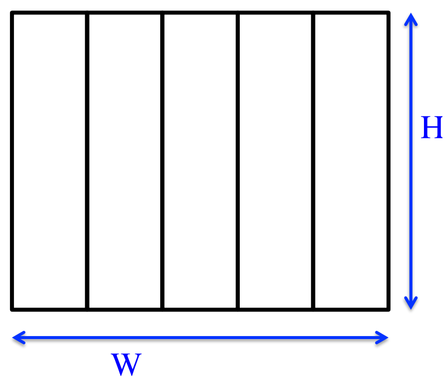
Problem 1.

Consider the following problem: A farmer with L meters of fencing wants to enclose a rectangular area and then divide it into five pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the five pens? To solve this optimization problem, use the following steps:

a. Draw a diagram illustrating the general situation. Introduce notation and label the diagram with your symbols.



b. Write an expression for the total area.

total area: $A = HW$

c. Use the given information to write an equation that relates the variables.

total length of the used fence: $L = 6H + 2W$

d. Identify the independent variable and the constant parameters. We can choose either H or W as the independent variable, but not both; e.g. once we pick the value of H, the value of W is determined by the equation given in part **c**. I choose H as the independent variable.

L is a constant parameter and does not change as we change H.

e. Use part (d) to write the total area as a function of one variable.

$$L = 6H + 2W \Rightarrow W = \frac{L - 6H}{2}$$

$$A = HW = H * \frac{L - 6H}{2}$$

f. Finish solving the problem.

We are trying to maximize A, so we should differentiate A with respect to the independent variable, H, and find the critical numbers.

$$\frac{dA}{dH} = \frac{d}{dH}\left(H * \frac{L-6H}{2}\right) \qquad \text{Use product rule}$$
$$\frac{dA}{dH} = \frac{dH}{dH} * \frac{L-6H}{2} + H * \frac{d}{dH}\left(\frac{L-6H}{2}\right) = \frac{L-6H}{2} + H * \frac{-6}{2} = \frac{L-6H-6H}{2}$$
$$\frac{dA}{dH} = \frac{L-12H}{2}$$
$$\frac{dA}{dH} = 0 \Rightarrow L = 12H \Rightarrow H = \frac{L}{12}$$

$\frac{dA}{dH}$ is defined everywhere. The domain of the function $A(H)$ is $0 \leq H \leq L/6$; i.e. H can take any value between 0 and $L/6$ (see the equation in part **c**). So there are 3 critical numbers: $H = 0$, $H = L/12$ and $H = L/6$.

Since we are looking for the absolute maximum, we can evaluate A at all the critical numbers and just pick the largest value as the absolute maximum of A:

$$H = 0 \Rightarrow W = \frac{L-6*0}{2} = \frac{L}{2} \Rightarrow A = 0 * \frac{L}{2} = 0$$
$$H = \frac{L}{6} \Rightarrow W = \frac{L-L}{2} = 0 \Rightarrow A = \frac{L}{6} * 0 = 0$$
$$H = \frac{L}{12} \Rightarrow W = \frac{L-L/2}{2} = \frac{L}{4} \Rightarrow A = \frac{L}{12} * \frac{L}{4} = \frac{L^2}{48}$$

Since $L^2/48 > 0$, The absolute maximum of the area is $A(L/12) = L^2/48$.

Also note that A has a local maximum at $L/12$.

	$0 < H < L/12$	$L/12 < H$
$\frac{dA}{dH}$	$+$	$-$
A	$\nearrow$	$\searrow$