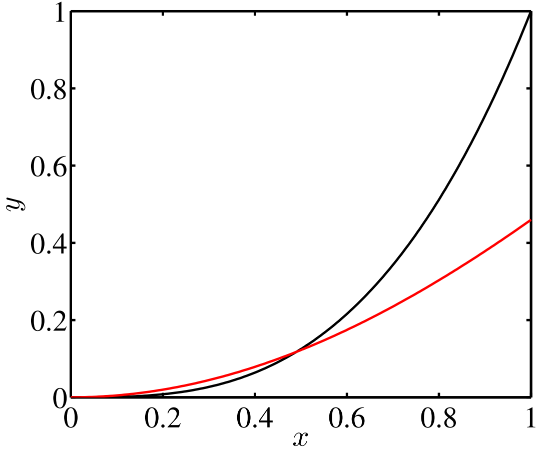
$f(x) = x.^3$

$g(x) = 1 - \cos(x)$



limit as $x \to 0$

$$\lim_{x \to 0} \frac{x^3}{1 - \cos(x)} = ?$$

$f(x) = x.^3$

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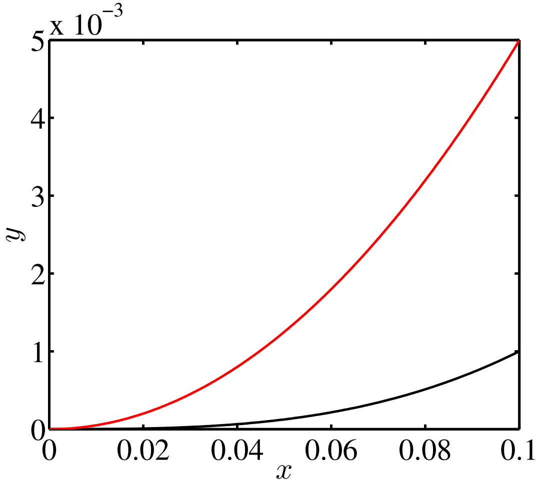


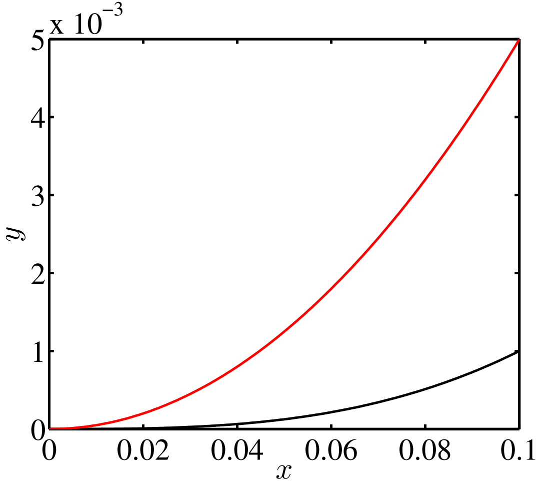
limit as $x \to 0$

$$\lim_{x\to 0} \frac{x^3}{1-\cos(x)} = ?$$

$f(x) = x.^3$

$g(x) = 1 - \cos(x)$

$$\lim_{x\to 0} \frac{x^3}{1-\cos(x)} = 0$$



limit as $x \to 0$

$$\lim_{x\to 0} \frac{\sin(x)}{x} \stackrel{?}{=} \frac{0}{0}$$

$$\lim_{x\to 0} \frac{\sin(x)}{x^2} \stackrel{?}{=} \frac{0}{0}$$

$$\lim_{x\to 0} \frac{x^3}{1-\cos(x)} \stackrel{?}{=} \frac{0}{0}$$

limit as $x \to 0$

$$\lim_{x\to 0} \frac{\sin(x)}{x} \stackrel{?}{=} \frac{0}{0}$$

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$$\lim_{x\to 0} \frac{x^3}{1-\cos(x)} \stackrel{?}{=} \frac{0}{0}$$

$$\lim_{x\to 0} \frac{\sin(x)}{x} = 1$$

$$\lim_{x\to 0} \frac{\sin(x)}{x^2} = \pm\infty$$

$$\lim_{x\to 0} \frac{x^3}{1-\cos(x)} = 0$$

limit as $x \to 0$

$$\lim_{x\to 0} \frac{\sin(x)}{x} \stackrel{?}{=} \frac{0}{0} \qquad \lim_{x\to 0} \frac{\sin(x)}{x} = 1$$

$$\lim_{x\to 0} \frac{\sin(x)}{x^2} \stackrel{?}{=} \frac{0}{0} \qquad \lim_{x\to 0} \frac{\sin(x)}{x^2} = \pm\infty$$

$$\lim_{x\to 0} \frac{x^3}{1-\cos(x)} \stackrel{?}{=} \frac{0}{0} \qquad \lim_{x\to 0} \frac{x^3}{1-\cos(x)} = 0$$

"0/0" is called an indeterminate form because knowing that $f(x)$ approaches 0 and $g(x)$ approaches 0 is not enough to determine the limit of $f(x)/g(x)$, even if it has a limit.

L'Hopital's rule

If $f(x)$ and $g(x)$ are differentiable on an interval I which contains the points $x = a$, $g'(x) \neq 0$ on I except possibly at a and

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0} \quad \text{or} \quad \frac{\infty}{\infty}$$

then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(a)}{g'(a)}$$

provided the limit on the right exists.
Note that a can represent a finite number or "∞".

limit as $x \to \infty$

$$\lim_{x\to\infty} \frac{x^5}{e^x} = ?$$

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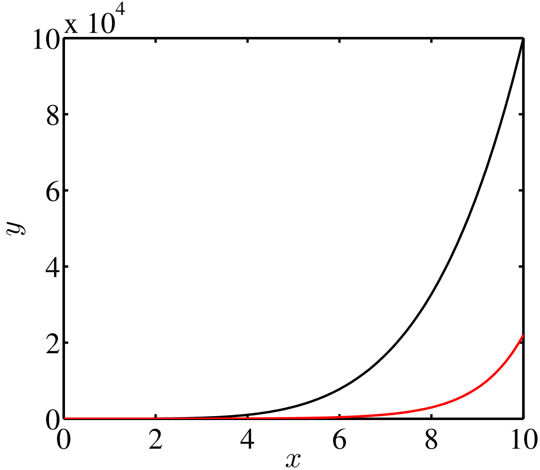
$$\lim_{x\to\infty} \frac{x^5}{e^x} = ?$$

$f(x) = x^5$

$g(x) = e^x$



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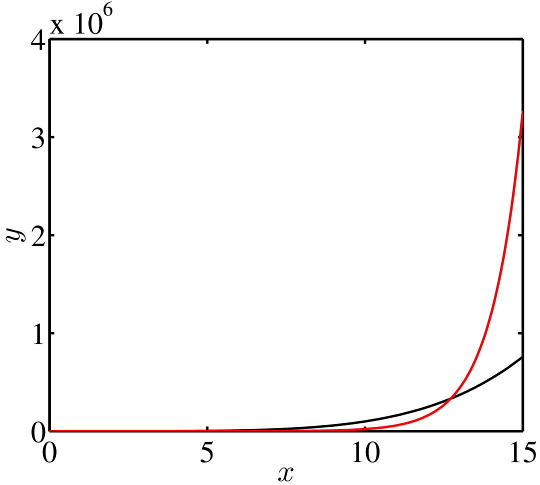


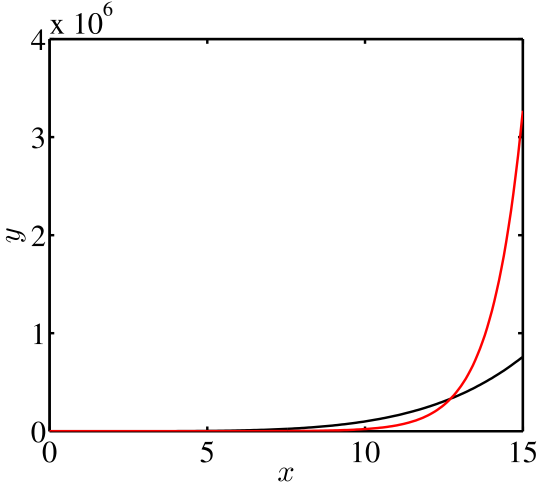
limit as $x \to \infty$

$$\lim_{x\to\infty} \frac{x^5}{e^x} = ?$$

$f(x) = x^5$

$g(x) = e^x$

$$\lim_{x\to\infty} \frac{x^5}{e^x} = 0$$



limit as $x \to \infty$

$$\lim_{x\to\infty} \frac{x}{\ln(x)} = ?$$

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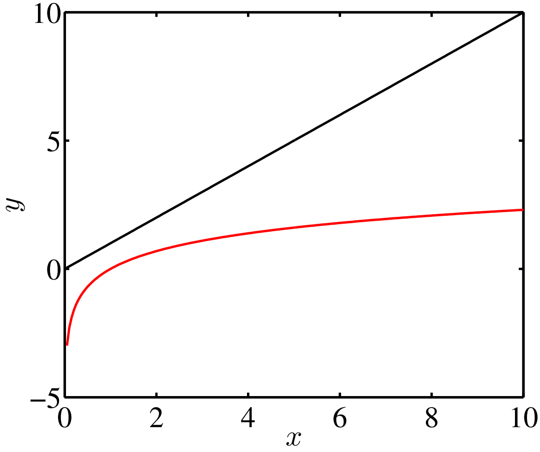
$f(x) = x$

$g(x) = \ln(x)$



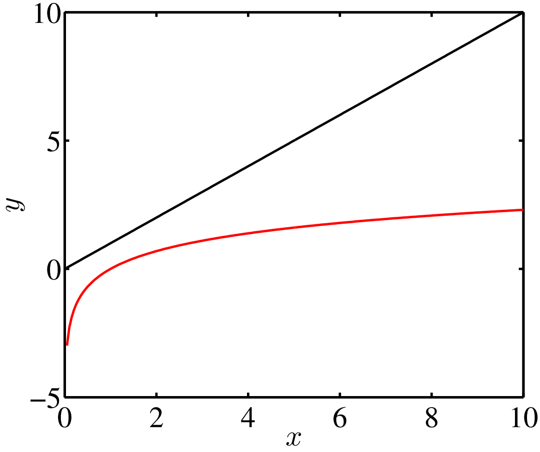
limit as $x \to \infty$

$$\lim_{x\to\infty} \frac{x}{\ln(x)} = ?$$

$f(x) = x$

$g(x) = \ln(x)$

$$\lim_{x\to\infty} \frac{x}{\ln(x)} = \infty$$



More exercises

$$\lim_{x\to 0} \frac{\sin(4x)}{\sin(x)} = ?$$

$$\lim_{x\to \infty} \frac{x\ln(x)}{x^2+1} = ?$$

$$\lim_{x\to 0} \frac{\sin(x)-x}{\cos(x)+x^2/2-1} = ?$$

$$\lim_{x\to -\infty} \frac{x^3+2x}{3x^3-5x^2+1} = ?$$

$$\lim_{x\to \infty} \frac{e^{x^2}+x}{e^{4x}-x^2+2} = ?$$

$$\lim_{x\to \infty} \frac{e^{x}+x^5}{x^6+3} = ?$$

$$\lim_{x\to 0} \left(\frac{1}{x}-\frac{1}{\sin(x)}\right) = ?$$

More exercises

- Find $\lim_{x\to\infty} \frac{x^n}{e^x}$ assuming that $n > 0$.
- Find $\lim_{x\to-\infty} \frac{x^n}{e^x}$ assuming that $n > 0$.
- Find $\lim_{x\to\infty} \frac{x^n}{\ln(x)}$ assuming that $n > 0$.
- Find $\lim_{x\to\infty} \frac{5x^n+2}{3x^m+1}$ assuming that $n > m > 0$.
- Find $\lim_{x\to-\infty} \frac{5x^n+2}{3x^m+1}$ assuming that $n > m > 0$.
- Find $\lim_{x\to\infty} \frac{5x^n+2}{3x^m+1}$ assuming that $m > n > 0$.
- Find $\lim_{x\to\infty} \frac{5x^n+2}{3x^m+1}$ assuming that $m = n > 0$.

More exercises

- Find $\lim_{x\to\infty} \dfrac{a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + b_{m-2}x^{m-2} + \cdots + b_1 x + b_0}$ assuming that $n > m > 0$ and that $a_n, a_{n-1}, \ldots, a_1, a_0$ and $b_m, b_{m-1}, \ldots, b_1, b_0$ are constant numbers.
- Find $\lim_{x\to\infty} \dfrac{a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + b_{m-2}x^{m-2} + \cdots + b_1 x + b_0}$ assuming that $m > n > 0$ and that $a_n, a_{n-1}, \ldots, a_1, a_0$ and $b_m, b_{m-1}, \ldots, b_1, b_0$ are constant numbers.
- Find $\lim_{x\to\infty} \dfrac{a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + b_{m-2}x^{m-2} + \cdots + b_1 x + b_0}$ assuming that $m = n > 0$ and that $a_n, a_{n-1}, \ldots, a_1, a_0$ and $b_m, b_{m-1}, \ldots, b_1, b_0$ are constant numbers.