MATH110-001, Polynomial fit

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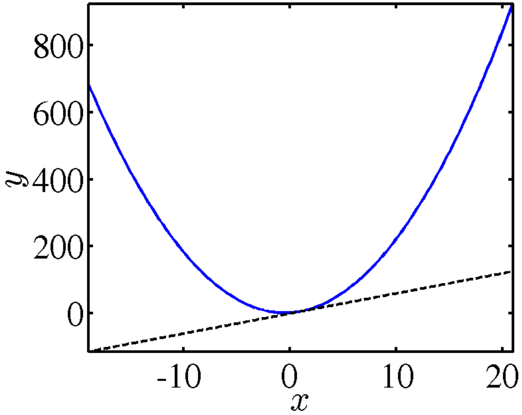
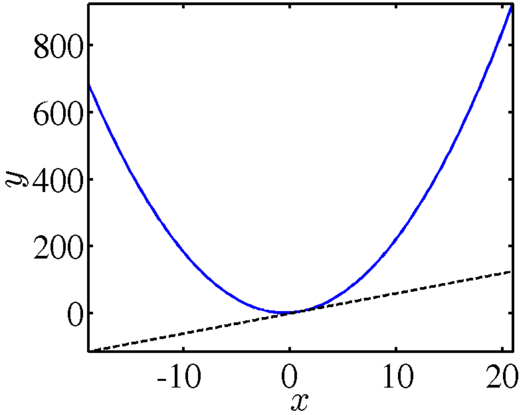
Linear fit



$y = 2x^2 + 2x + 1,$
$-20 < x - 1 < 20$

dashed line is the tangent line at $x = 1$

Linear fit

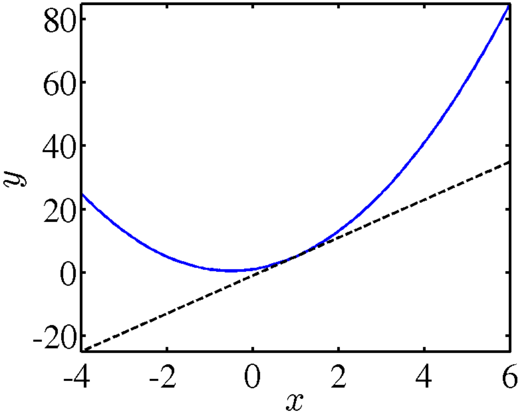


$y = 2x^2 + 2x + 1,$
$-20 < x - 1 < 20$

dashed line is the tangent line at $x = 1$

Let's zoom in around $x = 1$

Linear fit

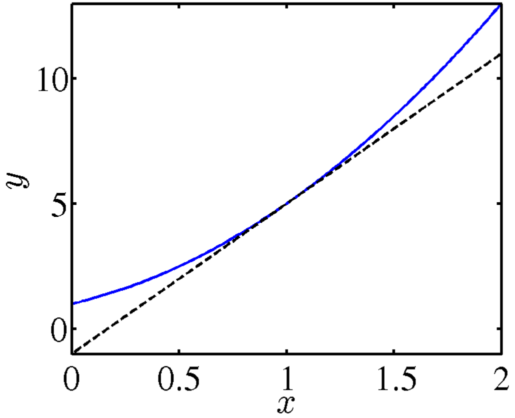


$y = 2x^2 + 2x + 1,$

$-5 < x - 1 < 5$

dashed line is the tangent line at $x = 1$

Linear fit

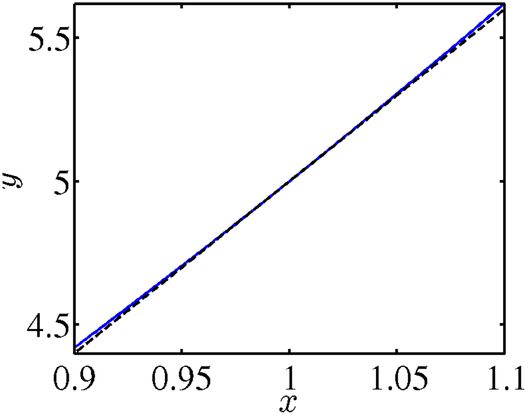


$y = 2x^2 + 2x + 1,$

$-1 < x - 1 < 1$

dashed line is the tangent line at $x = 1$

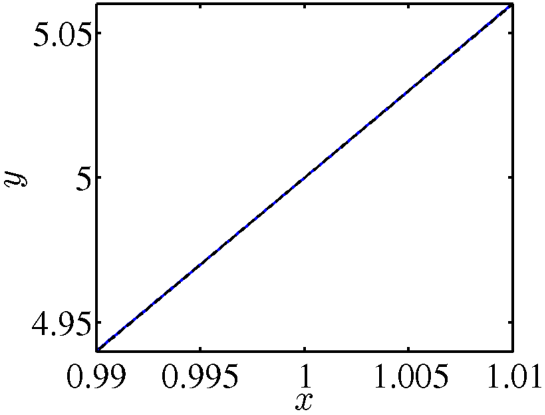
Linear fit



$$y = 2x^2 + 2x + 1,$$
$$-0.1 < x - 1 < 0.1$$

dashed line is the tangent line at $x = 1$

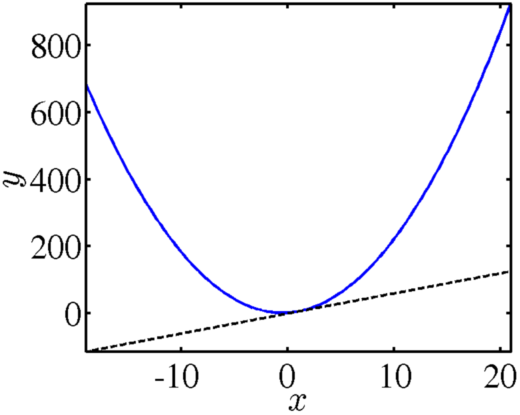
Linear fit



$y = 2x^2 + 2x + 1,$
$-0.01 < x - 1 < 0.01$

dashed line is the tangent line at $x = 1$

Linear fit



$y = 2x^2 + 2x + 1,$

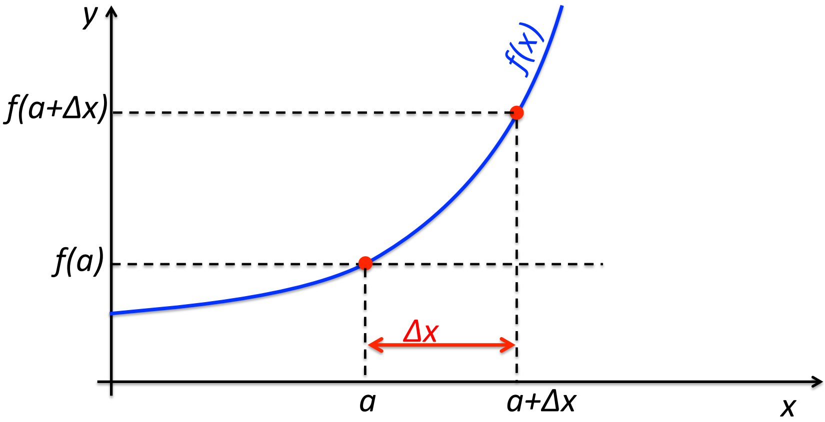
$\frac{dy}{dx} = 4x + 2$

$x = 1 \Rightarrow y = 5, y' = 6$

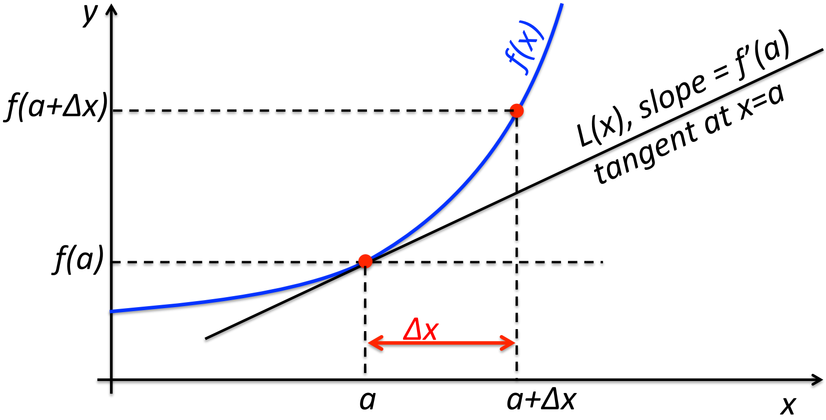
So the equation of the tangent line is:

$y - 5 = 6(x - 1)$

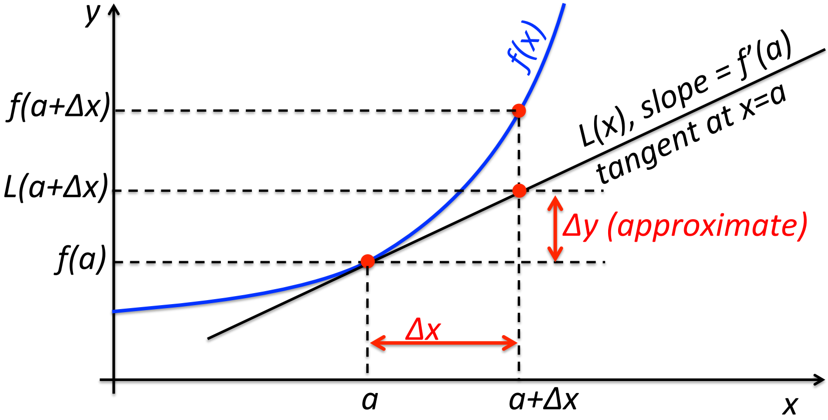
Relationship between Δx and Δy



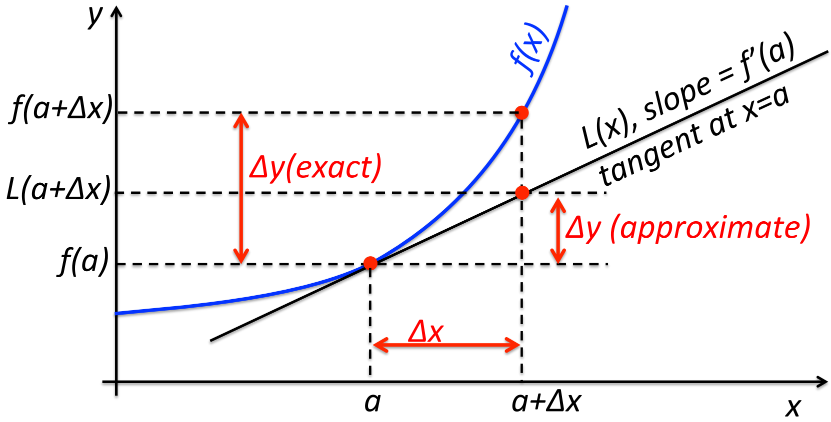
Relationship between Δx and Δy



Relationship between Δx and Δy



Relationship between Δx and Δy



Linear approximation and concavity

Relationship between Δx and Δy

Suppose f is differentiable on an interval I containing the point a. The change in the value of f between two point a and $a + \Delta x$ is approximately

$$\Delta y \approx f'(a)\,\Delta x$$

where $a + \Delta x$ is in I.

Note:

If f is concave up on I, using linear approximation of f at a we underestimate the value of the function; i.e. $L(a + \Delta x) < f(a + \Delta x)$. Alternatively, if f is concave down on I, using linear approximation of f at a we overestimate the value of the function; i.e. $L(a + \Delta x) > f(a + \Delta x)$.

Linear approximation and concavity

$y = 2x^2 + 2x + 1$

$y' = \frac{dy}{dx} = 4x + 2$

$y = x^2 + 4x$

$y' = \frac{dy}{dx} = 2x + 4$

At $x = 1$:

$y = 5$

$y' = 6$

$y = 5$

$y' = 6$

Linear approximation at $x = 1$:

$L(x) = 5 + 6(x - 1)$

$L(x) = 5 + 6(x - 1)$

Question: For which function does $L(1.1)$ give a more accurate estimate of y at $x = 1.1$?

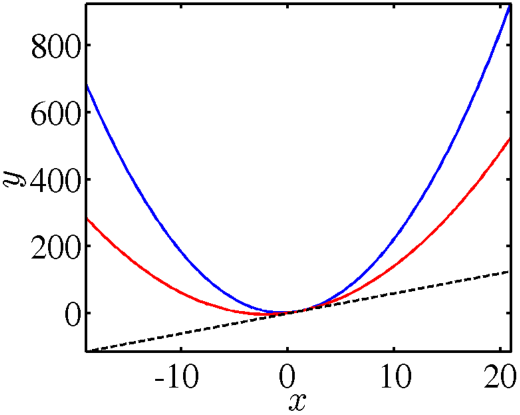
a) $y = 2x^2 + 2x + 1$

b) $y = x^2 + 4x$

c) It depends!

d) I don't know!

Linear approximation and concavity



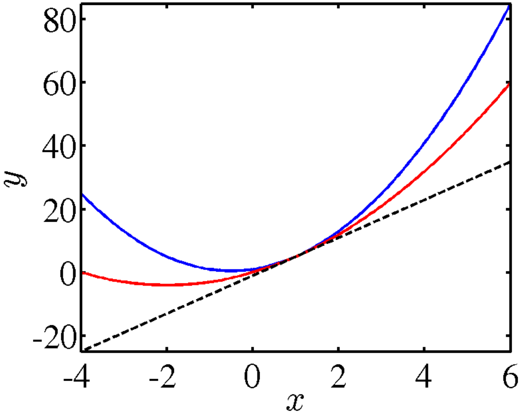
$y = 2x^2 + 2x + 1,$

$y = x^2 + 4x,$

$-20 < x - 1 < 20$

dashed line is the tangent line at $x = 1$

Linear approximation and concavity



$y = 2x^2 + 2x + 1$,

$y = x^2 + 4x$,

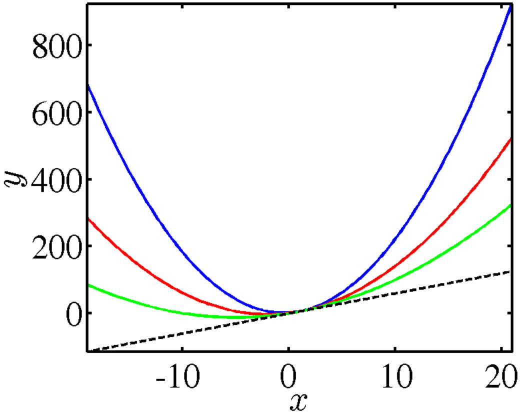
$-5 < x - 1 < 5$

dashed line is the tangent line at $x = 1$

$y'' = 4, \quad y'' = 2,$

The function with the smaller $|y''|$ gives a better estimate.

Linear approximation and concavity



$$y = 2x^2 + 2x + 1,$$

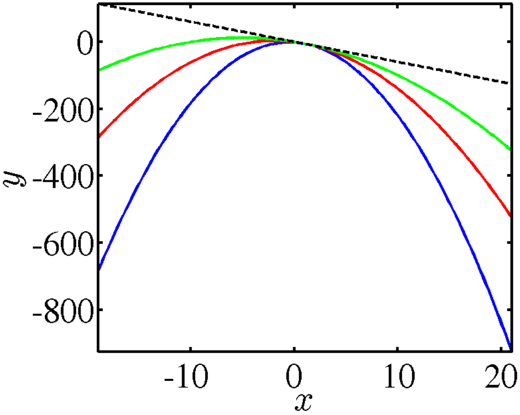
$$y = x^2 + 4x,$$

$$y = \frac{1}{2}x^2 + 5x - \frac{1}{2},$$

dashed line is the tangent line at $x = 1$

$y'' = 4, \quad y'' = 2, \quad y'' = 1$

Linear approximation and concavity



$y = -2x^2 - 2x - 1,$

$y = -x^2 - 4x,$

$y = -\frac{1}{2}x^2 - 5x + \frac{1}{2},$

dashed line is the tangent line at $x = 1$

$y'' = -4, \quad y'' = -2,$
$y'' = -1$