Matrix eigenvalue problems (7.1 to 7.4, Gilat 5.1 to 5.5)

Last time:

1. **Characteristic equation:** $\det(A-\lambda I) = 0$
2. **Power method**

Today:

1. **Inverse and shifted power methods**
2. **Gershgorin circle theorem**

The Power method

The power method is an iterative method for finding the dominant eigenvalue using the power method.

dominant eigenvalue: the eigenvalue with the largest magnitude

β_i: i^{th} estimate of the dominant eigenvalue

x_i : i^{th} estimate of the eigenvector of the dominant eigenvalue

Algorithm:

1. Choose a nonzero initial guess x_0
2. $i = 0$
3. $x_{i+1} = A x_i$
4. β_i : the element of x_{i+1} with the largest magnitude
5. $x_{i+1} = x_{i+1}/\beta_i$: normalization of x_{i+1}
6. $i = i+1$
7. *Go back to step 3 and repeat until enough precision accomplished; e,g,* $||x_{i+1}-x_i||< \varepsilon$ *OR* $||x_{i+1}-x_i||/||x_{i+1}||< \varepsilon$ *etc*

The Power method

- The power method generally converges very slowly unless the initial guess, x0, is very close to the eigenvector of the dominant eigenvalue

- The ratio of the two largest eigenvalues determines how quickly the power method converges

$$x_{k+1} = u1 + c_2/c_1\,(\lambda_2/\lambda_1)^k\,u_2 + \ldots + c_2/c_1\,(\lambda_n/\lambda_1)^k\,u_n$$

- The power method can only be used if the dominant eigenvalue is not a repeated root of the characteristic equation. This also implies that the dominant eigenvalue must be real.

Examples on Gershgorin theorem

Using Gershgorin theorem find the regions of eigenvalues for the following matrices:

a) $A = \begin{bmatrix} 3 & 2 & 1 \\ 5 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$

Circle1: R1

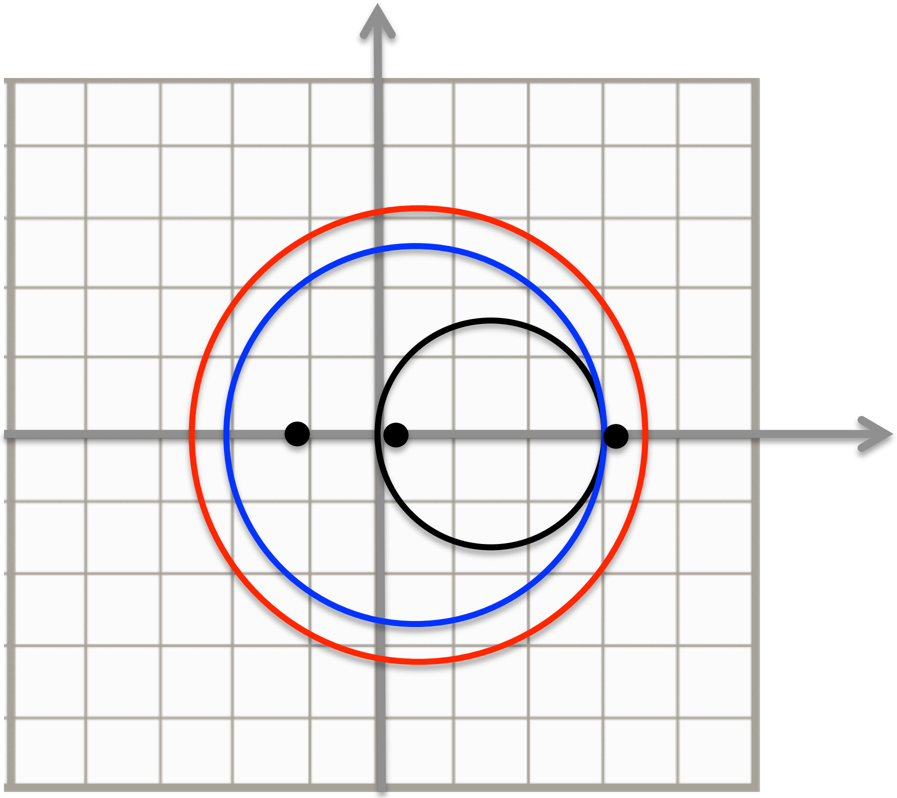
center: x = 3, radius: r = 2 + 1 = 3

Circle2: R2

Center: x = 1, radius: r = 5 + 0 = 5

Circle3: R3

Center: x = 1, radius: r = 4 + 2 = 6

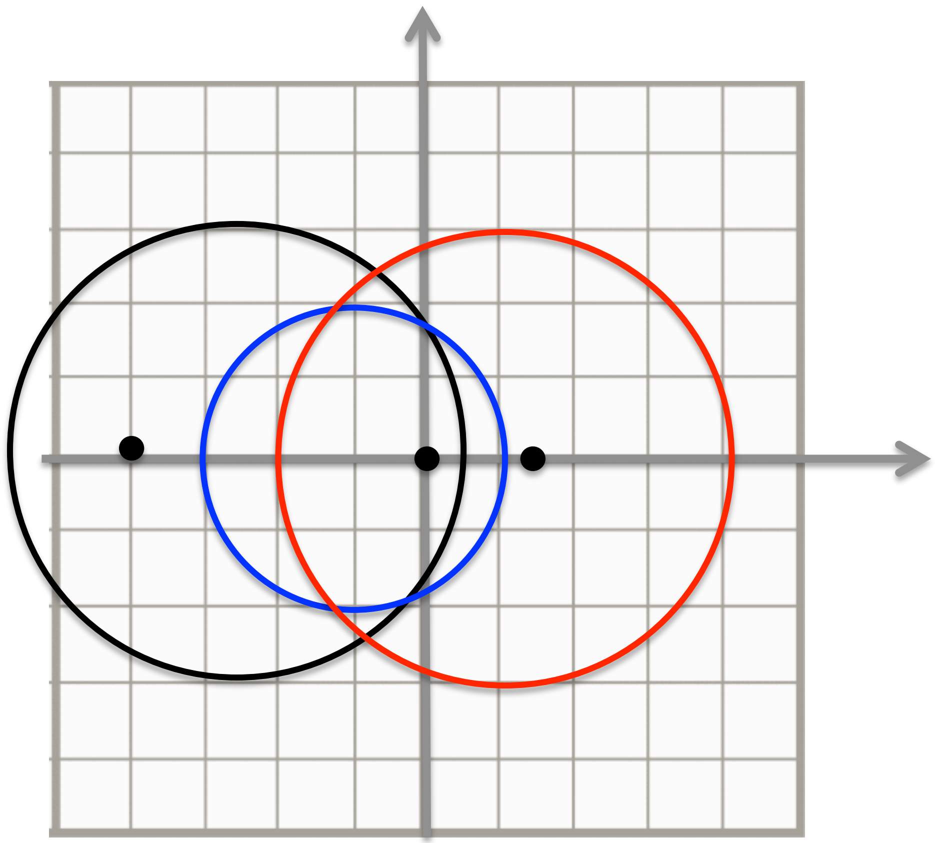


Black circles represent eigenvalues

Examples on Gershgorin theorem

Using Gershgorin theorem find the regions of eigenvalues for the following matrices:

b) $A = \begin{bmatrix} -5 & -4 & 2 \\ -2 & -2 & 2 \\ 4 & 2 & 2 \end{bmatrix}$



Circle1: R1

center: x = -5, radius: r = 4 + 2 = 6

Circle2: R2

Center: x = -2, radius: r = 2 + 2 = 4

Circle3: R3

Center: x = 2, radius: r = 4 + 2 = 6

Black circles represent eigenvalues