# CHAPTER II
# General Characteristics of Measurement Systems

Now that you have decided what to measure, you have to decide which measurement system is more suitable for your experiment.

## 2.1. Generalized measurement system

The objective of any experiment is to better understand a physical system or phenomenon. This can only be achieved through measuring some physical variables or properties of the system.

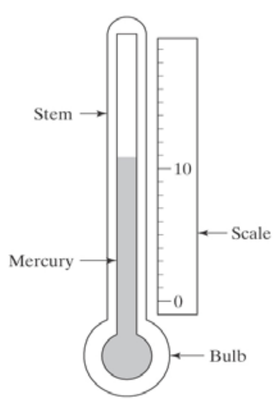
**What you want to measure $\rightarrow$ Measurands (e.g. temperature; pressure; speed, …)**

While planning for an experiment, you have to choose the best system allowing an accurate and unique determination of the measurands.

Any measurement system can be divided into three parts:

1- **Sensing element**: it has a physical characteristic that responds to the variation of the measurand.
   Exp: piezoelectric components respond to pressure variations
2- **Signal modification subsystem:** it will modify (without altering) the output (signal) of the sensing element to make it more suitable for reading or recording (e.g. amplification, filtering, averaging, conversion to physical units, … ).
3- **Indicator or recorder:** it will display the output (usually in physical units of measurand) and/or record it.

**Example:**




- Sensing element: Mercury (expends with T°).
- Signal modification subsystem: connecting the bulb to the stem to “amplify” the expansion of mercury.
- Indicator: reading scale.

## 2.2. Validity of measurement

While measuring, you have to be convinced that your measurement system accurately reflect the value or the change in the measurand.
Of course there is no infinite accuracy and high accuracy usually has a corresponding high cost.
Each measurement system has certain accuracy. In other words, it can reflect more (+) or less (-) the value or variation in the measurand. It is essential to keep these variations as small as possible (in a relative and not absolute sense). And

YOU ALWAYS HAVE TO KNOW THE ACCURACY OF YOUR MEASUREMENT SYSTEM

**- Measurement errors and related definitions:**

*Error = Measurand value - True value*

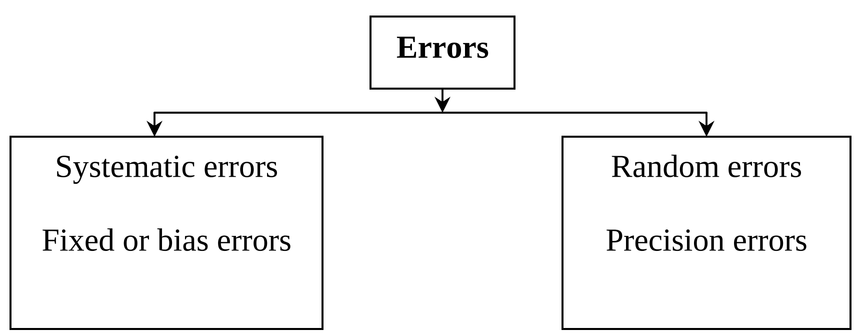
As is it is completely unrealistic to talk about such error, since the true value is obviously not known, in experimental procedure, we only can talk about uncertainty of the measurement.

**Uncertainty:** *by definition is an estimate (with some level of confidence) of the limits of error in the measurement.*

So, if you pretend that with 95% of confidence, the uncertainty of the T° measurement is ±1°C, this has to be interpreted as: if one performs 100 measurements with the temperature measurement system, in at least 95 cases, the error will be within 1°C. As a consequence, in maximum 5 cases, the error will be higher than 1°C.

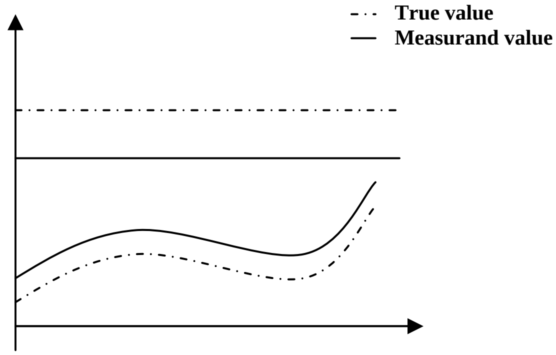
You can narrow the uncertainty interval by using calibrated, high quality measurement systems.

**- Systematic errors and random errors**

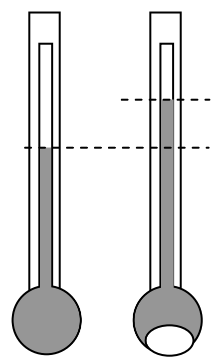


**Systematic errors:**

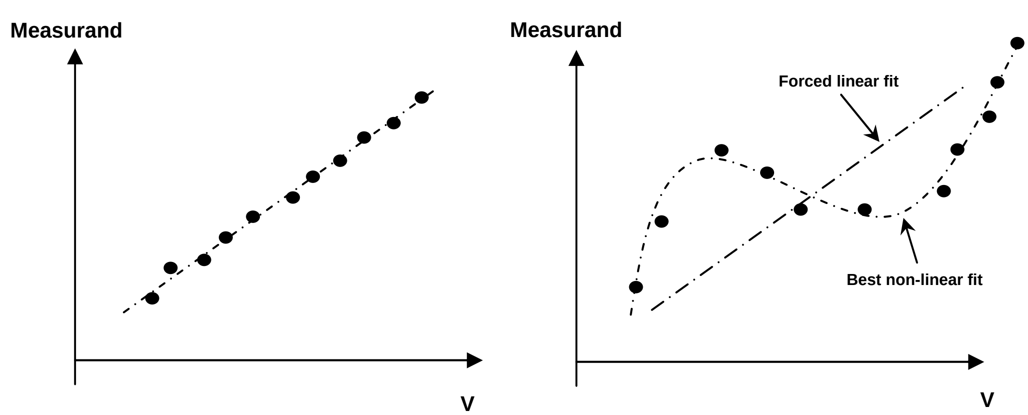
Systematic errors are consistent, keeping always the same value.




Example: the T° indicated by your thermometer will always be overestimated if the bulb is deformed.

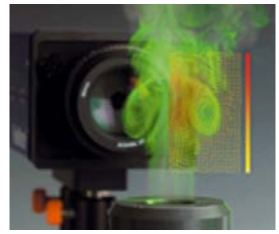


**1st major source of systematic errors:** It results from the calibration process of the measurement system: Calibration error. As no calibration is free of errors, this error will always affect the value of the measurand. Such calibration errors might come from the fact that we mainly try to force a linear behavior instead of truly non-linear behavior during calibration process.



**2$^{nd}$ major source of systematic errors:** Loading error due to intrusive measurement systems. Let say you want to measure the velocity of a fluid and you have the choice between hot wire anemometry and particle image velocimetry.


Hot wire anemometry


Particle image velocimetry

Hot wire anemometry will perturb the flow (intrusive method) leading a systematic error: Loading error. Particle image velocimetry is a non-intrusive method (does not disturb the flow). It does not induce, therefore, a loading error.

> ***Note:*** One of the most difficult measurand to accurately determine is the wall shear stress. This since any measurement system that you will put on the wall to measure the wall shear stress will modify its value by introducing a loading error.

**3$^{rd}$ major source of systematic errors:** An error occurs when only limited measurements in space are used to reflect the value of the measurand in a bigger domain. Such errors are called: Spatial errors.

| | | | |
|---|---|---|---|
| 30°C | 10°C | 15°C | 10°C |
| 20°C | 21°C | 7°C | 14°C |
| 18°C | 5°C | 9°C | 24°C |
| 3°C | 15°C | 20°C | 19°C |

Location of measurement

**Example:** Since the temperature is quite low in the north of Canada, most people around the world think that it is freezing in Montreal all year round.

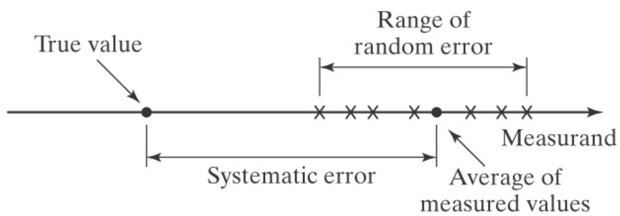
Systematic errors are hard to detect since the output will always follow the behavior of the true value (with only a certain bias). The main way to reduce systematic errors is to have the best possible calibration (exp: use high fidelity calibration devices, increase the number of points during the calibration process).

Systematic errors can be approximately expressed as: Average of readings – True value

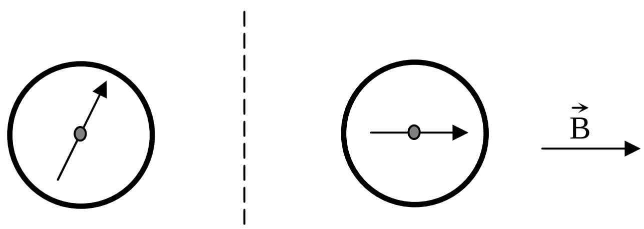
**Random errors:**

Random errors by definition are unpredictable and varying from one measurement to the other. This is why in order to determine an estimate of random errors, we need to record the measurand several times. Actually, the number of measurements N has at least to be large enough to limit the errors due to statistical averaging.

Random error = Reading – Average of readings




The main sources of random errors are the measuring system, the experimental setup or the environment surrounding the experiment.
For example the deviation of a compass is affected by the presence of an external magnetic field.




Another important source of random errors is electrical noise. Since the majority of measurement systems are composed of electrical components, these components can interfere with external magnetic or electric fields (like building wiring …). These perturbations can randomly alter the output of the measurement system. A very good way to limit such random errors is to use proper shielding or grounding.

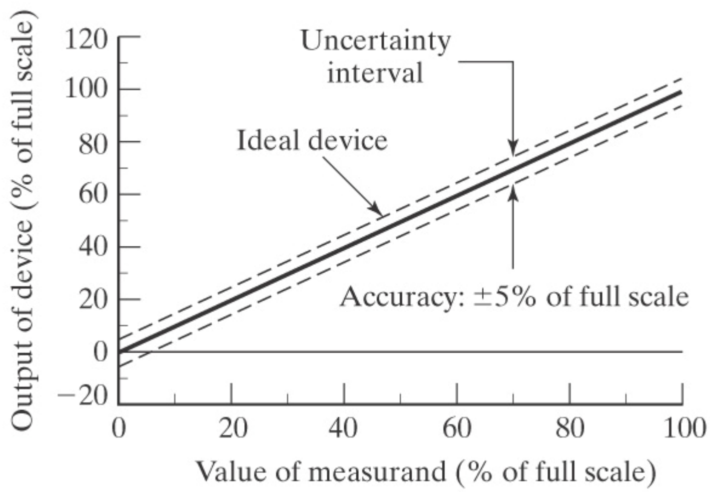
| **Example** |
| --- |
| In a calibration test, 10 measurements using a thermocouple have been made of the temperature at the inlet of small steam turbine. The true temperature is 400°C. The readings are: 400, 401, 398, 402, 402, 401, 399, 403, 402 and 399°C. *Estimate the systematic and maximum random errors caused by the thermocouple.* |

## Definitions

- **Range**: every measurement system is designed to operate over a certain specific range. Within the limits of this range, the response of the measurement system is optimal. You definitely can not imagine for example a measurement system capable of measuring the T° from 0 K to 300 K with the same accuracy.
We call the span of a measurement system the total length of the range. For example, a voltmeter with a range of ±3 V has a span of 6 V.

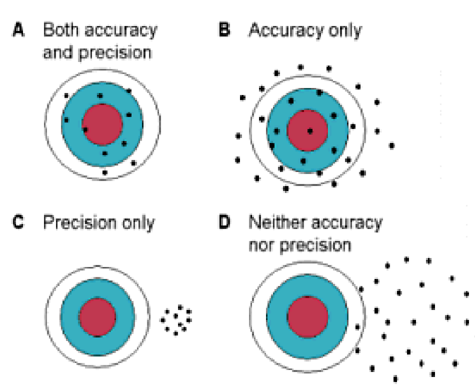
- **Accuracy - The most important concept**: accuracy is defined as the difference between the measured value and the true value. In this sense, its definition is close to uncertainty. Manufacturers give usually the accuracy (more often the inaccuracy) of their measurement system. They mean by this the residual error that persists even if you properly calibrate the system and it is used under optimal conditions.

Accuracy is usually given as percentage of the full scale output. This is normal since the manufacturer cannot guarantee the same error if the measurement system is used out of its range.
For example, when you read 5% accuracy of the full scale and the range of your system is 0 to 5 V, the uncertainty is ±0.25 V. This is whatever the value you get in the range of the measurement system.



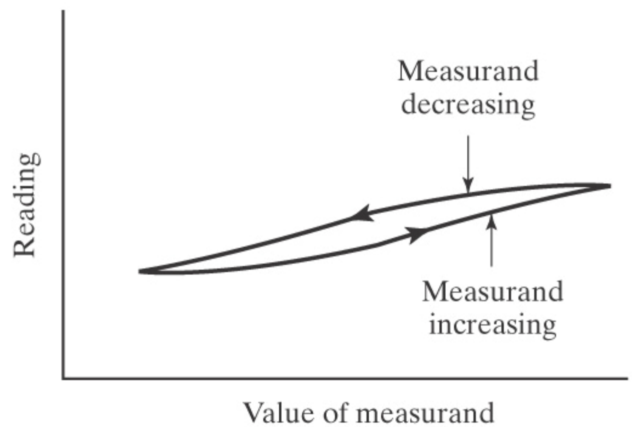

It is extremely important to choose a measurement system that has the best accuracy within the range of your expected values of the measurand.

- **Precision**: a measurement system is highly precise if it gives the same value each time it is used to read the measurand. The value obtained does not have to be close to the true value.



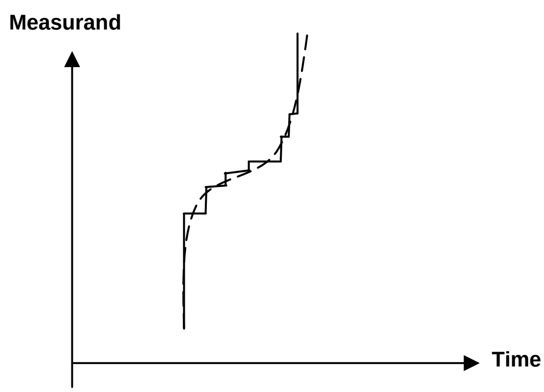
**You want a measurement system with the following characteristics:**
**<u>Highly accurate</u> (give a value close to the true value)**
**AND**
**<u>Highly precise</u> (always gives the same value)**

It should be noted that the accuracy of a measurement system can be affected by hysteresis errors. These errors are usually due to friction or electrical capacitance, for example. This will result in a lower precision, i.e. the measurement system will not give the same value dependent on if the measurand was increased or decreased prior to recording the measurand.



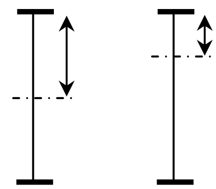

The hysteresis error is considered as a systematic error. The manufacturer usually provide you with an estimate for this error.

- **Resolution**: the discrete nature of must measurement systems will not allow them to follow exactly continuous changes in the measurand. This results in a resolution error which is treated as a random error.



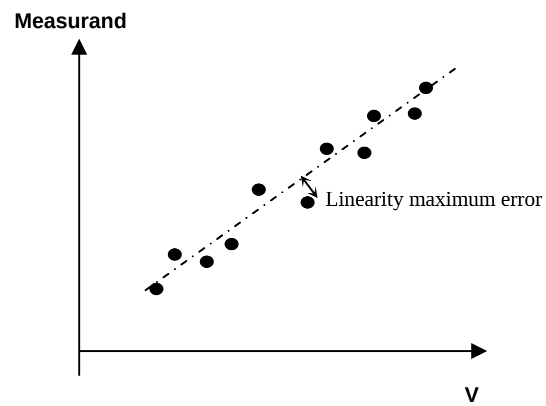

This error is also due to the systematic rounding of the recording of the measurand.

- **Scale readability**: this characteristic is specific to measurement systems where you have to read the value of the measurand on a scale (T°, height, mass, …). Normally the error induced in the reading process should be ±1/2 distance between two tick markers. The human eye can, however, interpolate between the marks lowering the error to around ±1/5 distance between two marks.



**Expected error** **Actual error**

- **Linearity error:** measurement systems with linear behavior are always more appropriate. Why? Because there is a linear (… obvious!!!) relationship between the input and the output. This simplify the calibration process, only two points are necessary (although we never use only 2 pts in practice) compared to several points required for a nonlinear system.
Furthermore, a linear measurement system means a proportional variation between the input and the output. As a consequence, it avoids large variations in the output as a result of small variations in the input.

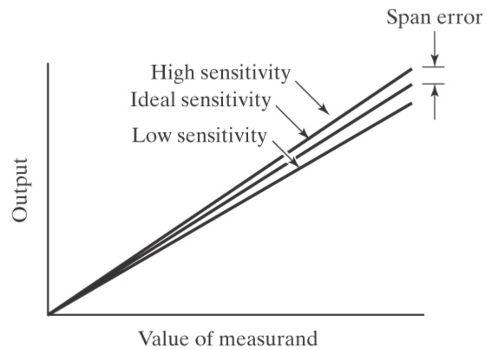



- **Zero error**: Let say you want to use a measurement system to measure the flow rate of a fluid flow in a pipe. Before starting your experiment, you have to make sure that at rest your measurement system indicates zero (0 L/min, for example). Otherwise, you will induce a systematic error on all the values that you will get using the measurement system.
Some manufacturer will indicate the maximum zero error (called also zero balance) of their measurement system. If you spend a long time trying to get a perfect zero recording at rest, this probably means that the measurement system is malfunctioning or maybe there is an external source affecting your device.

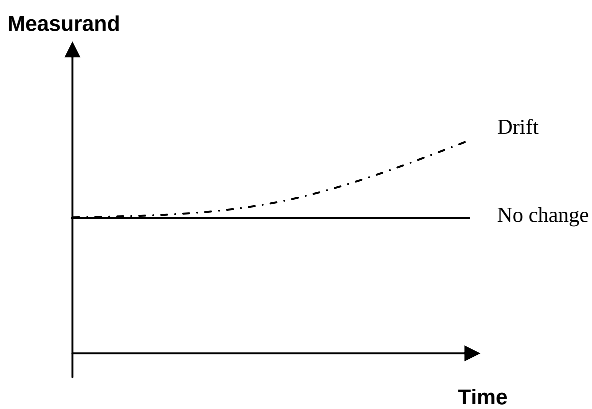
**- Sensitivity and span error**: the sensitivity of a measurement system is defined as:

$$sensitivity = \frac{d(output)}{d(input)} \cong \frac{\Delta\ output}{\Delta\ input}$$

It the measurement system is linear, the sensitivity is constant (usually the symbol given is K).




- **Drift and thermal stability**: The value recorded by a measurement system can change with time independently of change in all environmental factors. This characteristic is called: Drift. This will induce a systematic error on the values of the measurand.
Measurement systems are sensitive to temperature. It is important, therefore, to check thermal stability to avoid drift.




## Calibration of measurement systems

There is no escape from calibration process …
Calibration means comparing the output of your measurement system with an independent reference system that will give you the "true" value. This will allow the determination of the error. Here are some physical units and their standards.

| Physical variable | SI unit | Standard | Fixed or reproducible |
|---|---|---|---|
| Mass | kg | International prototype kilogram: a platinum-iridium cylinder. | Fixed |
| Time | second | The duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom. | Reproducible |
| Length | meter | The length of the path traveled by light in a vacuum during a time of 1/299,792,458 of a second. | Reproducible |
| Temperature | kelvin | The 1/273.16 of the thermodynamic temperature of the triple point of water. | Reproducible |
| Electric current | ampere | The constant current which, if maintained in two straight parallel conductors of infinite length and of negligible circular sections, and placed 1 meter apart in a vacuum would produce a force equal to 2 $10^{-7}$ newton per meter of length. | Reproducible |
| Amount of a substance | mole | The amount of a substance of a system which contains as many elementary entities as there are atoms in 0.0012 kg of carbon-12. | Reproducible |
| Light intensity | Candela | The luminous intensity, in a given direction, of the source that emits monochromatic radiation of frequency 540 $10^{-12}$ hertz and of which the radiant intensity in that direction is 1/683 watt per steradian. | Reproducible |

Although these standards are, in theory reproducible, it is unrealistic to use them in daily practice. It is necessary, therefore, to introduce secondary standards like accurately sized pieces of metal, quartz crystal clocks …
Since it is common to use the MLT system of units, all other physical variables can be deduced from the variables defined in the above table.

Practically speaking, measurement systems are usually calibrated by the manufacturer before selling them. Since your measurement system might experience some modifications with time, it is very good, if possible to re-calibrate the system after a certain period of time.

**- Static calibration**

By static calibration it is meant that time is not involved. Several known inputs feed the measurement system and the outputs are recorded. The inputs are changed slowly and the system is allowed to reach equilibrium before recording the output. Then, the outputs are plotted and used to get a calibration curve using curve fitting (I am sure you remember this very interesting chapter in ENGR 391). The process allows the determination of several systematic errors, but since time is not involved, this static calibration process will neither give thermal stability and drift for example nor spatial errors (application's dependent).

**Example [textbook p.19, but in SI units]**

A low-cost, nominally 0 to 5 lb spring scale has been calibrated by placing accurate weights on its platform. The values of the applied weights range from 0 to 5 lb in 0.5 lb increments. The weights are applied in a sequential manner, starting at a lowest value, increasing to the largest value (up data) and then decreasing to the lowest value (down data). Five such cycles were performed, and the results of the measurements are presented in Tab.1. As suggested by ANSI/ISA (1979), several cycles were completed before the data recording started. The data recording then started in the middle of the up portion of cycle 1 and ended in the up portion of cycle 6, giving five complete cycles.

Fit a straight line to the data and determine the accuracy, hysteresis, and linearity errors. Also, make estimates of the maximum systematic and random errors.

TABLE E2.3(a) Scale Calibration Data

| | Scale reading | | | | | |
|---|---|---|---|---|---|---|
| True weight (lb) | Cycle 1 | Cycle 2 | Cycle 3 | Cycle 4 | Cycle 5 | Cycle 6 |
| 0.5 | | 0.2 | 0.08 | 0.17 | 0.19 | 0.11 |
| 1 | | 0.7 | 0.78 | 0.64 | 0.61 | 0.7 |
| 1.5 | | 1.18 | 1.26 | 1.25 | 1.24 | 1.23 |
| 2 | | 1.81 | 1.93 | 1.81 | 1.93 | 1.88 |
| 2.5 | 2.62 | 2.49 | 2.46 | 2.46 | 2.58 | 2.53 |
| 3 | 3.15 | 3.18 | 3.24 | 3.28 | 3.13 | |
| 3.5 | 3.9 | 3.84 | 3.86 | 3.97 | 3.96 | |
| 4 | 4.59 | 4.71 | 4.61 | 4.6 | 4.6 | |
| 4.5 | 5.41 | 5.35 | 5.49 | 5.46 | 5.39 | |
| 5 | 6.24 | 6.27 | 6.1 | 6.24 | 6.16 | |
| 4.5 | 5.71 | 5.74 | 5.78 | 5.87 | 5.82 | |
| 4 | 4.96 | 5.11 | 5.08 | 5.03 | 5.03 | |
| 3.5 | 4.22 | 4.34 | 4.21 | 4.22 | 4.24 | |
| 3 | 3.57 | 3.64 | 3.66 | 3.55 | 3.67 | |
| 2.5 | 2.98 | 2.86 | 2.98 | 2.98 | 2.94 | |
| 2 | 2.22 | 2.23 | 2.26 | 2.29 | 2.26 | |
| 1.5 | 1.57 | 1.7 | 1.69 | 1.63 | 1.57 | |
| 1 | 1.07 | 1.07 | 1.11 | 1.16 | 1.11 | |
| 0.5 | 0.52 | 0.61 | 0.61 | 0.61 | 0.45 | |
| 0 | 0.02 | 0.08 | 0.08 | −0.03 | 0.06 | |

**Table. 1**

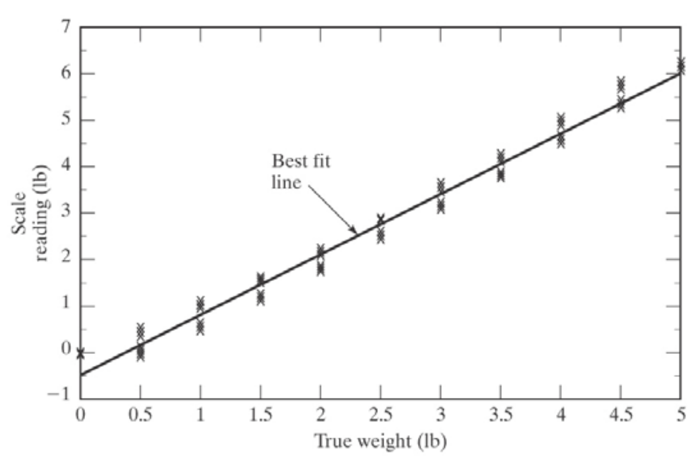
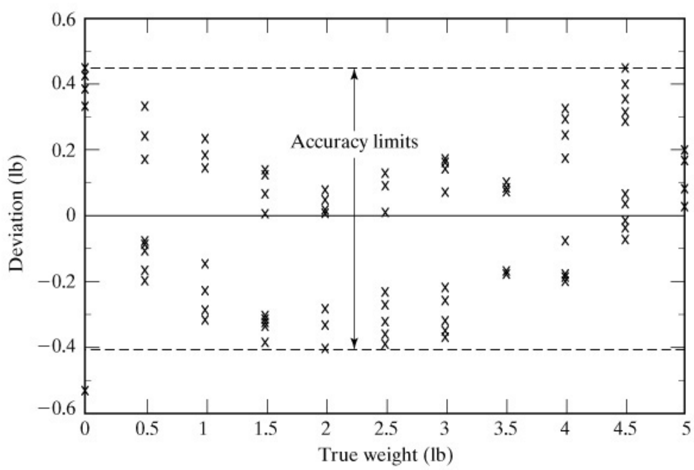
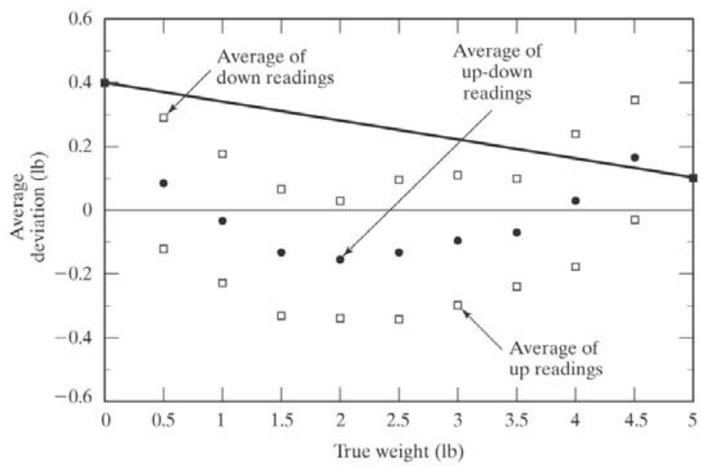


TABLE E2.3(b) Scale Deviation Data

| | Deviation | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| True weight (lb) | Cycle 1 | Cycle 2 | Cycle 3 | Cycle 4 | Cycle 5 | Cycle 6 | Average of cycles | Average up–down | Repeat |
| 0 | | | | | | | | 0.41 | |
| 0.5 | | −0.07 | −0.19 | −0.1 | −0.08 | −0.16 | −0.12 | 0.085 | 0.12 |
| 1 | | −0.22 | −0.14 | −0.28 | −0.31 | −0.22 | −0.23 | −0.025 | 0.17 |
| 1.5 | | −0.38 | −0.3 | −0.31 | −0.32 | −0.33 | −0.33 | −0.13 | 0.08 |
| 2 | | −0.4 | −0.28 | −0.4 | −0.28 | −0.33 | −0.34 | −0.15 | 0.12 |
| 2.5 | −0.23 | −0.36 | −0.39 | −0.39 | −0.27 | −0.32 | −0.35 | −0.125 | 0.16 |
| 3 | −0.35 | −0.32 | −0.26 | −0.22 | −0.37 | | −0.3 | −0.09 | 0.15 |
| 3.5 | −0.24 | −0.3 | −0.28 | −0.17 | −0.18 | | −0.23 | −0.06 | 0.13 |
| 4 | −0.2 | −0.08 | −0.18 | −0.19 | −0.19 | | −0.17 | 0.04 | 0.12 |
| 4.5 | −0.02 | −0.08 | 0.06 | 0.03 | −0.04 | | −0.01 | 0.17 | 0.14 |
| 5 | 0.16 | 0.19 | 0.02 | 0.16 | 0.08 | | 0.12 | 0.12 | 0.17 |
| 4.5 | 0.28 | 0.31 | 0.35 | 0.44 | 0.39 | | 0.35 | | 0.16 |
| 4 | 0.17 | 0.32 | 0.29 | 0.24 | 0.24 | | 0.25 | | 0.15 |
| 3.5 | 0.08 | 0.2 | 0.07 | 0.08 | 0.1 | | 0.11 | | 0.13 |
| 3 | 0.07 | 0.14 | 0.16 | 0.05 | 0.17 | | 0.12 | | 0.12 |
| 2.5 | 0.13 | 0.01 | 0.13 | 0.13 | 0.09 | | 0.1 | | 0.12 |
| 2 | 0.01 | 0.02 | 0.05 | 0.08 | 0.05 | | 0.04 | | 0.07 |
| 1.5 | 0.01 | 0.14 | 0.13 | 0.07 | 0.01 | | 0.07 | | 0.13 |
| 1 | 0.15 | 0.15 | 0.19 | 0.24 | 0.19 | | 0.18 | | 0.09 |
| 0.5 | 0.25 | 0.34 | 0.34 | 0.34 | 0.18 | | 0.29 | | 0.16 |
| 0 | 0.39 | 0.45 | 0.45 | 0.34 | 0.43 | | 0.41 | | 0.11 |




**Deviation plot**




**Steps for static calibration**

1. get a measurement system with know accuracy.
2. record the outputs from your measurement system for several values of the measurand and several cycles.
3. compare your results with the reference measurement system.
4. curve fit the data
5. compare the curve fitting and the measured values (deviation plots).

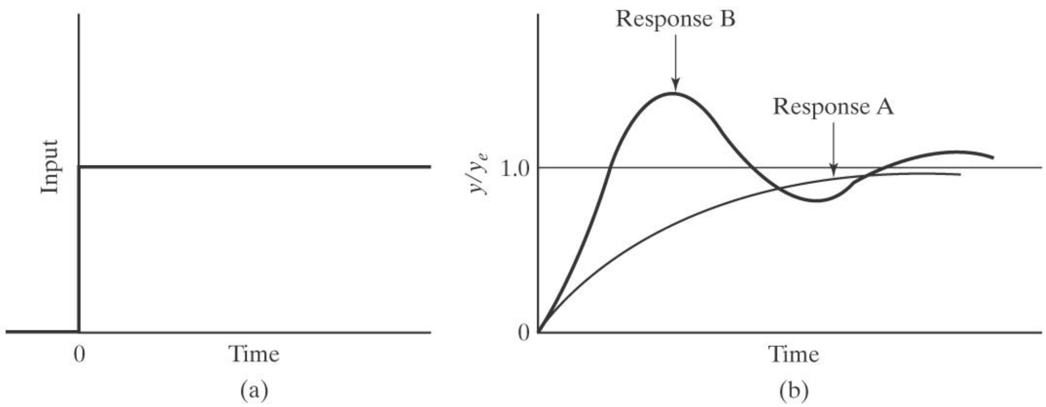
**Dynamic calibration:**

In dynamic calibration, the notion of time response has to be taken into account. This is the case for systems where it takes time to reach equilibrium (example: oral thermometer). In this case, the dynamic characteristics of the measurement system must be taken into account, usually by considering its time response.

The time response can be classified as: zero order; $1^{st}$ order and $2^{nd}$ order, dependent on the order of the differential equation needed to describe the dynamic response of the system.

- zero order response systems: respond instantaneously to the variation in the measurand.
- $1^{st}$ order response systems: show capacitance-type energy storage effects (or spring effects).
- $2^{nd}$ order response systems: have not only capacitance energy storage effects, but also inertial effects. They are characterized by damping effects (dissipation of energy). $2^{nd}$ order systems can show underdamped response (oscillatory behavior) or overdamped response (non-oscillatory behavior).

To characterize dynamically a measurement system, two common inputs are usually used: step change (gives transient response) and sinusoidal change (gives frequency response).




1st order systems and overdamped 2nd order systems show an asymptotic response. For first order systems, the response can be written under the form:

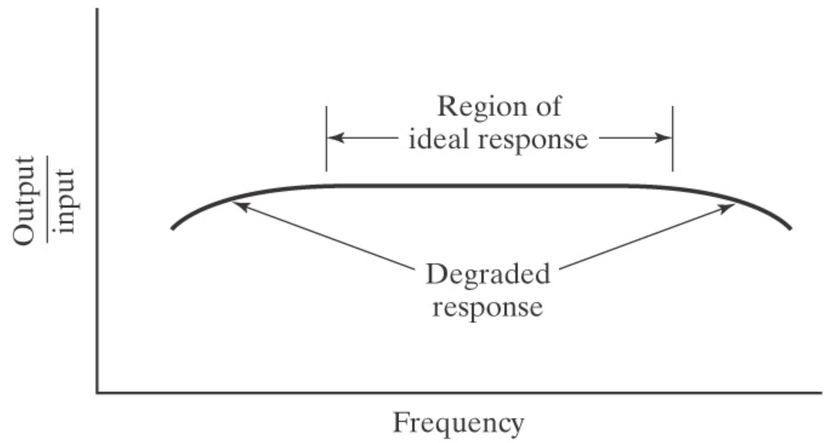
$$\frac{y}{y_e} = 1 - e^{-\frac{t}{\tau}}$$

Where $\tau$ is the time constant, it is the time at which the response has a value y=1-1/e = 0.632.
For overdamped 2nd order systems, the response is more complex and the concept of time response can be hardly applicable. So other parameters can be used to characterize both 1st and 2nd order systems:

- 95% response time: or the time at which $y/y_e$ reaches 0.95.
- Rise time: or the time it takes for the response to increase from 0 to 0.9.

For second order underdamped systems, the settling (x%) time is an important characteristic. It is defined as the time until the amplitude of the oscillations reach x% of the equilibrium response.
Practically speaking, the time of dynamic response of your system should be small compared to the time change in the measurements in the experiments.

For frequency response, the system is fed with pure sine wave inputs at different frequencies. As a result, there will be a flat range called bandwidth of the device where (Amplitude output / Amplitude input) is constant.
However, since real inputs are not sinusoidal, it is important to decompose the real signal into sine waves (Fourier transforms) and to apply, then, the appropriate frequencies.




It is also possible to characterize the system using its natural frequency.

These characteristics are not always provided by the manufacturer, since the response can depend on several parameters associated with the experiment such T°, type of material used in the experiment (solid, fluid, …).