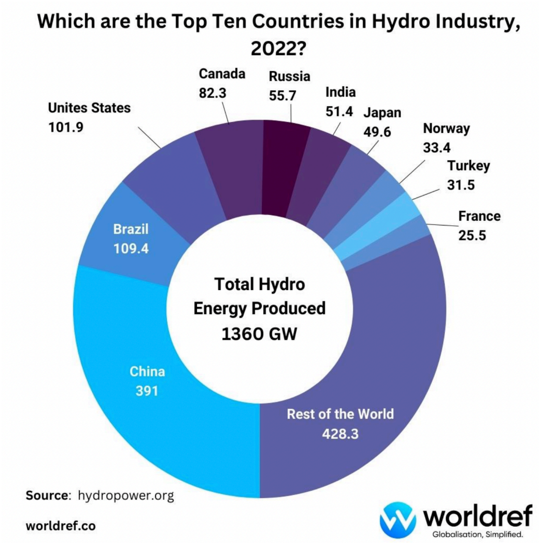
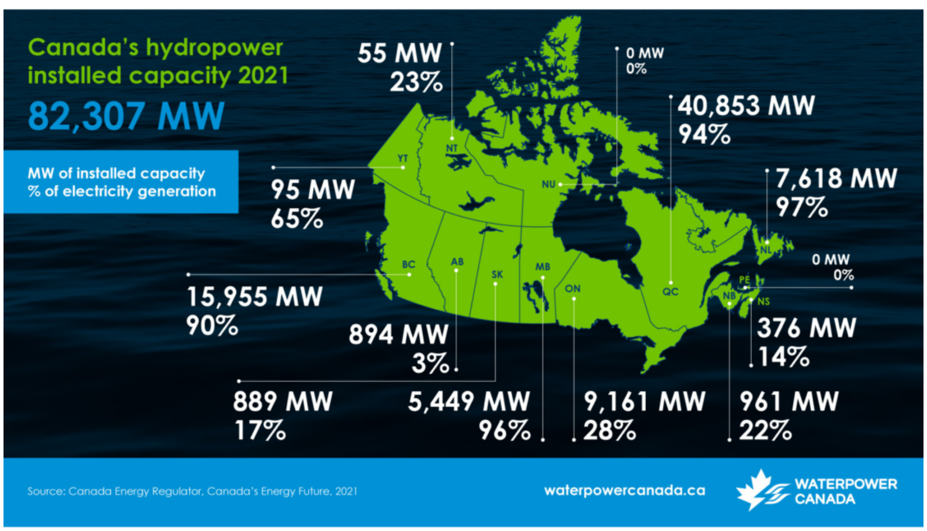
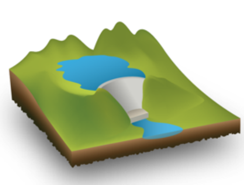
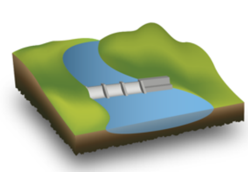
CHAPTER 5
HydroPower







Accumulation
Hydroelectric Power Plant



Run-of-the-river
Hydroelectric Power Plant

Derivational
Hydroelectric Power Plant

Pumped-storage
Hydroelectric Power Plant

Hydropower has been used for centuries even millennia as a way to convert the power of water to mechanical energy. Dams are usually used to store water potential energy that is converted into mechanical energy and then electricity through turbines. A large dam takes a long time to build and requires important investment in capital. But, the cost of producing electricity by hydropower is much lower than producing electricity using fossil fuels.

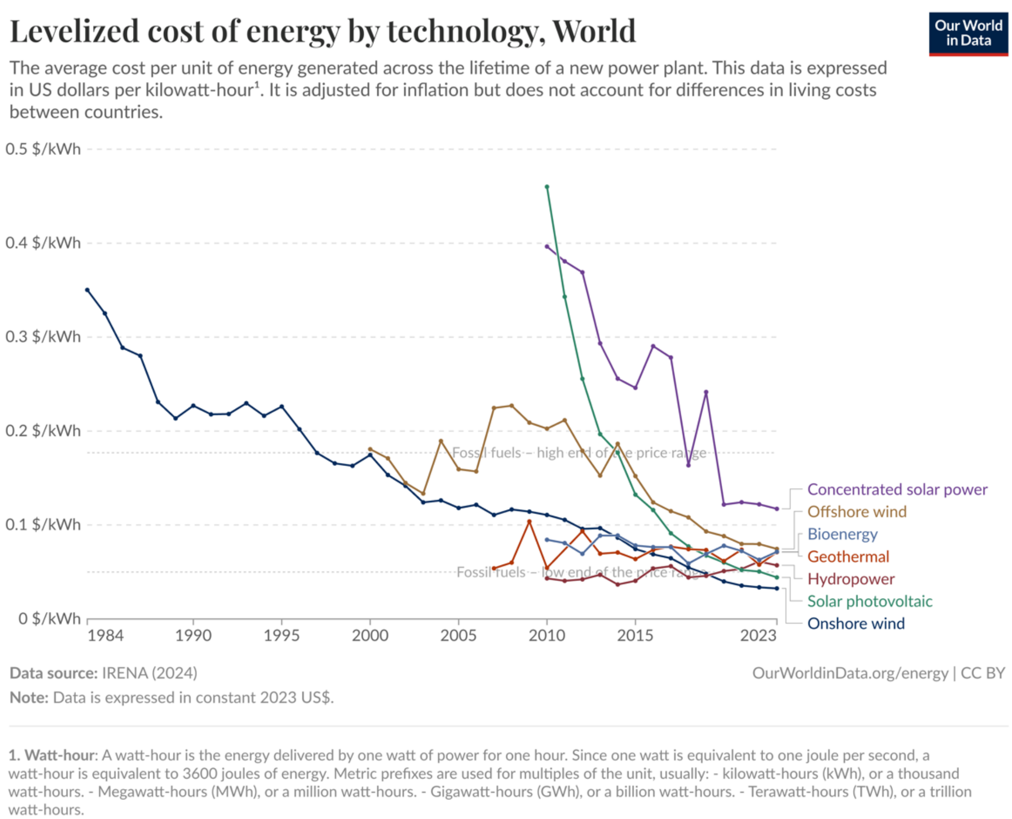
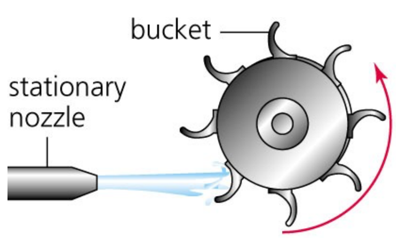


Figure 5.1. Levelized cost of energy by technology in the world.

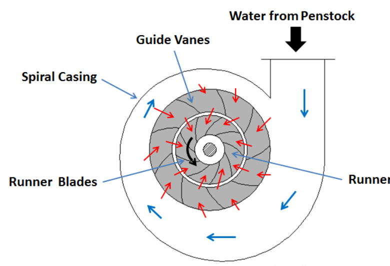
The first hydraulic turbine power generation set was developed around 1882 and generated around 6 kW of power to light bulbs. The first known hydroelectric power plant was operated in 1883 in Northern Ireland. It produced 39 kW using two turbines. Most large hydroelectric power plants have several turbines working in parallel. This gives a significant flexibility since some turbines can be turned off during low demand or for maintenance. An example of a major historical engineering achievement is the Hoover dam that includes 17 turbines, each producing about 140 MW of electricity. Currently, the largest hydroelectric power dam is the Three Gorges Dam, in China, producing around 22.5 GW of energy.
In hydroelectric power plants, large turbines are used to produce electricity. There are two basic types:

Impulse turbines



Requires a higher head but can operate with a smaller volume rate.

Reaction turbine



Can operate with much less head but requires a higher volume flowrate.

5.1. Analysis of hydroelectric power plant

Consider an ideal turbine (absence of irreversible losses) and an incompressible flow. The mechanical energy is the sum the flow energy, the kinetic energy and the potential energy and gives:

$$e_{\text{mech}} = \frac{P}{\rho} + \frac{V^2}{2} + gz$$

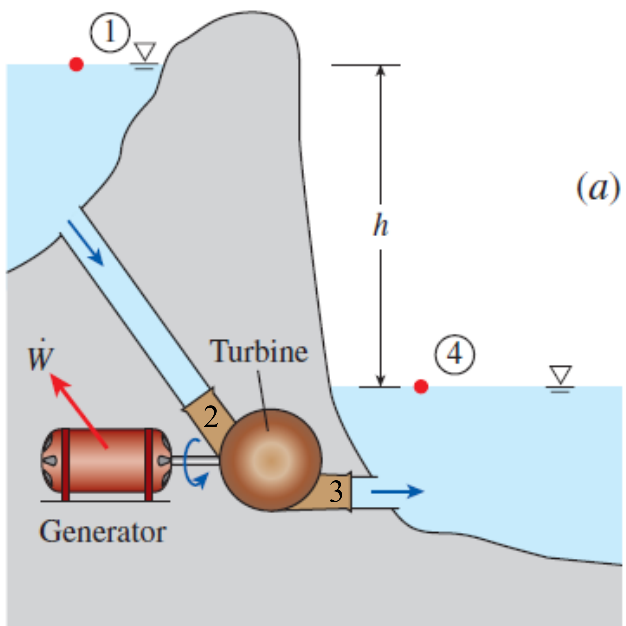


Figure 5.2. Schematic representation of electric power generation using a hydraulic turbine.

Now between two states:

$$\Delta e_{\text{mech}} = \frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

The ideal (maximum) power generated by a turbine is:

$$\dot{W}_{\max} = \dot{m}\Delta e_{\text{mech}}$$

Let us consider now two points, (1) at the surface of the water reservoir and (4) at the surface of the discharge water from the turbine, then:

$$\dot{W}_{\max} = \dot{m}\Delta e_{\text{mech}} = \dot{m}g(z_1 - z_4) = \dot{m}gh$$

Since

$$P_1 \approx P_4 = P_{\text{atm}} \quad \text{and} \quad V_1 = V_4 \approx 0$$

And through the turbine

$$\dot{W}_{\max} = \dot{m}\Delta e_{\text{mech}} = \dot{m}\frac{P_2 - P_3}{\rho} = \dot{m}\frac{\Delta P}{\rho}$$

Since

$$V_2 \approx V_3 \quad \text{and} \quad z_2 \approx z_3$$

We can define the turbine efficiency as the ratio between the shaft power and the maximum power:

$$\eta_{\text{turbine}} = \frac{\dot{W}_{\text{shaft}}}{\Delta\dot{E}_{\text{mech,fluid}}} = \frac{\dot{W}_{\text{shaft}}}{\dot{m}\Delta e_{\text{mech}}} = \frac{\dot{W}_{\text{shaft}}}{\dot{m}gh} = \frac{\dot{W}_{\text{shaft}}}{\dot{W}_{\max}}$$

This efficiency can approach 100% (but never attain) if the frictional effects are minimized.

We can also define the generator efficiency as:

$$\eta_{\text{generator}} = \frac{\dot{W}_{\text{electric}}}{\dot{W}_{\text{shaft}}}$$

Since the turbine is usually packaged together with its generator, we can define the overall efficiency as:

$$\eta_{\text{turbine-generator}} = \eta_{\text{turbine}}\eta_{\text{generator}} = \left(\frac{\dot{W}_{\text{shaft}}}{\dot{W}_{\max}}\right)\left(\frac{\dot{W}_{\text{electric}}}{\dot{W}_{\text{shaft}}}\right) = \frac{\dot{W}_{\text{electric}}}{\dot{W}_{\max}}$$

Most turbines have an efficiency approaching 90% with large hydro turbines achieving an overall efficiency of about 95%.

A more in-depth analysis of the hydroelectric power plant should consider the penstock, the piping system between the upper and lower water levels.

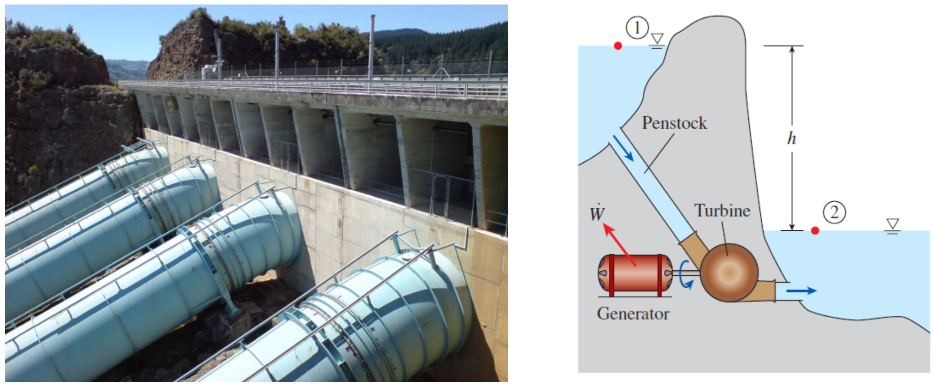


Figure 5.3. Penstock in a hydroelectric power plant.

Then, considering the system including the penstock and the turbine:

$$e_{\text{mech, in}} = e_{\text{mech, out}} + e_{\text{mech, loss}}$$

or

$$\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 + w_{\text{turbine}} + e_{\text{mech, loss}}$$

Or in terms of power:

$$\dot{m}\left(\frac{P_1}{\rho_1} + \frac{V_1^2}{2} + gz_1\right) = \dot{m}\left(\frac{P_2}{\rho_2} + \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech, loss, total}}$$

Where $\dot{E}_{\text{mech,loss,total}}$ represents the total mechanical power losses including the loss due to the turbine as well as the loss due to frictional effects in the piping network:

$$\dot{E}_{\text{mech, loss, total}} = \dot{E}_{\text{mech, loss, turbine}} + \dot{E}_{\text{mech, loss, piping}}$$

We can rewrite the conservation of energy, to express it in terms of head, by dividing both sides by $\dot{m}g$ as:

$$\frac{P_1}{\rho_1 g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho_2 g} + \frac{V_2^2}{2g} + z_2 + h_{\text{turbine},e} + h_L$$

With $h_{turbine,e}$ represents the extracted head from the fluid by the turbine.

$$h_{\text{turbine},e} = \frac{w_{\text{turbine},e}}{g} = \frac{\dot{W}_{\text{turbine},e}}{\dot{m}g} = \frac{\dot{W}_{\text{turbine}}}{\eta_{\text{turbine}}\dot{m}g}$$

And h_L is the irreversible head loss between (1) and (2) due to piping system (including the penstock):

$$h_L = \frac{e_{\text{mech, loss, piping}}}{g} = \frac{\dot{E}_{\text{mech, loss, piping}}}{\dot{m}g}$$

For the penstock, this can be calculated as:

$$h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = f\frac{L}{D}\frac{V^2}{2g} + \sum K_L \frac{V^2}{2g} = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$

where f = Darcy friction factor. It can be determined from the Moody chart (or Colebrook equation) for turbulent flow. For laminar flow, f = 64/Re, where Re is the Reynolds number.

L = length of the penstock

D = diameter of the penstock

V = velocity of water in the penstock

K_L = loss coefficient for minor losses in the piping system

The figure below shows a typical hydroelectric power plant.

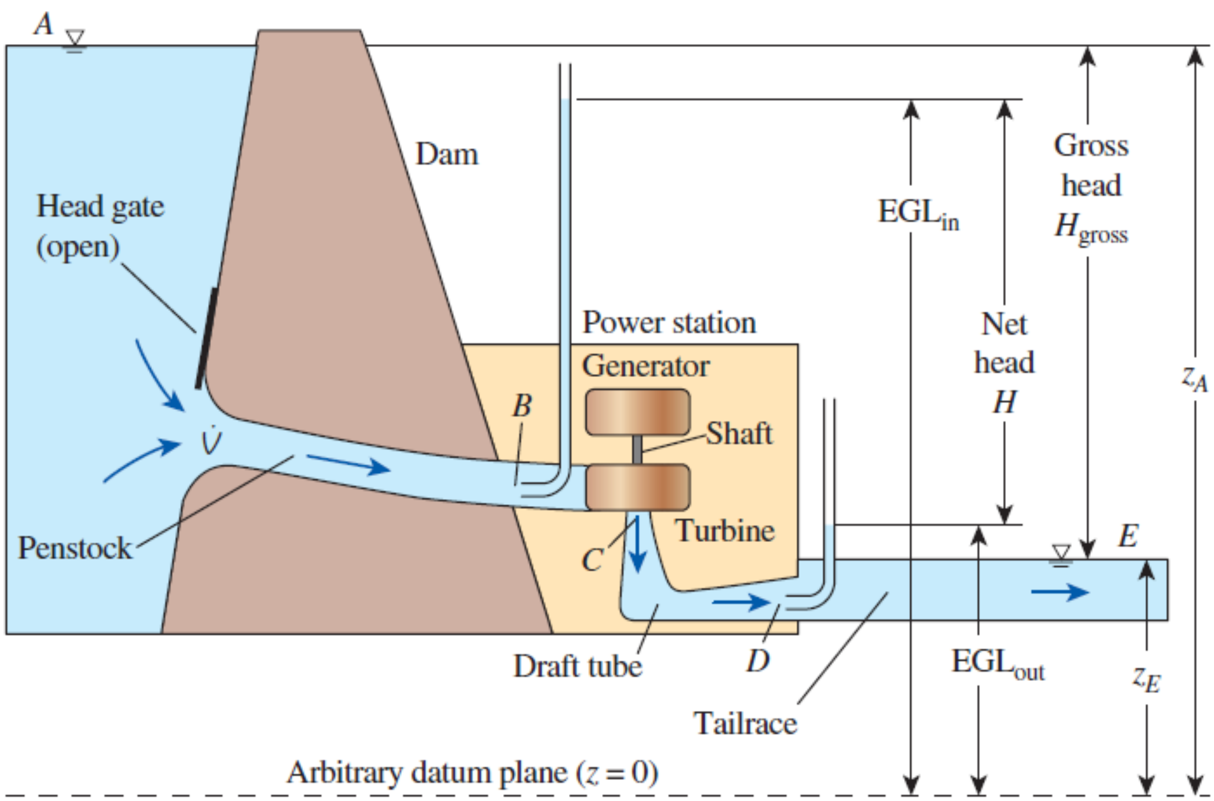


Figure 5.4. Typical hydroelectric power plant and typical terminologies used.

The overall elevation difference between the reservoir surface upstream of the dam and downstream of the dam is called the **gross head**:

$$H_{\text{gross}} = z_A - z_E$$

Assuming no irreversible losses anywhere in the system, the maximum power that could be generated by the turbine would be:

$$\dot{W}_{\max} = \rho g \dot{V} H_{\text{gross}}$$

With $\dot{V}$ is the volumetric flowrate.

However, the actual power produced will be lower than the one predicted by the gross head. This is because of different irreversible losses occurring along the path of the water. Starting from the penstock and the head gate (a valve that can control the flow entering the turbine). If one puts a pitot tube just before the turbine, the water will rise up to a height equal to the energy grade line EGL_{in}. Note that $EGL_{in} < H_{gross}$ because of the losses in the penstock and its inlet.
After passing through the turbine, in the context of a Francis reaction turbine, the water enters an expanding area diffuser called a **draft tube** which turns the flow horizontally and converts the kinetic energy of the flow to pressure before discharging into the downstream water called **the tailrace**.
Again, if one puts a pitot tube at the exit of the draft tube, the water will rise up to an elevation EGL_{out}. Note that EGL_{out} is higher than Z_E because the remaining velocity. Then one can write:

$$H_{\text{net}} = \text{EGL}_{\text{in}} - \text{EGL}_{\text{out}}$$

So we can then express the efficiency of the turbine in a more realistic way based on H_{net} and not H_{gross} as:

$$\eta_{\text{turbine}} = \frac{\dot{W}_{\text{shaft}}}{\rho g \dot{V} H_{\text{net}}}$$

Since the turbine is not responsible for the losses generated by the penstock and for those at the exit of the draft tube.

If the head loss h_L in the piping is known, then, the corresponding power loss can be determined as:

$$\dot{E}_{\text{mech,loss,piping}} = \rho g \dot{V} h_L$$

The turbine efficiency becomes:

$$\eta_{\text{turbine}} = \frac{\dot{W}_{\text{shaft}}}{\dot{W}_{\max} - \dot{E}_{\text{mech,loss,piping}}} = \frac{\dot{W}_{\text{shaft}}}{\rho g \dot{V} H_{\text{gross}} - \rho g \dot{V} h_L} = \frac{\dot{W}_{\text{shaft}}}{\rho g \dot{V} (H_{\text{gross}} - h_L)}$$

We can also define an efficiency for the piping system as:

$$\eta_{\text{piping}} = 1 - \frac{\dot{E}_{\text{mech, loss, piping}}}{\dot{W}_{\max}}$$

And knowing that:

$$\eta_{\text{generator}} = \dot{W}_{\text{electric}} / \dot{W}_{\text{shaft}}$$

This gives the following expression for the overall efficiency of the power plant:

$$\eta_{\text{plant}} = \eta_{\text{generator}}\eta_{\text{turbine}}\eta_{\text{piping}}$$

$$= \left(\frac{\dot{W}_{\text{electric}}}{\dot{W}_{\text{shaft}}}\right)\left(\frac{\dot{W}_{\text{shaft}}}{\dot{W}_{\text{max}} - \dot{E}_{\text{mech,loss,piping}}}\right)\left(1 - \frac{\dot{E}_{\text{mech,loss,piping}}}{\dot{W}_{\text{max}}}\right)$$

$$= \left(\frac{\dot{W}_{\text{electric}}}{\dot{W}_{\text{shaft}}}\right)\left[\frac{\dot{W}_{\text{shaft}}}{\dot{W}_{\text{max}}(1 - \dot{E}_{\text{mech,loss,piping}} / \dot{W}_{\text{max}})}\right]\left(1 - \frac{\dot{E}_{\text{mech,loss,piping}}}{\dot{W}_{\text{max}}}\right)$$

$$= \frac{\dot{W}_{\text{electric}}}{\dot{W}_{\text{max}}}$$

Example 5.1

The water in a large dam is to be used to generate electricity by installing a Voith Francis turbine, a widely used and reliable hydraulic turbine. The elevation difference between the free surfaces upstream and downstream of the dam is 320 m. Water is supplied to the turbine at a rate of 8000 L/s. The turbine efficiency is 93% based on the net head, and the generator efficiency is 96%. The total irreversible head loss (including major and minor losses) in the piping system and penstock is estimated to be 7.5 m.

Determine:
1- The overall efficiency of the hydroelectric plant
2- The electric power produced
3- The turbine shaft power

5.2. Types of turbines

5.2.1. Impulse turbines

In an impulse turbine, the fluid is sent through a nozzle so that most of its mechanical energy is converted into kinetic energy. This high-speed, high-energy jet impacts bucket-shaped vanes, transferring energy to the turbine shaft.
The most commonly used impulse turbine was invented by Lester A. Pelton around 1878, and it is still in use today. To honor this invention by Pelton, the rotating wheel is now called a Pelton wheel. The Pelton wheel includes buckets designed to split the flow in half and turn it nearly 180° (with respect to a frame of reference moving with the bucket).
According to legend, Pelton modeled the splitter shape after the nostrils of a cow's nose.

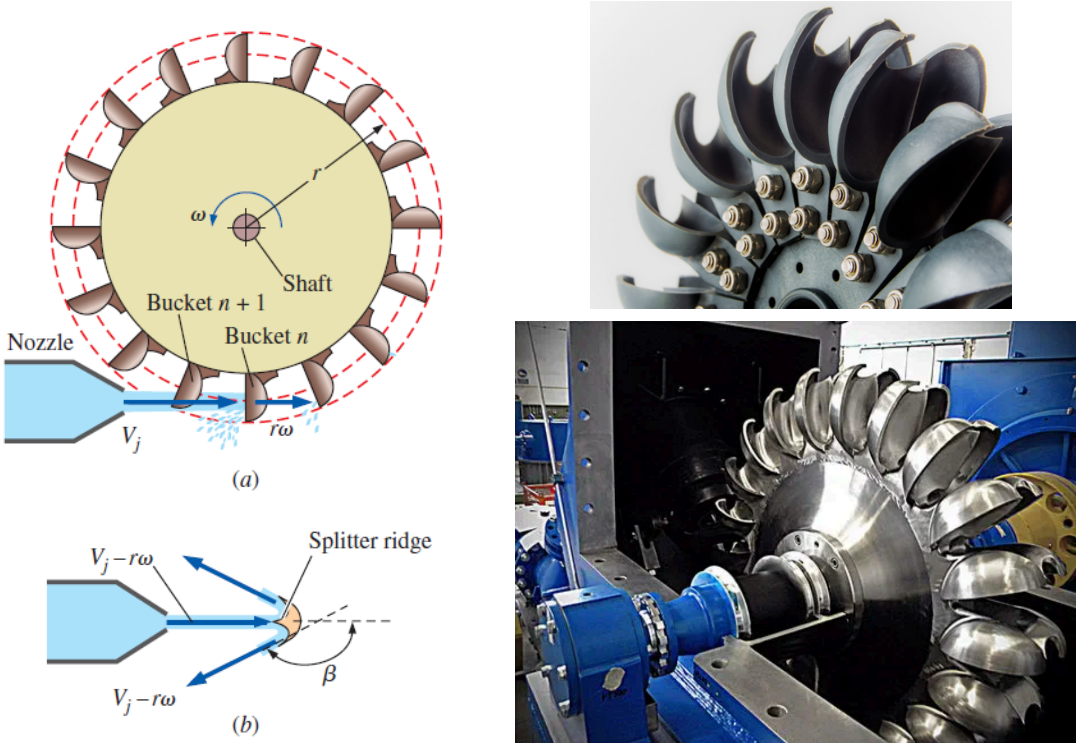


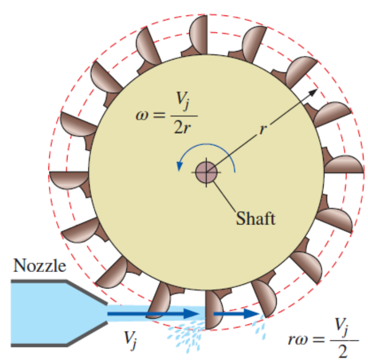
Figure 5.5. Schematic diagram of a Pelton-type impulse turbine

Analysis of the power output of a Pelton wheel

$$\dot{W}_{\text{shaft}} = \omega T_{\text{shaft}} = \rho \omega \dot{V}(r_2 V_{2,t} - r_1 V_{1,t})$$

Where, by convention (2) is the inlet and (1) is the outlet.
The angle β of the deflected jet can not reach 180° otherwise it will hit the bucket before it. In practice, the maximum power is achieved for β = 160°- 165°.
For a velocity of the jet V_J, the shaft power is given by:

$$\dot{W}_{\text{shaft}} = \rho r \omega \dot{V}(V_j - r\omega)(1 - \cos\beta)$$



The maximum shaft power is obtained differentiating the shaft power with respect to rω. This gives that the theoretical maximum power achievable by a Pelton turbine occurs when the wheel rotates at $\omega = V_j/(2r)$, that is, when the bucket moves at half the speed of the water jet.
Losses in energy conversion include hydrodynamic drag, mechanical friction, non-alignment of the jet and the bucket. Even with this, the efficiency of a Pelton wheel can approach 90%, defined as the ratio of the rotating shaft energy over the mechanical energy of the water jet.

5.2.2. Reaction turbines

The other type often used is the reaction turbines. They include:

Stay vanes	fixed guide vanes
Wicket gates	adjustable guide vanes

Runner blades	rotating blades

Momentum is exchanged between the fluid and the runner as it rotates. In the case of reaction turbines, the water completely fills the casing, unlike impulse turbines where water flows in jets. As a result, a reaction turbine can produce more power than an impulse turbine of the same diameter, net head, and volume flow rate.

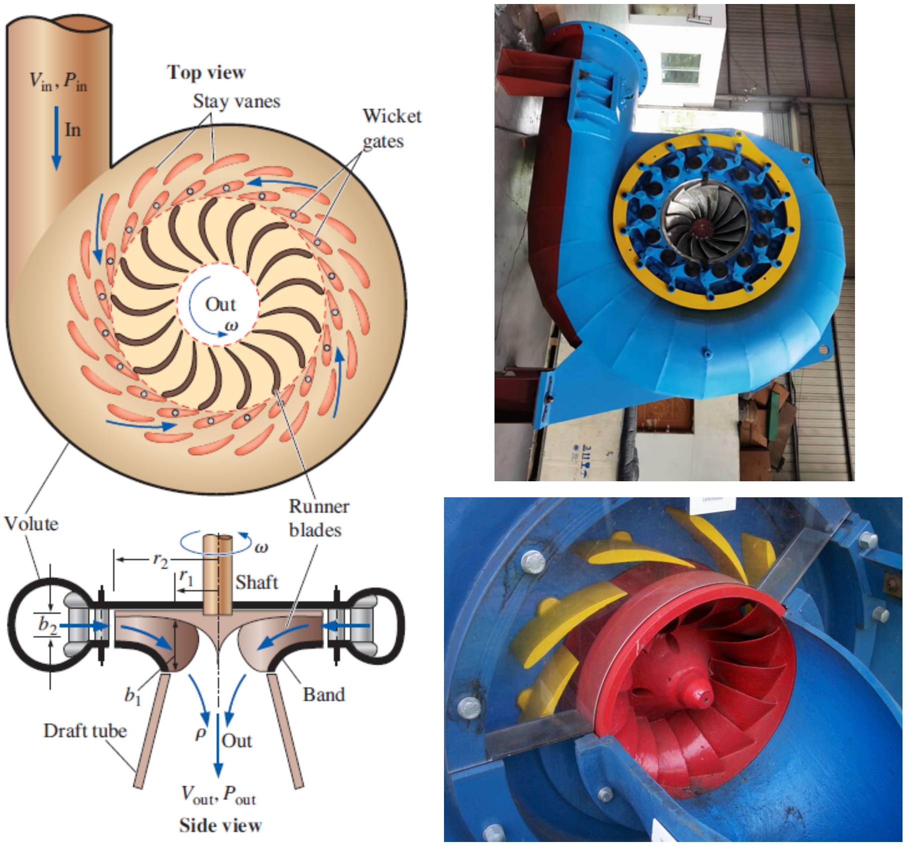
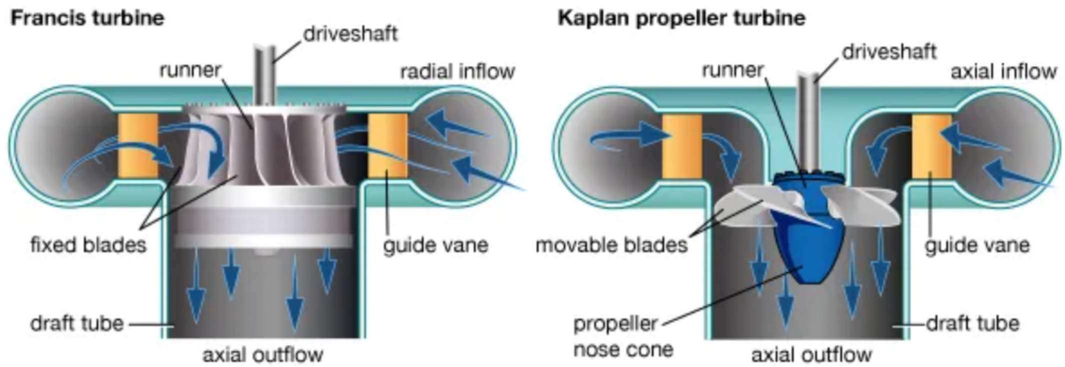


Figure 5.6. Schematic diagram of a reaction turbine

The angle of the wicket gates can be adjusted to control the volume flow rate through the runner. The wicket gates can even close completely to shut off the flow. To avoid vibrations and simultaneous impacts of two or more wicket gate wakes on the leading edge of the runner blades, the number of wicket gates and runner blades are chosen to avoid a common denominator (e.g., 17 runner blades and 20 wicket gates). The number of stay gates is typically the same as the number of wicket gates, as neither rotates.

There are two main types of reaction turbines: Francis and Kaplan.

- **Francis turbine:** This turbine closely resembles a centrifugal pump but operates in the reverse direction. It is named after James B. Francis (1815–1892), who developed the design in the 1840s.
- **Kaplan turbine:** This turbine resembles an axial flow fan running in reverse. It is named after its inventor Viktor Kaplan (1876–1934).



Reaction turbines are classified based on the angle at which the flow enters the runner.

- Francis radial-flow turbine: If the flow enters the runner radially.
- Francis mixed-flow turbine: If the flow enters the runner at some angle between radial and axial. This design is more common.
  A typical large Francis turbine may have 16 or more runner blades and can reach an efficiency between 90% and 95%.
- Propeller mixed-flow turbine: The runner has no band, and the flow enters the runner partially turned.
- Axial flow turbine: The flow is turned completely axially before entering the runner. In this case, the runner usually has 3 to 8 blades, far fewer than a Francis turbine.

The Kaplan turbines are called double-regulated because the flow rate can be regulated using the wicket gates or by adjusting the pitch of the blades.
Propeller turbines are very similar to Kaplan turbines, but they possess fixed blades, meaning the flow rate cannot be adjusted by changing the pitch of the blades.
The efficiency of Kaplan turbines is also high and can reach up to 95%.

5.2.3. Turbine specific speed (N_{St})

How to select which turbine is the most suitable? For this, one can compute the turbine specific speed. The turbine specific speed is a function of the power coefficient C_p and the head coefficient C_H as:

$$N_{St} = \frac{C_P^{1/2}}{C_H^{5/4}} = \frac{\left(\dfrac{\text{bhp}}{\rho\omega^3 D^5}\right)^{1/2}}{\left(\dfrac{gH}{\omega^2 D^2}\right)^{5/2}} = \frac{\omega(\text{bhp})^{1/2}}{\rho^{1/2}(gH)^{5/4}}$$

ω	Rotational speed
bhp	Brake horsepower (or shaft power $\dot{W}_{shaft}$)
ρ	Fluid density
H	The net head

N_{St} should be dimensionless, but in practice engineers with expertise in hydro turbines use different and inconsistent units. For example:

$$N_{St,\,US} = \frac{(\dot{n},\,\text{rpm})(\text{bhp},\,\text{hp})^{1/2}}{(H,\,\text{ft})^{5/4}}$$

Note that:

$N_{St,\,US} = 43.46 \times N_{St}$

While

$$N_{St,\,SI} = \frac{(\dot{n},\,\text{rpm})(\dot{V},\,\text{m}^3/\text{s})^{1/2}}{(H,\,\text{m})^{3/4}}$$

This can be called capacity specific speed to distinguish it from the original definition-based power and called power specific speed.

The best choices in terms of turbines type can be selected using the figure below.

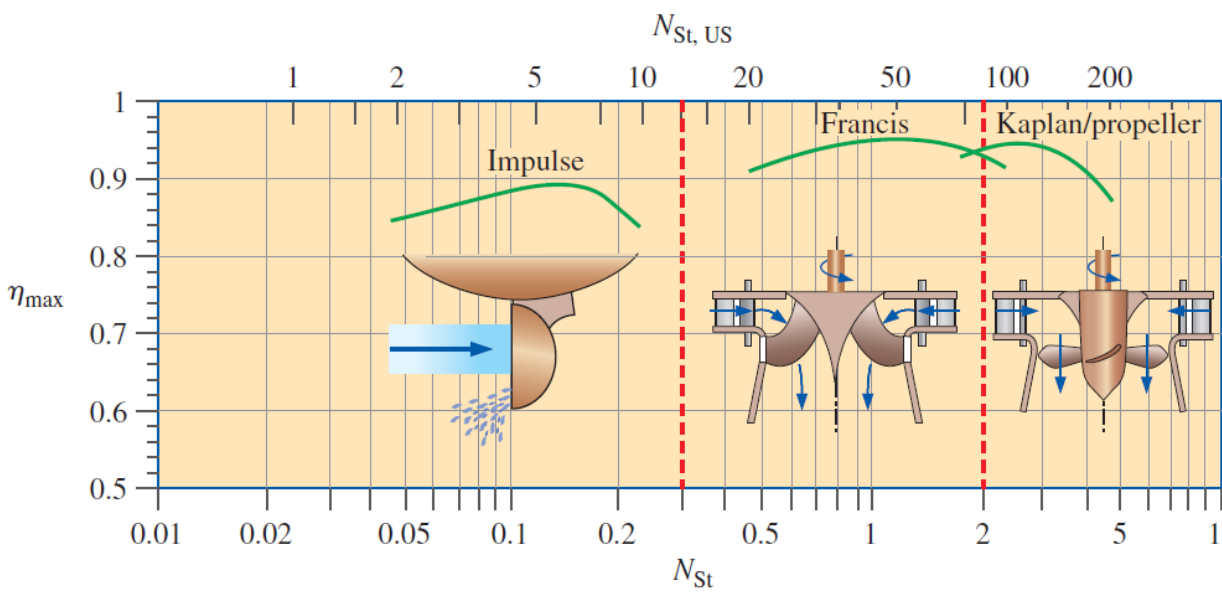


Figure 5.7. Maximum efficiency as a function of turbine specific speed for the three main types of dynamic turbine. Horizontal scales show nondimensional turbine specific speed (N_{St}) and turbine specific speed in customary U.S. units ($N_{St,US}$). Sketches of the blade types are also provided on the plot for reference.

Example 5.2

Calculate the turbine specific speed given the following parameters:

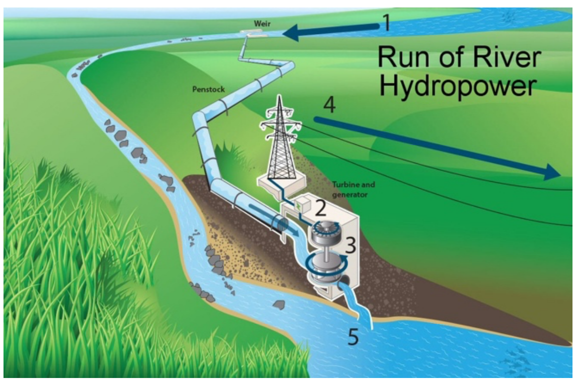
- Angular velocity, ω=20.42 rad/s
- Brake horsepower (bhp) = 2.958×10^{8} W
- Head, H=85.5 m

Provide the answer in both dimensionless form and US units.
Based on the calculated specific speed, determine the most appropriate turbine type.

5.3. Run-of-river plants and water wheels

The majority of electricity generated by hydropower comes from accumulation reservoirs, or dams. However, other methods of generating electricity from water also exist, such as using run-of-river plants or water wheels.

In the case of run-of-river plants (also called small power plants), instead of relying on a dam, they are built along a water stream, such as a river. The water is diverted to a turbine generation unit, and the used water is then returned to the river.



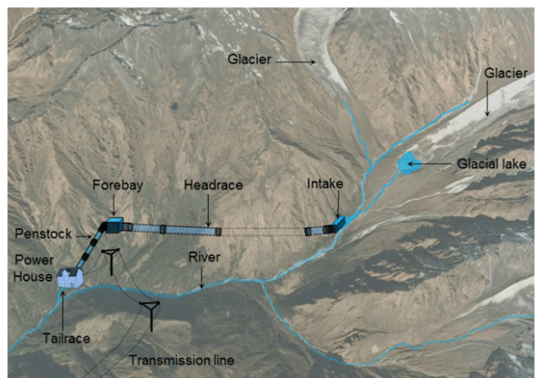


Figure 5.8. Run-of-river hydropower.

One specificity of this type of setup is that the inflow from the river is not constant and changes with the seasons, months, and weather conditions. Since the turbine is designed for a certain rated flow, if the flow rate exceeds the rated flow, the excess water is diverted from the turbine. Selecting a larger turbine wouldn't be optimal because it would operate at lower capacity and performance most of the time.

Since this type of system relies on the energy of flowing water, it is important to estimate the power potential of the river:

$$\dot{W}_{\text{available}} = \dot{m}\frac{V^2}{2} = \rho A V \frac{V^2}{2} = \rho A \frac{V^3}{2}$$

Note that typically, not all of this energy can be used, and only around 60% is actually available.

A water wheel is installed directly into falling or freely flowing water to convert the energy of the water into mechanical power. Water flows through the upper buckets, filling them and forcing the wheel to rotate due to the weight acting on the buckets. The power potential of a water wheel can be estimated as:

$$\dot{W}_{\text{available}} = \dot{m}gH = \rho\dot{V}gH$$

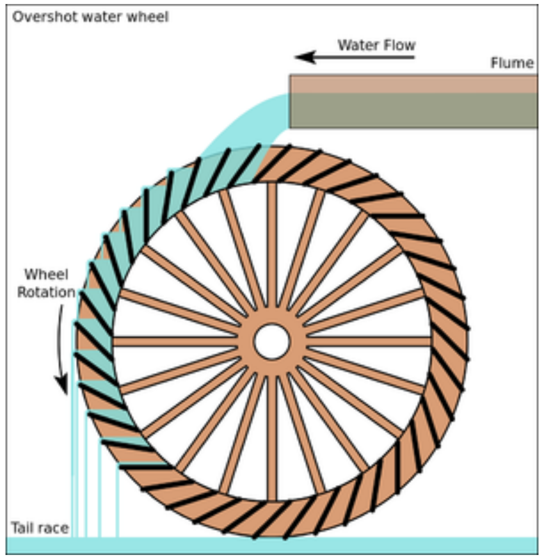


Figure 5.9. Schematic of a water wheel.

Water wheels are very simple devices that have been used since ancient times for milling flour, lifting water for irrigation, hammering iron, and other applications. One distinctive advantage of water wheels is that their operation is not affected by dirty water. However, their overall performance and efficiency remain very low for electricity generation due to their low speed, which requires multiple gear multipliers, thereby increasing irreversible losses.

5.4. Pumped storage hydropower (PSH)

It provides peak-load supply, harnessing water which is cycled between a lower and upper reservoir by pumps which use surplus energy from the system at times of low demand. When electricity demand is high, water is released back to the lower reservoir through turbines to produce electricity.

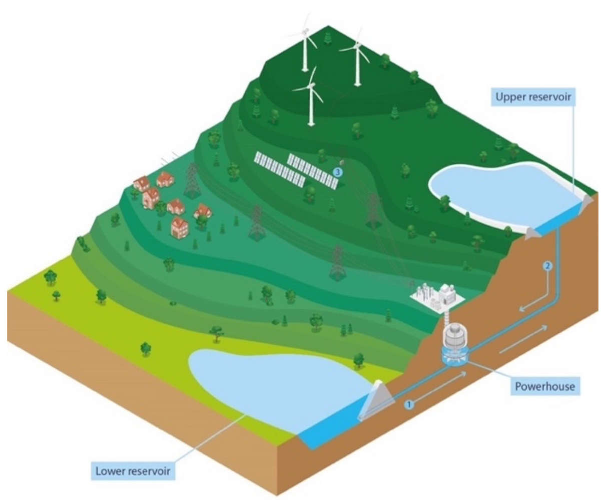


Figure 5.10. Pumped storage hydropower