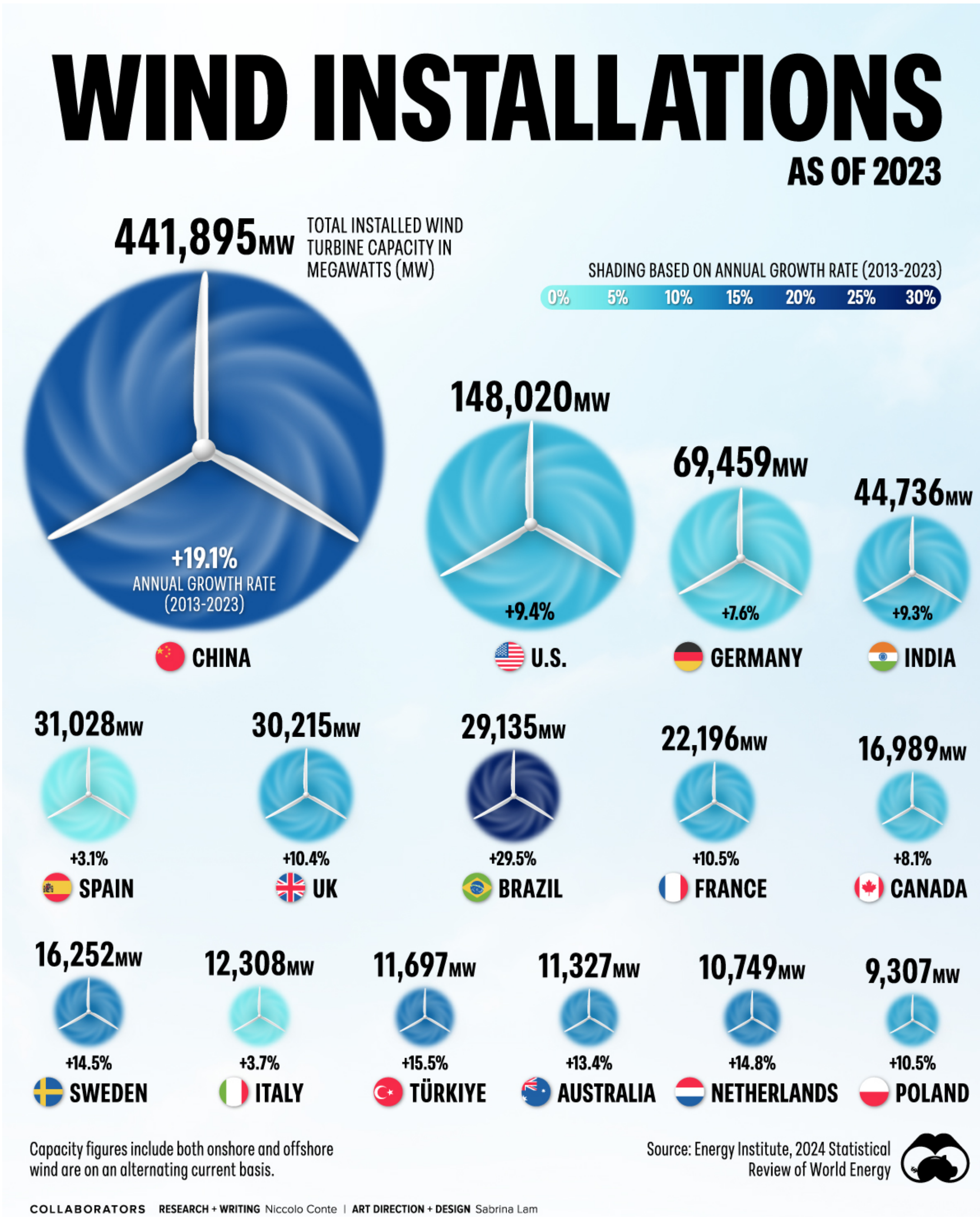
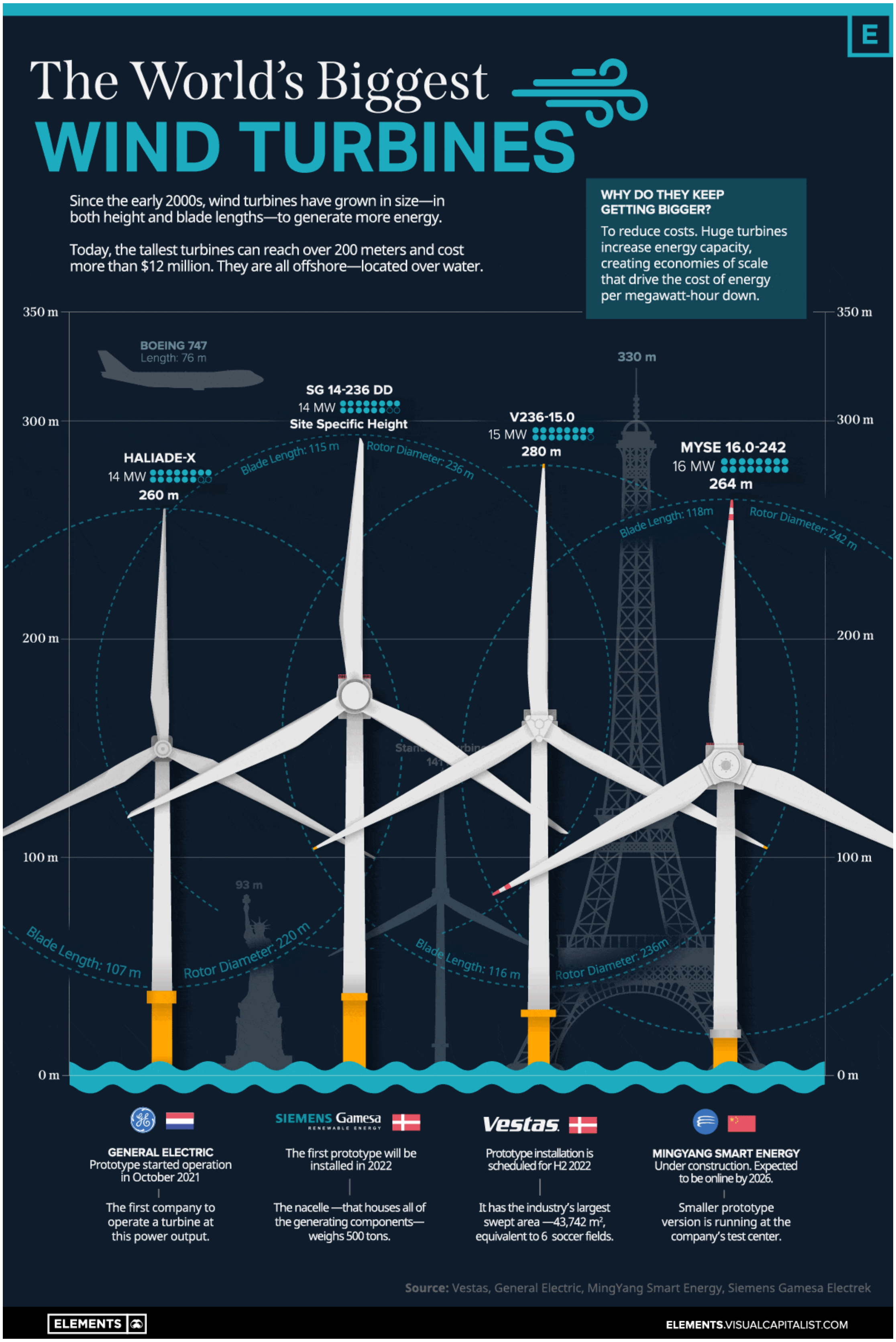
CHAPTER 3
Wind Energy




The World's Biggest
WIND TURBINES
Since the early 2000s, wind turbines have grown in size—in both height and blade lengths—to generate more energy.
Today, the tallest turbines can reach over 200 meters and cost more than $12 million. They are all offshore—located over water.
WHY DO THEY KEEP GETTING BIGGER?
To reduce costs. Huge turbines increase energy capacity, creating economies of scale that drive the cost of energy per megawatt-hour down.
350 m
300 m
200 m
100 m
0 m
BOEING 747
Length: 76 m
330 m
93 m
HALIADE-X
14 MW
260 m
Blade Length: 107 m
Rotor Diameter: 220 m
SG 14-236 DD
14 MW
Site Specific Height
Blade Length: 115 m
Rotor Diameter: 236 m
V236-15.0
15 MW
280 m
Blade Length: 116 m
Rotor Diameter: 236m
MYSE 16.0-242
16 MW
264 m
Blade Length: 118m
Rotor Diameter: 242 m
GENERAL ELECTRIC
Prototype started operation in October 2021
The first company to operate a turbine at this power output.
SIEMENS Gamesa
The first prototype will be installed in 2022
The nacelle —that houses all of the generating components— weighs 500 tons.
Vestas
Prototype installation is scheduled for H2 2022
It has the industry's largest swept area —43,742 m², equivalent to 6 soccer fields.
MINGYANG SMART ENERGY
Under construction. Expected to be online by 2026.
Smaller prototype version is running at the company's test center.
Source: Vestas, General Electric, MingYang Smart Energy, Siemens Gamesa Electrek
ELEMENTS
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Wind energy is not new and has been used for more than 6,000 years to power sailboats, grind grains, and pump water. It is important to distinguish between the terms windmill and wind turbine.

Windmill	Wind energy is used generate mechanical energy
Wind Turbine	Wind energy is used to generate electricity

Areas with an average wind speed of 6 m/s or higher are considered potential sites for economical wind power generation.

Power production (at peak design conditions)	1 to 15 MW
Rotation speed of the rotor	10 to 30 rpm

The main advantage of wind turbines is that they extract energy from the wind, a renewable form of energy. The main disadvantage is that the wind does not blow constantly; as a result, wind energy is an unsteady source of energy. Furthermore, the ideal sites for wind power generation are usually far from locations where there is a major need for electricity and power grids. This requires the construction of new high-voltage power lines.

3.1. Wind turbine types and power performance curves

Several designs of wind turbines exist (see Figure 3.1 for some examples). They are usually classified based on the orientation of their axis of rotation:

1. Horizontal-axis wind turbines (HAWT)
2. Vertical-axis wind turbines (VAWT)

Another classification is based on the mechanism for power generation: lift or drag wind turbines.

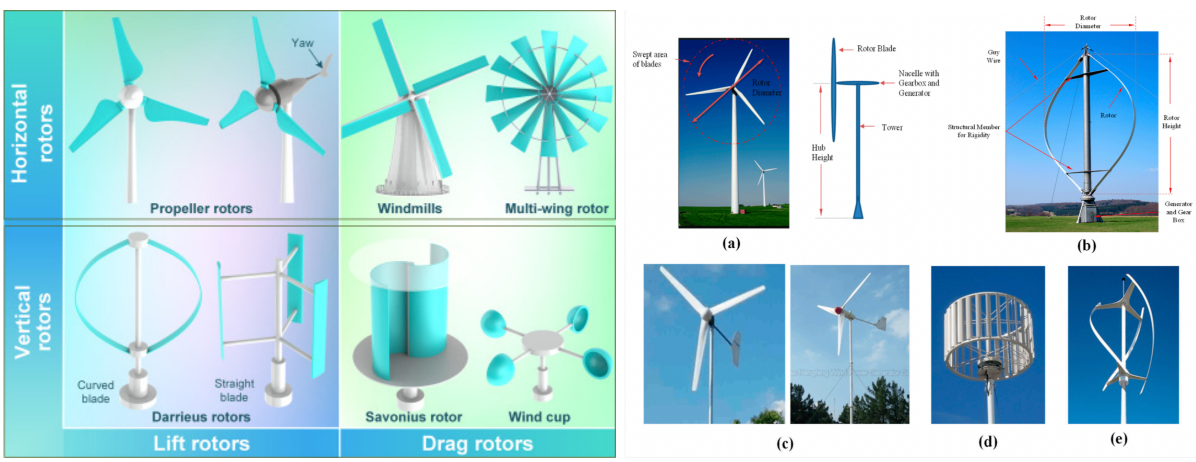


Figure 3.1. Classification of wind turbines and different types of wind turbines. (Left panel) (a) Horizontal-axis wind turbine (HAWT); (b) Vertical-axis wind turbines (VAWT); (c) Domestic (small-scale) HAWTs; (d) Domestic (small-scale) VAWT (Savonius type); (e) Domestic (small-scale) VAWT (Darrieus type).

As of now, the most performant wind turbines are of the lift-type HAWT. Note that wind turbines can be isolated or installed in clusters called wind farms. In this chapter, we will only discuss lift-type HAWTs.

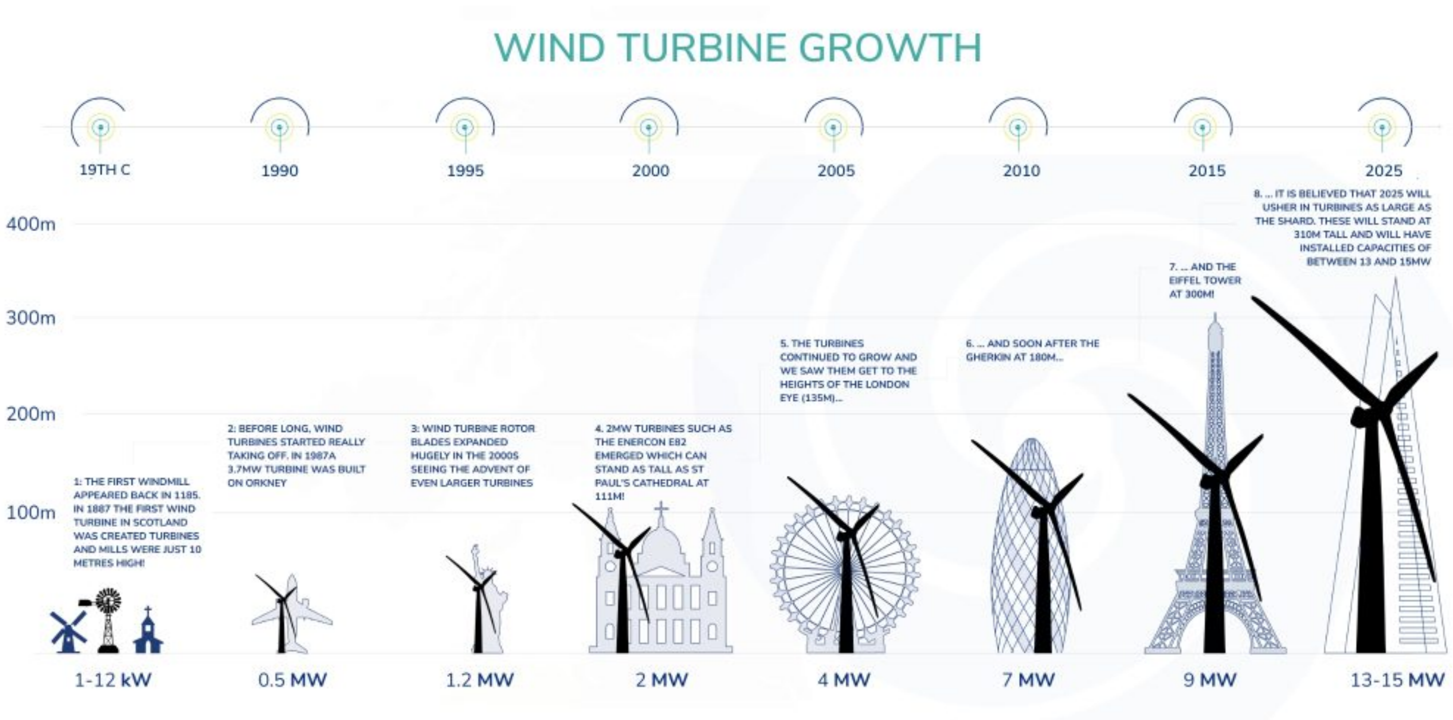


Figure 3.2. Wind turbine growth.

3.2. Wind turbine performance curve

Each wind turbine has its specific performance curve. See example below.
This curve provides essential information on the electric power production as a function of the wind speed.

Cut-in-speed	This is the minimal wind speed required to produce useful electric power
Rated speed	This corresponds to the wind speed that delivers the rated power, usually the maximal power
Cut-out-speed	This corresponds to the maximum wind speed at which the turbine is designed to produce power. Beyond this speed, the turbine blades are stopped to avoid damage.

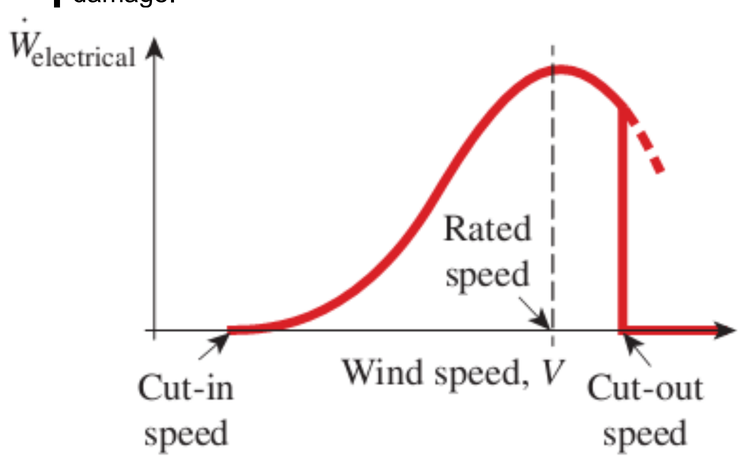


Figure 3.3. Wind turbine performance curve.

3.2. Wind turbine operations and aerodynamics

Wind turbines convert the kinetic energy of the wind into electricity. Fluid dynamics and aerodynamics play a critical role in the design and performance of the wind turbine. Wind turbine blades behave similarly to the wings of an airplane, producing lift and drag.
Different designs exist depending on whether the wind turbine uses lift or drag to generate power.
In most cases, the blades face the wind (upwind turbines), but in some designs, the blades are positioned downstream (downwind turbines). Downwind turbines are subject to alternating wind, causing noise and fatigue. However, their advantage is that they do not require a yaw system.
The rotor of the wind turbine can be directly connected to the generator (direct drive) or to a gearbox. Direct drive is usually used offshore. When connected to a gearbox, the speed is increased from 20-100 rpm to 200-300 rpm to be more compatible with electric power generation, allowing the use of smaller generators.
The turbine structure is supported by a steel tower. The tower is designed in multiple sections, and the taller, the better to capture high winds. The figure below shows the evolution of wind speed with height. However, this comes at the cost of more complex maintenance.
Modern wind turbines typically have three blades, though this was not the case for early designs, which included one, two, or even more blades for windmills. Three blades are optimal for power generation. The blades have an aerodynamic shape with varying geometry along their length to reduce drag at the blade tip.

3.3. Main components of a wind turbine

Tower	Made of steal
Blades	Made of fiberglass
Yaw drive	Orient the wind turbine with the wind by rotating the nacelle
Wind vane	Detects the wind direction
Anemometer	Measures the wind speed
Controller	Ensures a safe operation of the wind turbine by stopping it a high wind speed (usually > 25 m/s)
Pitch system	Adjust the angle of the blade with respect to the wind

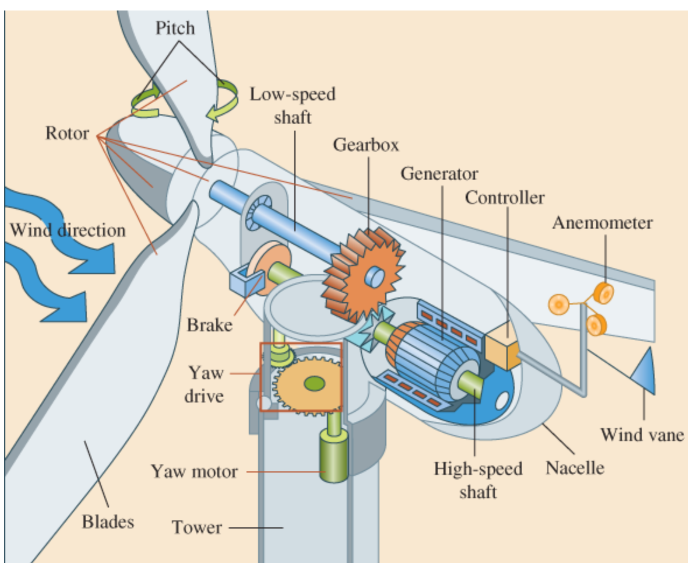


Figure 3.3. Main components of a horizontal axis wind turbine.

3.4. Wind power potential

For an ideal wind turbine, we can write:

$$\dot{W} = \dot{m}\left[\left(h_1 + \frac{1}{2}V_1^2 + gz_1\right) - \left(h_2 + \frac{1}{2}V_2^2 + gz_2\right)\right]$$

The pressure and temperature remain the same upstream and downstream of the wind turbine. There is no change in height between upstream and downstream, so the changes in potential energy can be neglected.

The available wind power is then (considering V_2 << V_1 and simply calling (V_1) V)

$$\dot{W}_{\text{available}} = \frac{1}{2}\dot{m}V^2$$

This represents the max power that can be generated for a given wind velocity or the max power energy that can be extracted from the wind.
We know that $\dot{m} = \rho AV$, with A being the circular area swept by the blades of the wind turbine, then:

$$\dot{W}_{\text{available}} = \frac{1}{2}\rho AV^3$$

This is important since it is proportional to ρ, proportional to A, but proportional to V^3. So, if the wind speed is doubled, the available wind power increases by a factor of 8. The choice of the location or positioning of the wind turbine or wind farms becomes critical. Another approach is to increase the height of the tower, but this will expose the components of the wind turbine to higher forces, potentially reducing durability and also complicating maintenance.

Considering ideal gas law and a disk area of $A = \frac{\pi D^2}{4}$, where D is the blade diameter, then,

$$\dot{W}_{\text{available}} = \frac{\pi}{8}\frac{PD^2V^3}{RT}$$

So $\dot{W} = f(P, \frac{1}{T}, D^2, V^3)$

Note that increasing D requires non-conventional manufacturing and increases the stress on the blades.

Example 3.1

A wind turbine, such as the Vestas V90-2.0 MW (with a blade diameter of 90 meters), is to be installed in a location where the average wind speed is 6 m/s. The average temperature and pressure of ambient air at this location are 25°C and 101.3 kPa, respectively.

Determine the wind power potential available to the turbine.

3.5. Wind power density

For a fair comparison between various wind turbines and locations, one has to remove the dependence on the wind turbine characteristics and determine the available wind power per unit area. This is called the wind power density (WPD):

$$\mathrm{WPD} = \frac{\dot{W}_{\text{available}}}{A} = \frac{1}{2}\rho V^3$$

This can give us the instantaneous WPD. However, what is preferable—given that wind speed changes with time and throughout the day—is to determine an average value. More specifically, hourly wind speed averages for the entire year:

$$\mathrm{WPD}_{\text{avg}} = \frac{\overline{\dot{W}}_{\text{available}}}{A} = \frac{1}{2}\rho \bar{V}^3$$

This is important information since it allows us to define some rules of thumb for site selection with:

$\mathrm{WPD}_{avg} < 100$ W/m^2	Poor site
$\mathrm{WPD}_{avg} \approx 400$ W/m^2	Good site
$\mathrm{WPD}_{avg} > 700$ W/m^2	Great site

Example 3.2

Consider two locations, location A and location B with average wind power densities of 250 W/m^2 and 500 W/m^2, respectively. Determine the average wind speed in each location. Take the density of air to be 1.18 kg/m^3.
If a turbine with a diameter of 40 m is to be installed in location A and a turbine with a diameter of 20 m is to be installed in location B, what is the ratio of wind power potentials in location A and B?

3.5. Wind turbine efficiency

Wind turbine efficiency defines the percentage of wind power captured by the wind turbine and converted into shaft power.

$$\eta_{\text{wt}} = \frac{\dot{W}_{\text{shaft}}}{\dot{W}_{\text{available}}} = \frac{\dot{W}_{\text{shaft}}}{\frac{1}{2}\rho A V^3}$$

The endpoint for using wind turbines is not to create shaft work ($\dot{W}_{shaft}$) but instead to create electric work ($\dot{W}_{electric}$). For this we use a gearbox and a generator with each having a certain efficiency so:

$$\dot{W}_{\text{electric}} = \eta_{\text{gearbox/generator}}\dot{W}_{\text{shaft}}$$

With η_{gearbox} and $\eta_{\text{generation}}$ typically higher than 80%.

We can also define the overall wind turbine efficiency as:

$$\eta_{\text{wt, overall}} = \frac{\dot{W}_{\text{electric}}}{\dot{W}_{\text{available}}} = \frac{\dot{W}_{\text{electric}}}{\frac{1}{2}\rho A V^3}$$

The efficiency of a wind turbine is usually referred to as the **power coefficient** (Cp).

Let us now consider a control volume around the wind turbine and let us try to evaluate the relationship between the velocity at the inlet V_1 and the velocity at the exit V_2.

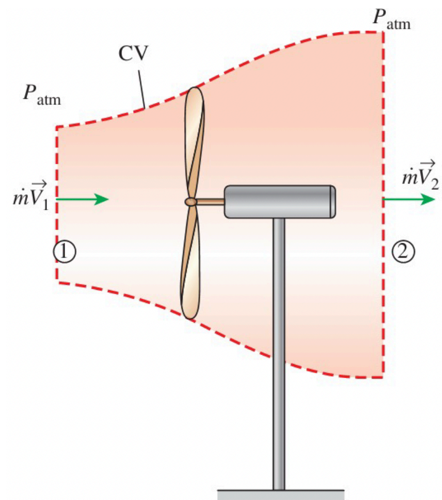


Figure 3.4. Control volume around the wind turbine.

From the 1st law, we get:

$$\dot{m}\frac{V_1^2}{2} = \dot{W}_{\text{shaft}} + \dot{m}\frac{V_2^2}{2}$$

But,

$$\eta_{\text{wt}} = \frac{\dot{W}_{\text{shaft}}}{\dot{m}\dfrac{V_1^2}{2}}$$

So,

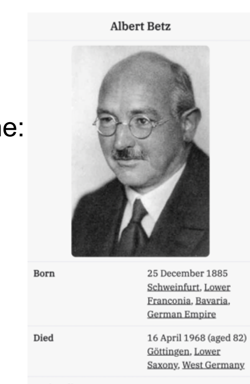
$$\dot{m}\frac{V_2^2}{2} = \dot{m}\frac{V_1^2}{2}(1 - \eta_{\text{wt}})$$

and the exit velocity V_2 is related to the inlet velocity V_1 by:

$$V_2 = V_1\sqrt{1 - \eta_{\text{wt}}}$$

This is the exit velocity when the frictional effects are neglected.

3.6. Betz limit for wind turbine efficiency



Looking at the 1st law of thermodynamics applied to the control volume around the wind turbine:

$$\dot{m}\frac{V_1^2}{2} = \dot{W}_{\text{shaft}} + \dot{m}\frac{V_2^2}{2}$$

We can see that for getting the maximal work, V_2 should be equal to zero.

$$\eta = 100\% \quad \Longrightarrow \quad \dot{W}_{shaft} = \dot{m}\frac{V_1^2}{2}$$

So, for a wind turbine to 100% efficient, the wind turbine must extract all the energy from the wind leading to a fluid at rest $V_2 = 0$ at the exit.
But in practice, this is impossible since air must be removed at the wind turbine exit to maintain the mass flow rate.

Conservation of mass:

$$\cancel{\frac{dm}{dt}} = \dot{m}_1 - \dot{m}_2$$

So, $\dot{m}_1 = \dot{m}_2 = \rho \cancel{V_2} A_2$

Then $\dot{m}_1 = 0$

If reaching 100% efficiency is not possible, is there a maximum efficiency that cannot be exceeded? (An analogy can be drawn here with Carnot efficiency, although Carnot efficiency represents a fundamental limit, not a practical one.) This question was answered by Albert Betz (1885–1968).

Consider two control volumes surrounding the wind turbine.

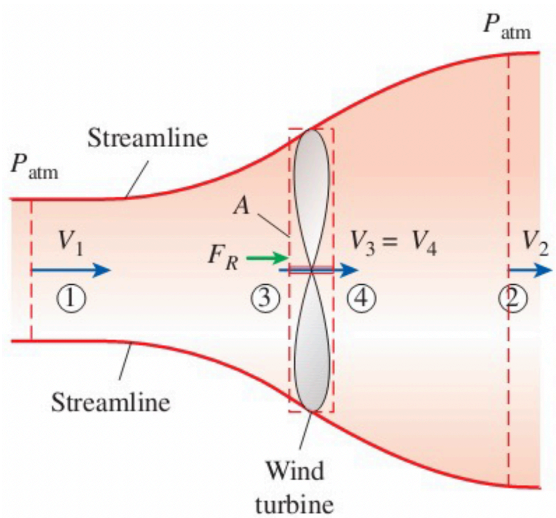


Figure 3.5. Two control volumes around the wind turbine.

Let us consider $P_1 = P_2 = P_{atm}$. This means that P_1 and P_2 are sufficiently far from the wind turbine.
The momentum equation gives for the reaction force on the turbine:

$$F_R = \dot{m}(V_2 - V_1)$$

We assume the wind turbine to be as a disk with $A_3 = A_4$. Also, assuming an incompressible flow then:

$$\dot{m}_3 = \dot{m}_4 \Leftrightarrow \rho_3 V_3 A_3 = \rho_4 V_4 A_4 \Longrightarrow V_3 = V_4$$

But the wind turbine extracts energy from the wind so $P_3 \neq P_4$ (something must drop for the work to increase and to be extracted).

Then,

$$F_R + P_3 A - P_4 A = 0 \quad \rightarrow \quad F_R = (P_4 - P_3)A$$

Now let's apply the Bernoulli equation between (1) and (3) and between (2) and (4). Note that we can't apply Bernoulli between (1) and (2) since the wind turbine is extracting energy from the air.

So, between (1) and (3):

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_3}{\rho g} + \frac{V_3^2}{2g} + z_3$$

And between (2) and (4):

$$\frac{P_4}{\rho g} + \frac{V_4^2}{2g} + z_4 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

So, the pressure increases between (1) and (3), drops suddenly between (3) and (4) across the wind turbine and then recovers to (2) (equal the atmospheric pressure).

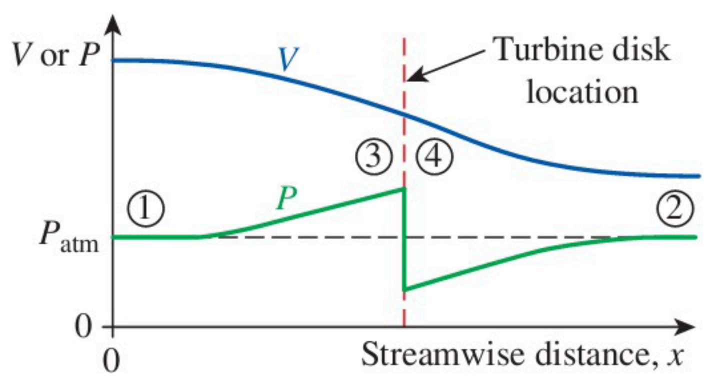


Figure 3.6. Variation of the velocity and pressure along the wind turbine.

Now knowing that $P_1 = P_2 = P_{atm}$ and $V_3 = V_4$ leads to:

$$\frac{V_1^2 - V_2^2}{2} = \frac{P_3 - P_4}{\rho}$$

We also have that: $\dot{m}_3 = \rho_3 V_3 A_3 = \rho V_3 A$, so:

$F_R = \rho V_3 A(V_2 - V_1)$ and $F_R = (P_4 - P_3)A$

This leads to:

$(P_4 - P_3)A = \rho V_3 A(V_2 - V_1)$ or $(P_3 - P_4) = \rho V_3(V_1 - V_2)$

Replacing in

$\frac{V_1^2 - V_2^2}{2} = \frac{P_3 - P_4}{\rho}$ leads $\frac{V_1^2 - V_2^2}{2} = \frac{\rho V_3(V_1 - V_2)}{\rho}$

So,

$$V_3 = \frac{V_1 \ + V_2}{2}$$

Then the velocity of the air through an ideal wind turbine is the arithmetic average of the far upstream and far downstream velocities.

Let us define a new variable (a) as the relative fractional loss of velocity upstream of the wind turbine:

$$a = \frac{V_1 - V_3}{V_1}$$

Then $V_3 = V_1(1-a)$

And $\dot{m} = \rho V_3 A = \rho V_1 (1-a) A$

Now replacing in the expression of V_3

$$V_3 = \frac{V_1 \ + V_2}{2} \Leftrightarrow V_2 = V_1(1-2a)$$

So, for an ideal wind turbine (no irreversible losses):

$$\dot{W}_{ideal} = \dot{m}\frac{V_1^2 - V_2^2}{2} = \rho A V_1 (1-a)\frac{V_1^2 - V_1^2(1-2a)^2}{2} = 2\rho A V_1^3 a(1-a)^2$$

And the efficiency of a wind turbine:

$$\eta_{WT} = \frac{\dot{W}_{shaft}}{\frac{1}{2}\rho V_1^3 A} = \frac{\dot{W}_{ideal}}{\frac{1}{2}\rho V_1^3 A} = \frac{2\rho A V_1^3 a(1-a)^2}{\frac{1}{2}\rho V_1^3 A} = 4a(1-a)^2$$

So, to get the maximum efficiency, we need to calculate $\frac{d\eta}{da} = 0$ this gives a=1 which is the trivial solution (leading to V_3 = 0, no power generation) and a=1/3.

Replacing the value of a=1/3 in the expression of the efficiency of the wind turbine leads to:

$$\eta_{WT}\Big|_{max}^{a=\frac{1}{3}} = 4a(1-a)^2 = \frac{16}{27} = 0.5926$$

This is the maximum reachable efficiency of any wind turbine and then is called the Betz limit.

Existing wind turbines have a lower efficiency because primary: 1) rotation of the wake behind the rotor; 2) finite number of rotor blades; 3) tip vortices and drag.

Example 3.2

A Siemens Gamesa SWT-2.3-50 wind turbine with a rotor diameter of 50 meters is operating at 25 rpm under steady wind conditions, with an average wind velocity of 10 m/s. The turbine generates an electrical power output of 375 kW, and the combined efficiency of the gearbox and generator is 90%. The density of air is assumed to be 1.2 kg/m³. Determine:
(a) The overall efficiency of the wind turbine.
(b) The tip speed of the blade in km/h.
(c) The air velocity at the turbine exit if the turbine operates ideally at the Betz limit.

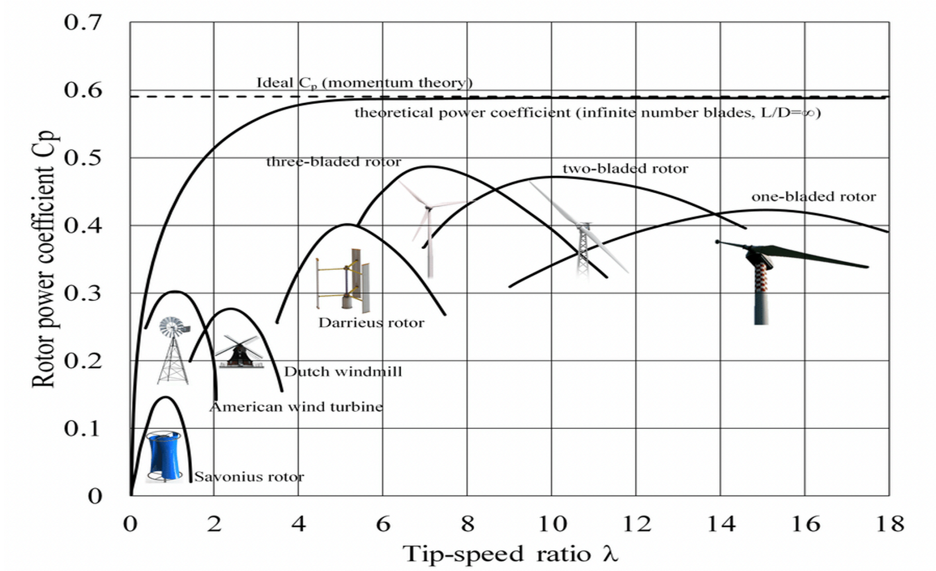


Figure 3.7. Power coefficient (or efficiency) as a function of tip-speed ratio. The dotted line represents Betz limit.