

## Assignment 5

Due April 1, 2016

Hand in full solutions to the questions below. Make sure you justify all your work and include complete arguments and explanations. Your answers must be clear and neatly written, as well as legible.

### Problem 1.

This problem is a variant of the “silly walks” problem from the workshop. Suppose you are in a room that is  $a$  meters wide and 10 meters long. You can “wall walk” along the  $a$  meter long wall at  $s$  meters per second and “floor walk” in any direction at 1 meter per second. You start at one corner of the room, wall walk along the wall of length  $a$  meters for some distance, and then floor walk straight to the corner opposite from where you started.

Suppose that the fastest route to the opposite corner of the room is to directly floor walk from one corner of the room to the other (i.e. you wall walk for 0 distance). What can be said about the relationship between  $a$  and  $s$ ?

### Problem 2.

You can solve multivariable optimization problems by first treating one of the variables as a parameter. This problem will illustrate how this is done.

We want to solve the following optimization problem: suppose  $x$ ,  $y$ , and  $z$  are positive numbers that sum to 10. What is the largest possible value of  $xy + xz + yz$ ?

To solve this optimization problem, use the following steps:

- a) First, suppose that  $z$  is a fixed parameter. Then we have to find nonnegative numbers  $x$  and  $y$  (depending on the fixed value  $z$  such that  $x + y = 10 - z$ ), such that  $xy + xz + yz$  is maximized.
- b) To find the global maximum, plug in the values for  $x$  and  $y$  obtained in (a) in order to rewrite  $xy + xz + yz$  as a function of a single variable. Then maximize this function.