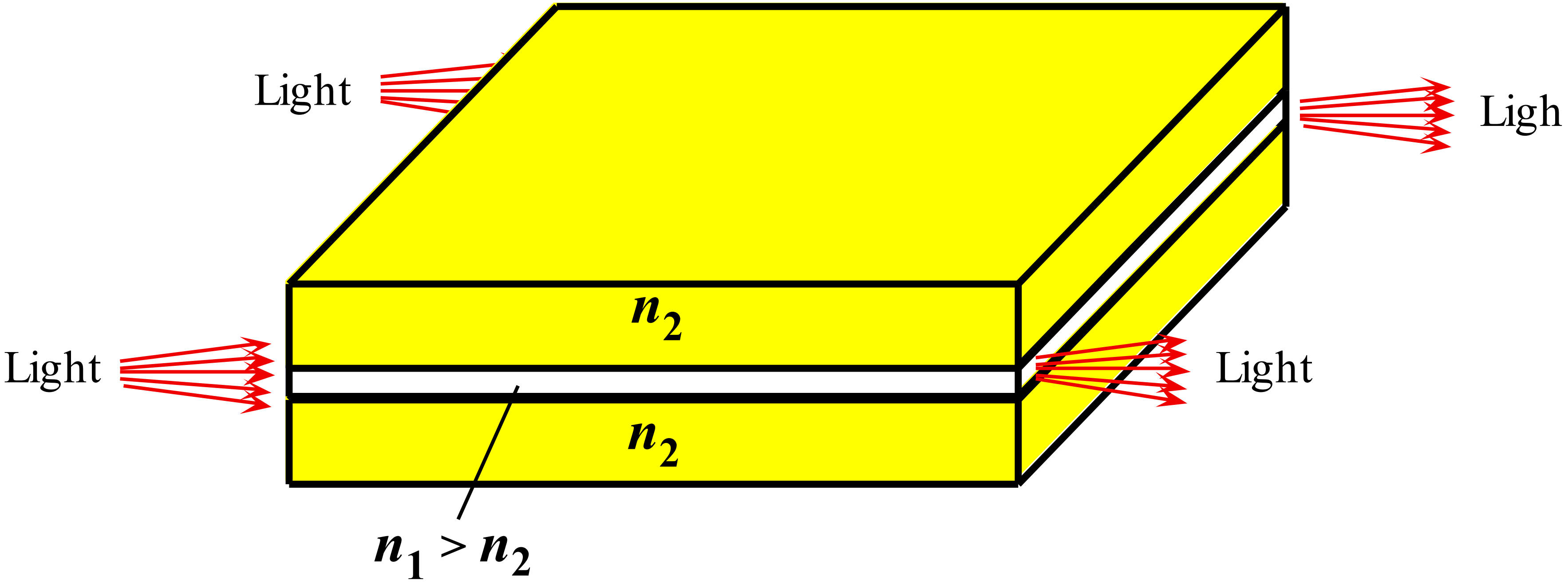
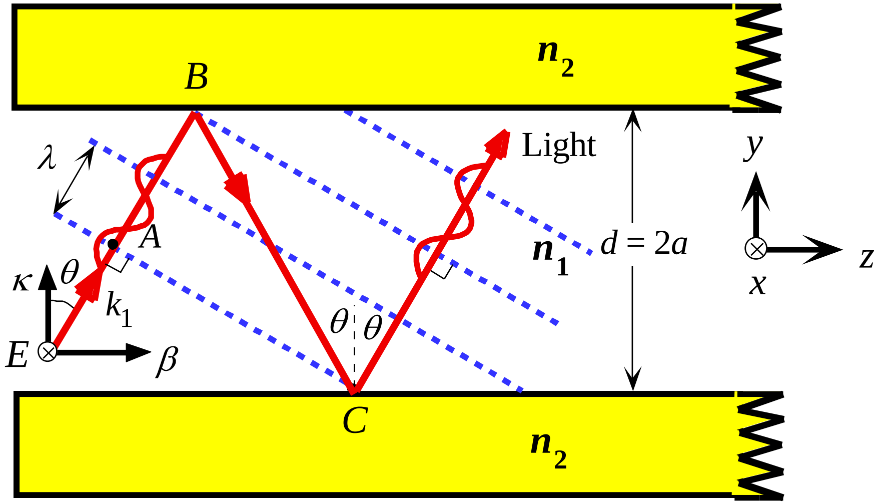
Waveguides and Optical Fibers

Dielectric Waveguides



A planar dielectric waveguide has a central rectangular region of higher refractive index n_1 than the surrounding region which has a refractive index n_2. It is assumed that the waveguide is infinitely wide and the central region is of thickness 2 a. It is illuminated at one end by a monochromatic light source.

A plane wave propagate a waveguide with a n1/n2 core-cladding, along Z-direction.



A light ray travelling in the guide must interfere constructively with itself to propagate successfully. Otherwise destructive interference will destroy the wave.

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Waveguide condition

$\Delta\Phi\ (AC) = k_1(AB + BC) - 2\Phi = m(2\pi)$, $k_1 = kn_1 = 2\pi n_1/\lambda$

$m = 0, 1, 2, \ldots$

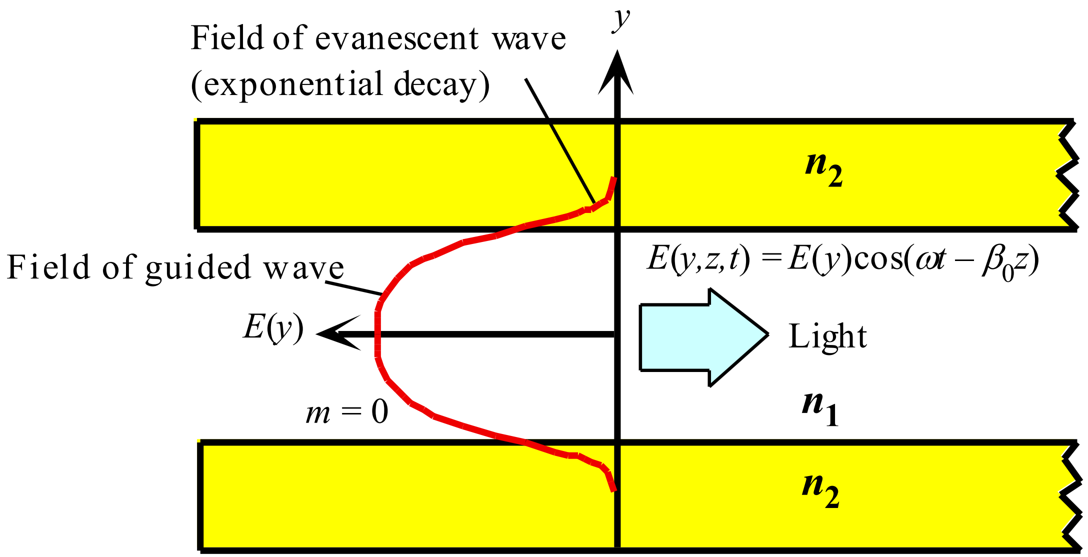
BC = d/cosθ and AB = BC cos(2θ)

$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

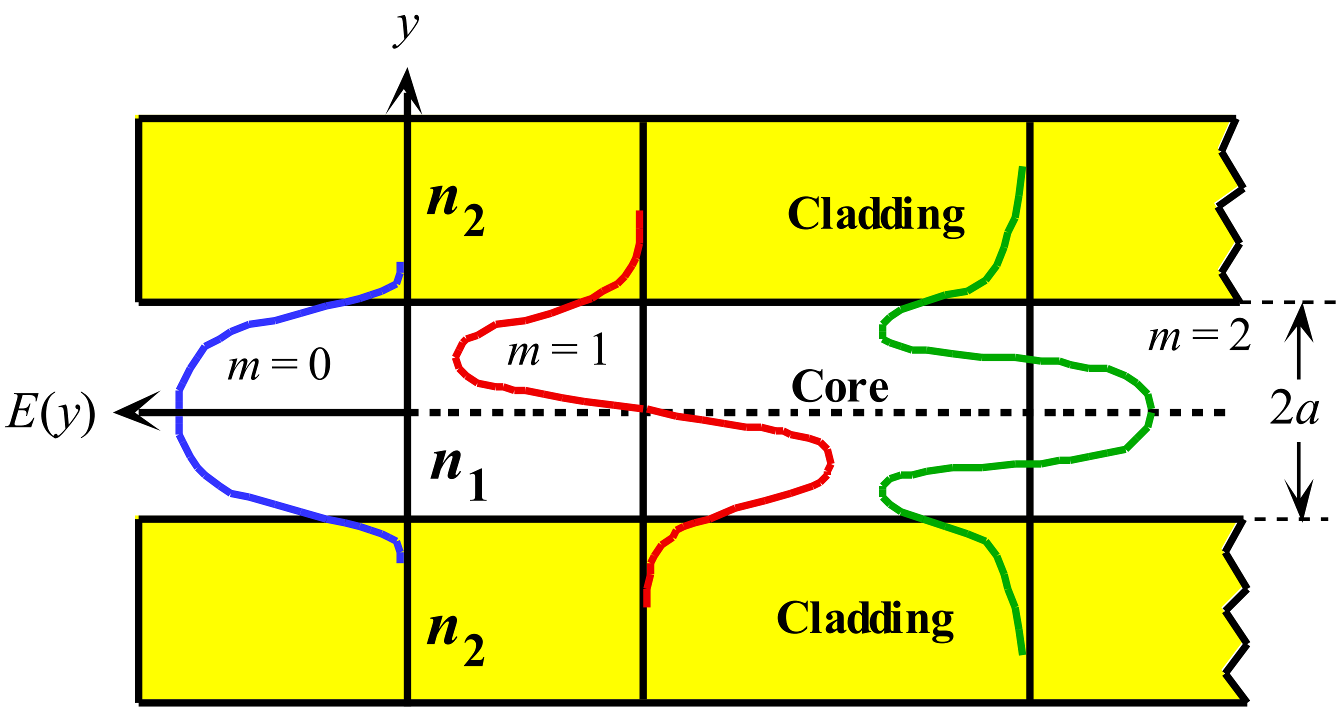
$$\beta_m = k_1 \sin\theta_m = \left(\frac{2\pi n_1}{\lambda}\right)\sin\theta_m \qquad k_m = k_1\cos\theta_m = \left(\frac{2\pi n_1}{\lambda}\right)\cos\theta_m$$

$$E_1(y,z,t) = 2E_0 \cos\left(k_m y + \frac{1}{2}\phi_m\right)\cos\left(\omega t - \beta_m z + k_m y + \frac{1}{2}\phi_m\right)$$

$$E_1(y,z,t) = 2E_m(y)\cos(\omega t - \beta_m z)$$



The electric field pattern of the lowest mode traveling wave along the guide. This mode has $m = 0$ and the lowest θ. It is often referred to as th glazing incidence ray. It has the highest phase velocity along the guide.

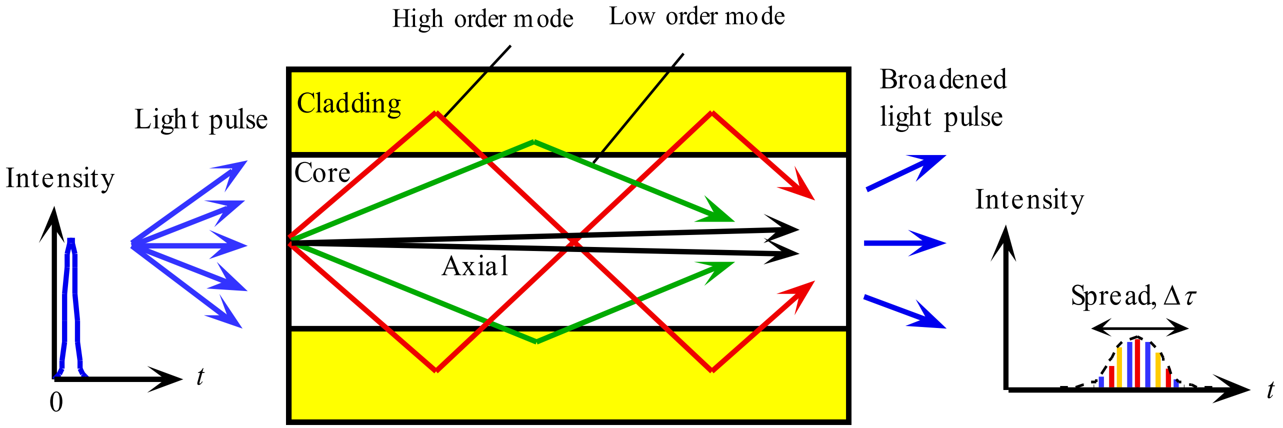


The electric field patterns of the first three modes (m = 0, 1, 2) traveling wave along the guide. Notice different extents of field penetration into the cladding.

*Ey (m) is the field distribution along y axis and constitute **a mode of propagation**.*

***m is called mode number**. Defines the number of modes traveling along the waveguide. For every value of m we have an angle θm satisfying the waveguide condition provided to satisfy the TIR as well. Considering these condition one can show that the number of modes should satisfy:*

$$m \le (2V - \Phi)/\pi$$



Schematic illustration of light propagation in a slab dielectric waveguide. Light pulse entering the waveguide breaks up into various modes which then propagate at differen group velocities down the guide. At the end of the guide, the modes combine to constitute the output light pulse which is broader than the input light pulse.

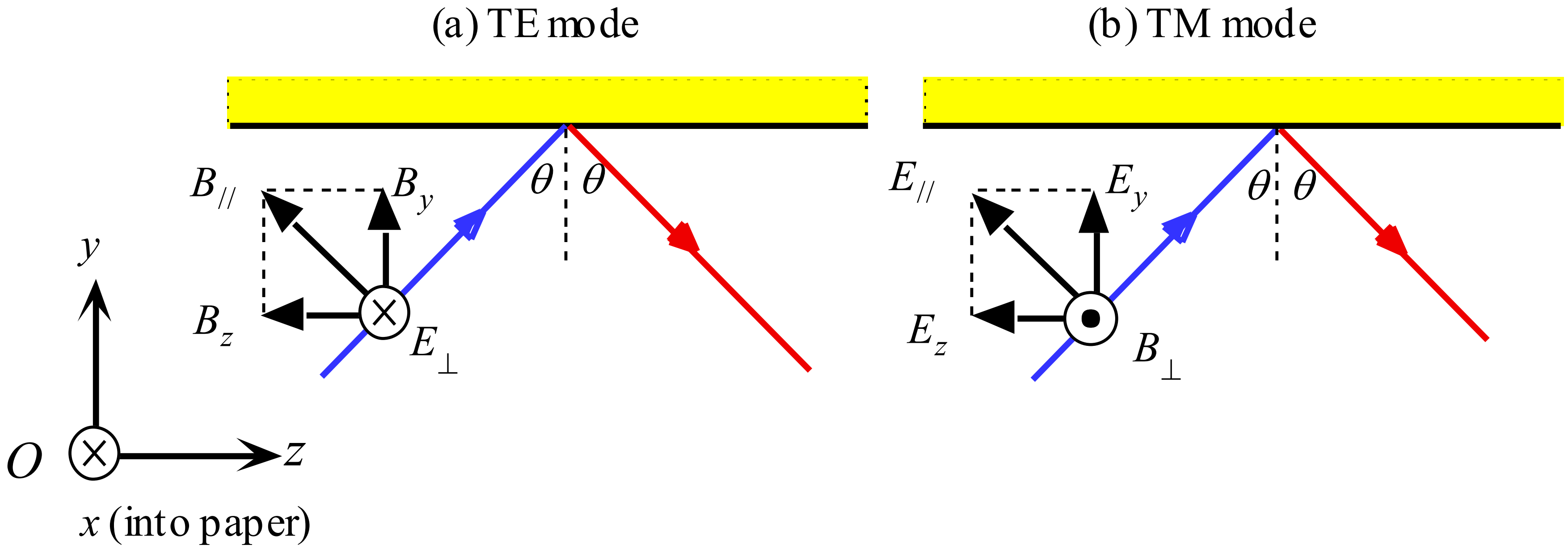
$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{\frac{1}{2}}$$

V is called V-number and it is a characteristic parameter of the waveguide

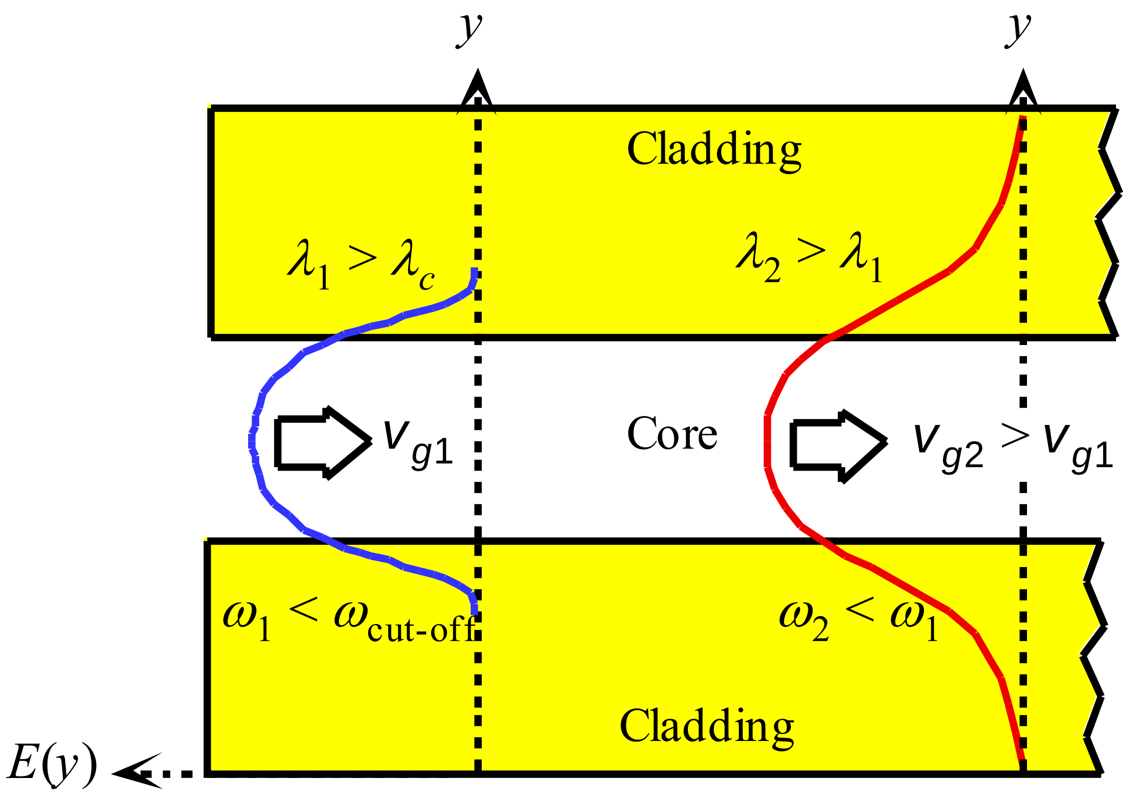
For V ≤ π/2, m= 0, it is the lowest mode of propagation referred to single mode waveguides.

The cut-off wavelength (frequency) is a free space wavelength for v = π/2

For V=π/2, The wavelength is the cut-off wavelength, above this wavelength only one-mode (m=0) will propagate

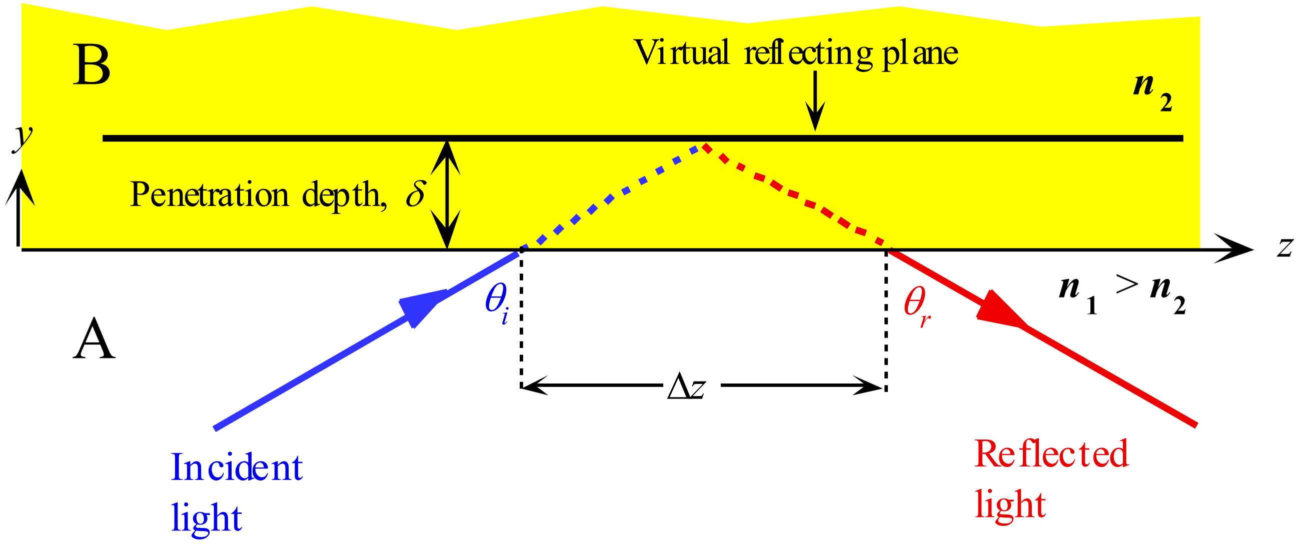


Possible modes can be classified in terms of (a) transelectric field (TE
and (b) transmagnetic field (TM). Plane of incidence is the paper.



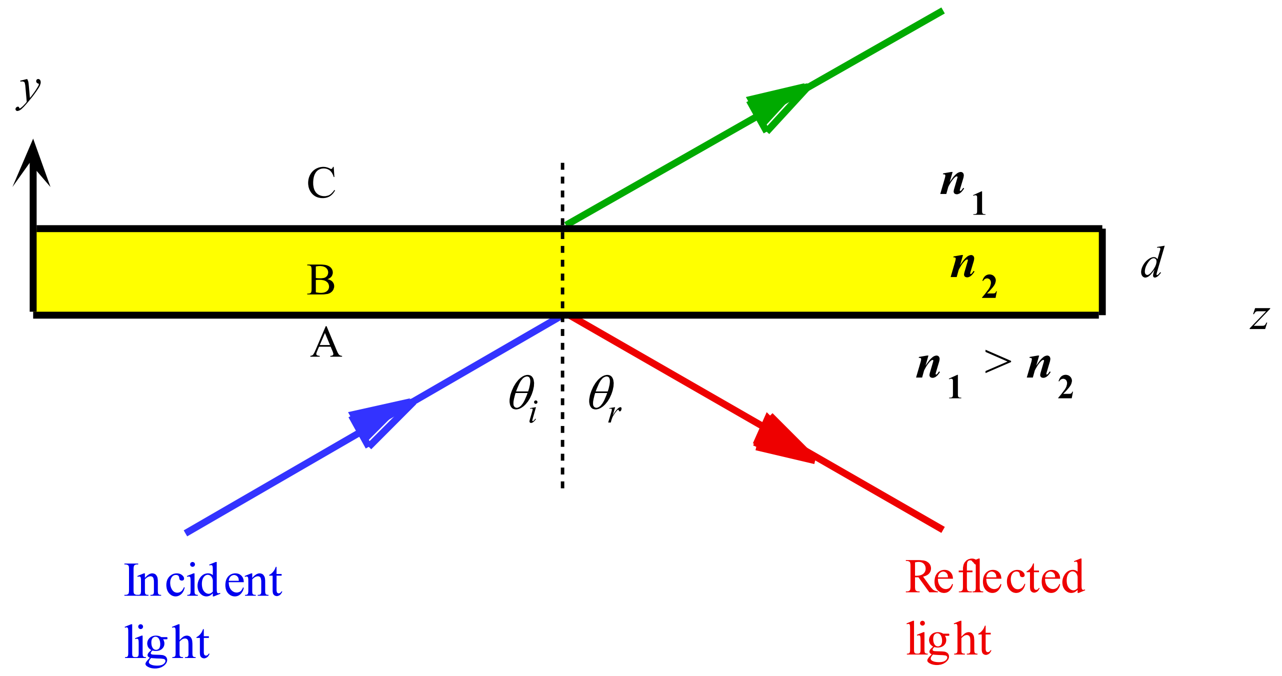
The electric field of TE_0 mode extends more into the cladding as the wavelength increases. As more of the field is carried by the cladding, the group velocity increases.

Goos-Hanchen shift



The reflected light beam in total internal reflection appears to have been laterally shifted by an amount Δz at the interface.

Optical Tunneling



When medium B is thin (thickness *d* is small), the field penetrates to the BC interface and gives rise to an attenuated wave in medium C. The effect is the tunnelling of the incident beam in A through B to C.

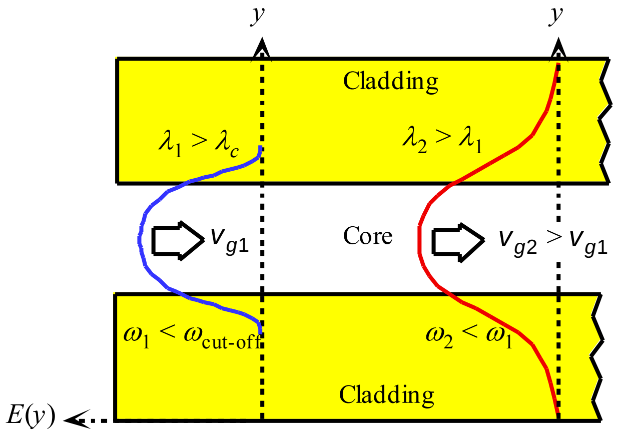
Waveguide Dispersion

Intermodal dispersion: In multimode waveguides the lowest mode has the slowest group velocity, the highest mode has the highest group velocity

$$\frac{\Delta\tau}{L} = \frac{1}{V_{g\,\min}} - \frac{1}{V_{g\,\max}} \approx \frac{n_1 - n_2}{c}$$

Intramodal Dispersion; In single mode waveguide:

- Waveguide Dispersion: as there is no prefect monochromatic light
- Material Dispersion: due to the n(λ)



The electric field of TE_0 mode extends more into the cladding as the wavelength increases. As more of the field is carried by the cladding, the group velocity increases.

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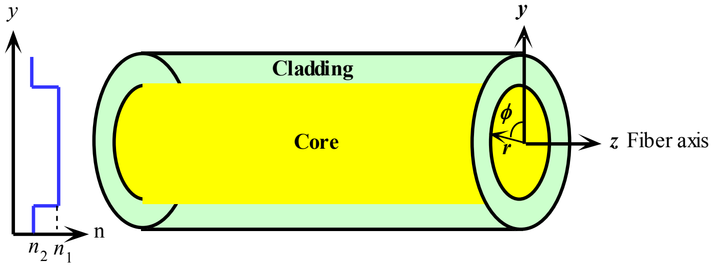

The step index optical fiber. The central region, the core, has greater refractive index than the outer region, the cladding. The fiber has cylindrical symmetry. We use the coordinates r, ϕ, z to represent any point in the fiber. Cladding is normally much thicker than shown.

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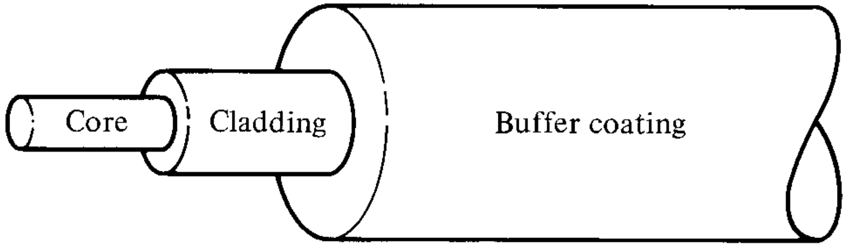

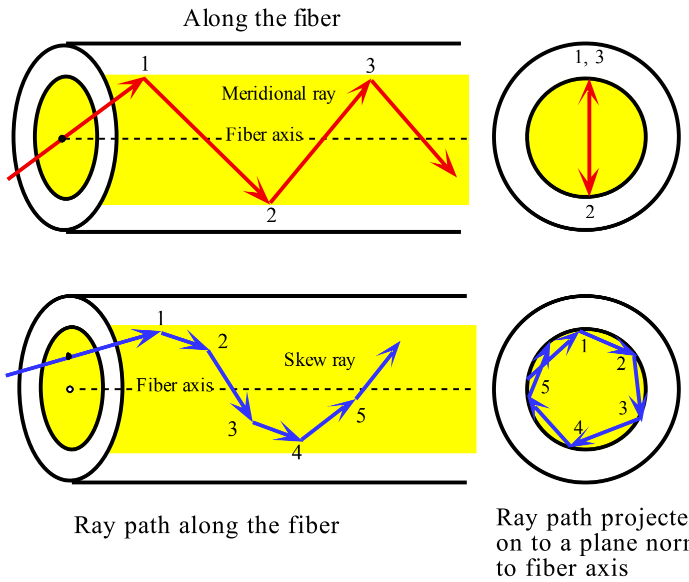

(a) A meridiona ray always crosses the fibe axis.

(b) A skew ray does not have to cross the fiber axis. It zigzags around the fiber axis.

Illustration of the difference between a meridional ray and a skew ray. Numbers represent reflections of the ray.

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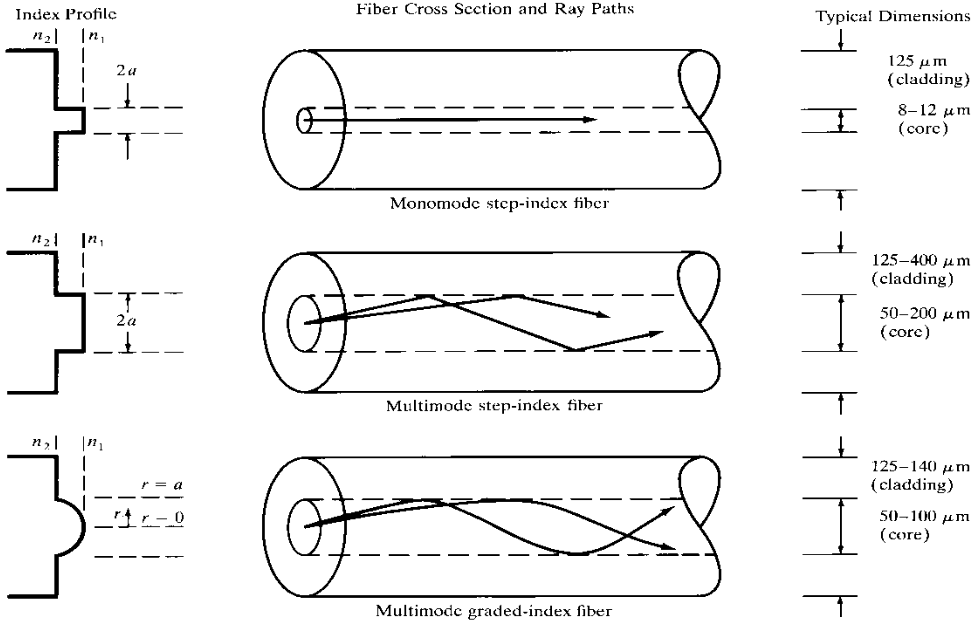


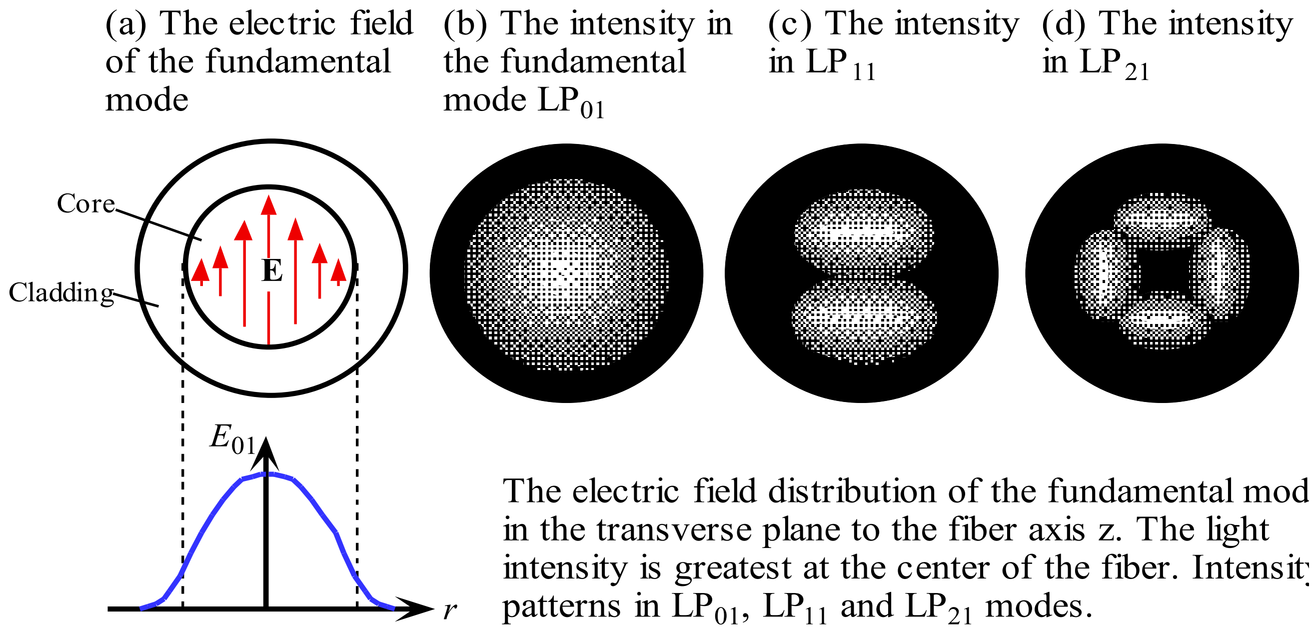
For the step index optical fiber $\Delta = (n_1 - n_2)/n_1$

is called *normalized index difference*

For weakly guided fibers, i.e.:

$$\Delta = \frac{n_1 - n_2}{n_1} = \frac{n_1^2 - n_2^2}{2n_1^2} << 1$$

The guide modes are visualized by traveling waves that are almost polarized, called linearly polarized (LP)



(a) The electric field of the fundamental mode

(b) The intensity in the fundamental mode LP_{01}

(c) The intensity in LP_{11}

(d) The intensity in LP_{21}

The electric field distribution of the fundamental mod in the transverse plane to the fiber axis z. The light intensity is greatest at the center of the fiber. Intensit patterns in LP_{01}, LP_{11} and LP_{21} modes.

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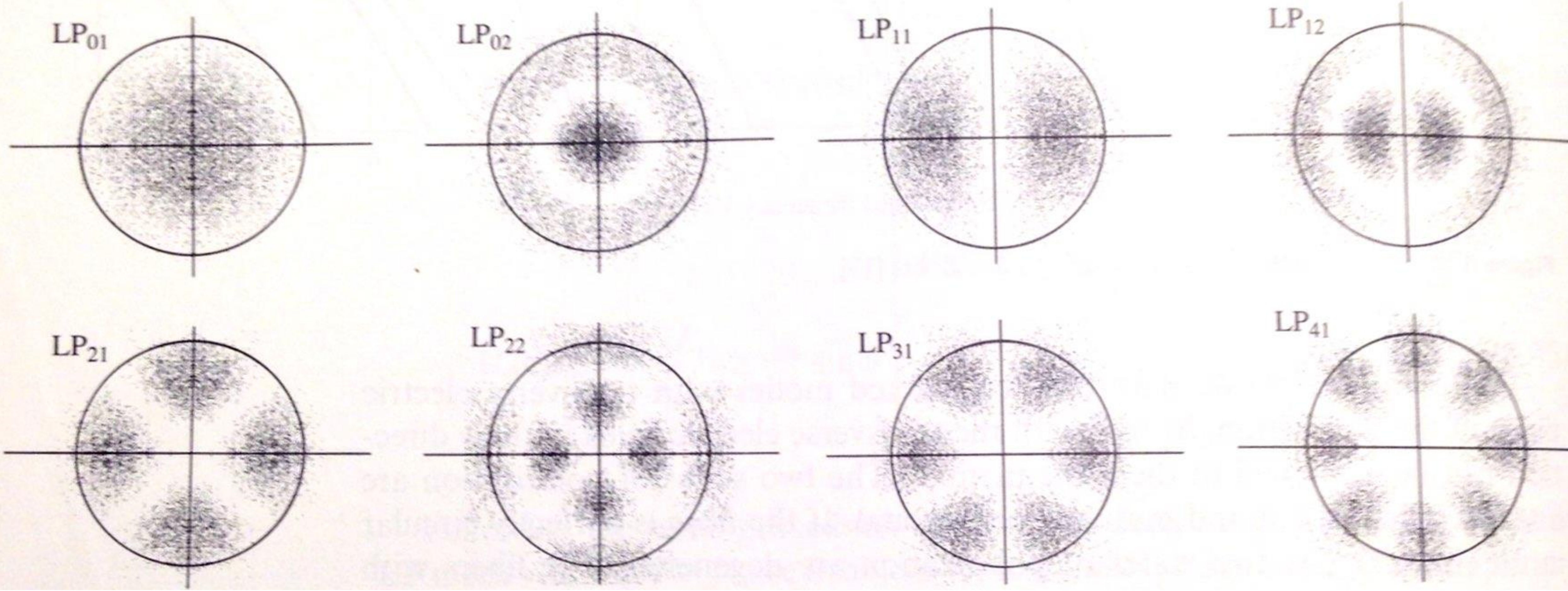
LPs (linearly polarized waves) propagating along the fiber have either TE or TM type represented by the propagation of an electric field distribution $E_{lm}(r,\phi)$ along z.

$$E_{LP} = E_{lm}(r.\varphi)\exp j(\omega t - \beta_{lm} z)$$

E_{LP} is the field of the LP mode and β_{lm} is its propagation constant along z.


LP_{01}
LP_{02}
LP_{11}
LP_{12}
LP_{21}
LP_{22}
LP_{31}
LP_{41}

V-number

$$\Delta = (n_1 - n_2)/n_1 \approx (n_1^2 - n_2^2)/2n_1^2$$

Normalized index difference

$$V_{cut-off} = \frac{2\pi a}{\lambda_c}\left(n_1^2 - n_2^2\right)^{\frac{1}{2}} = 2.405$$

For weakly guided fiber Δ=0.01, 0.005,..

For V = 2.405, the fiber is called single mode (only the fundamental mode propagate along the fiber). For V > 2.405 the number of mode increases according to approximately

$$M = \frac{V^2}{2}$$

Most SM fibers designed with 1.5<V<2.4

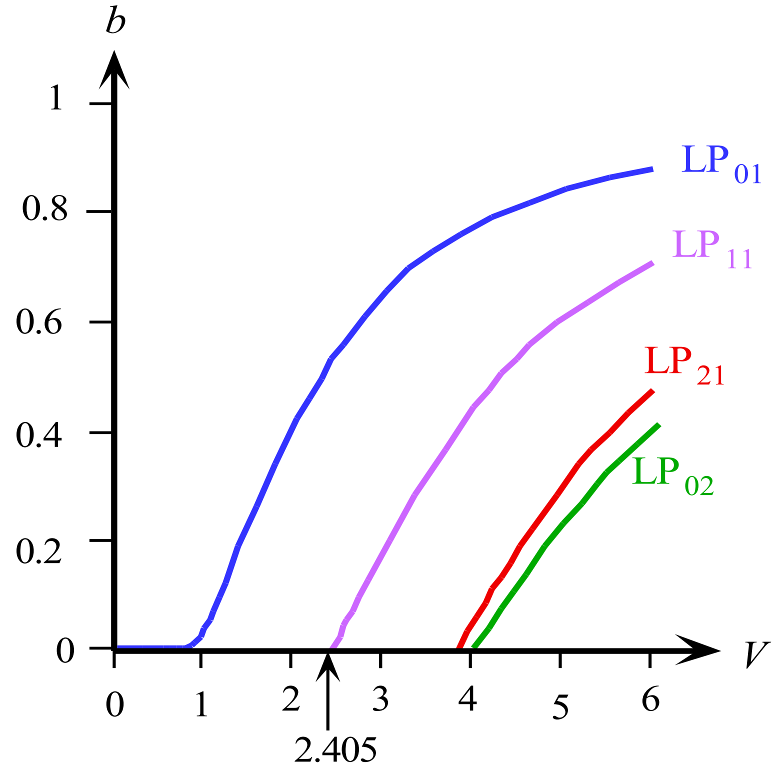
Since β_{lm} of an LP mode depends on the waveguide properties and the source wavelength, the light Propagation is described in terms of a "normalized propagation" constant that depends only on the V-number.

$$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{\frac{1}{2}} = \frac{2\pi a}{\lambda}\left(2n_1 n\Delta\right)^{\frac{1}{2}}$$

$$b = \frac{(\beta / k)^2 - n_2^2}{n_1^2 - n_2^2}$$

$kn_2 < \beta < kn_1$ Propagation condition

b changes between 0 and 1



Normalized propagation constant *b* vs. *V*-number for a step index fiber for various LP modes.

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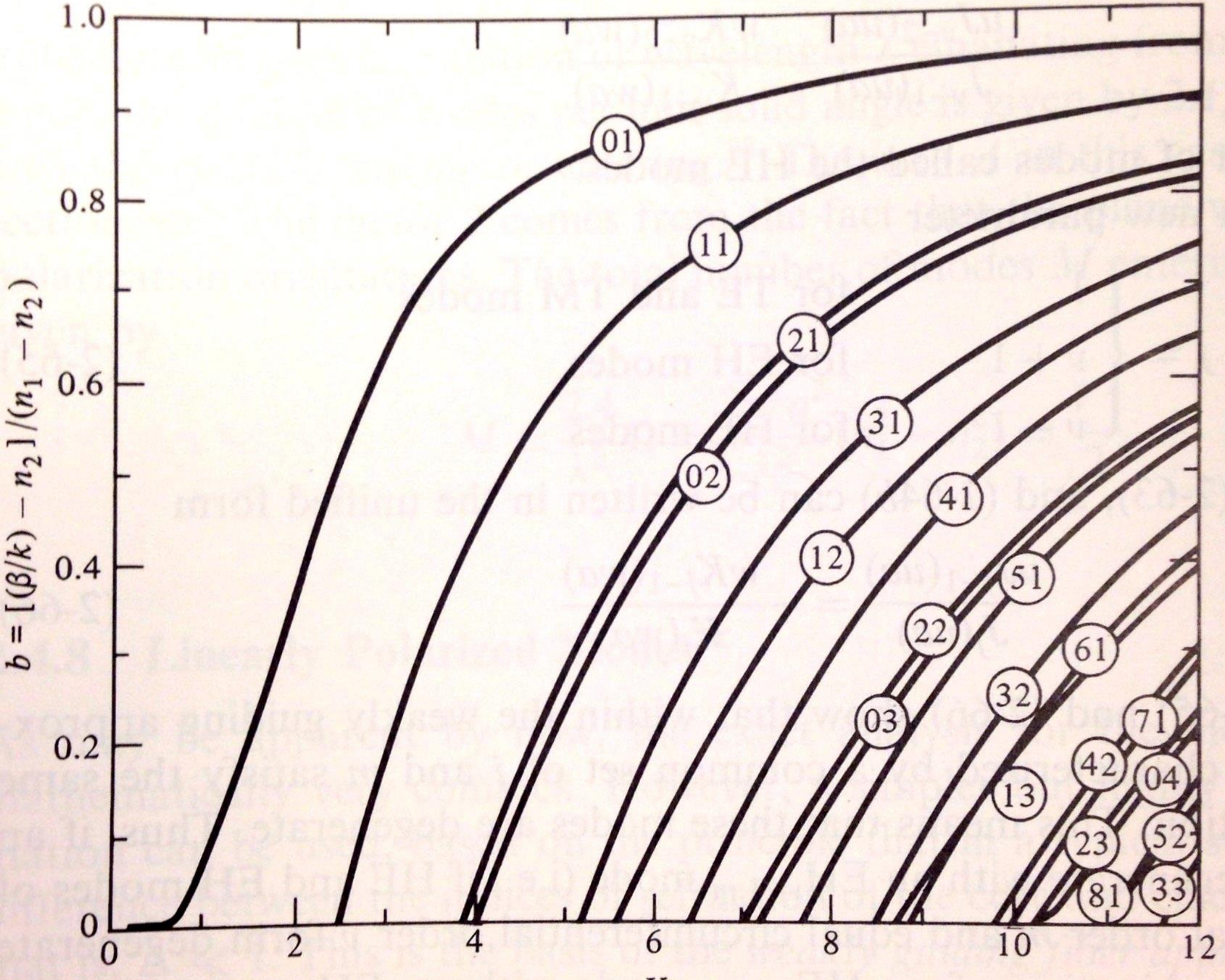

1.0
0.8
0.6
0.4
0.2
0
$b = [(\beta/k) - n_2]/(n_1 - n_2)$
0
2
4
6
8
10
12
01
11
21
02
31
41
12
51
22
61
32
03
71
42
04
13
23
52
81
33

Numerical Aperture---Maximum Acceptance Angle

$$\frac{\sin\alpha_{\max}}{\sin(90^o-\theta_c)}=\frac{n_1}{n_0}$$

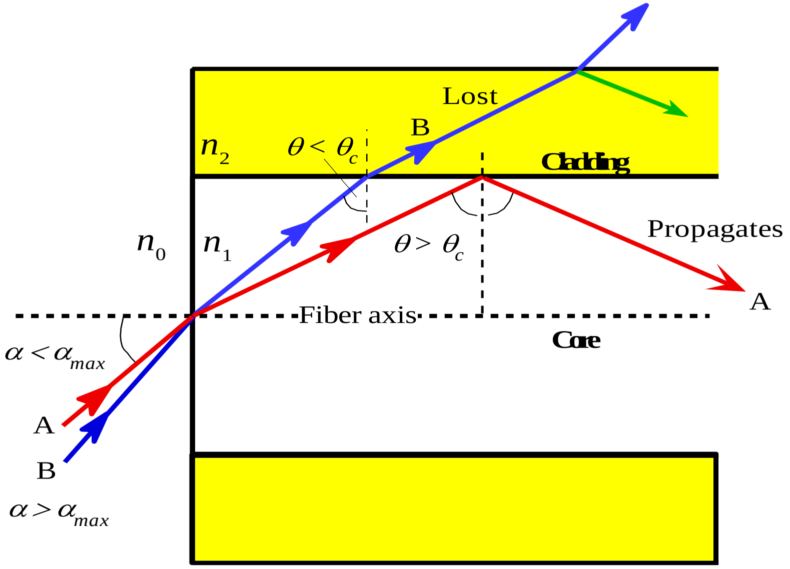
$$\sin\theta_c=\frac{n_2}{n_1}$$

$$\sin\alpha_{\max}=\frac{\left(n_1^2-n_2^2\right)^{\frac{1}{2}}}{n_0}$$

$$NA=(n_1^2-n_2^2)^{1/2}$$ Numerical aperture

$$\sin\alpha_{\max}=\frac{NA}{n_0}$$

$$V=\frac{2\pi a}{\lambda}NA$$



Maximum acceptance angle α_{max} is that which just gives total internal reflection at the core-cladding interface, i.e. when $\alpha=\alpha_{max}$ then $\theta=\theta_c$. Rays with $\alpha>\alpha_{max}$ (e.g. ray B) become refracted and penetrate the cladding and are eventually lost.

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Example values for $n_1=1.48$, $n_2=1.47$; very close numbers

Typical values of NA = 0.07…,0.25

Optical waveguides display 3 types of dispersion:

These are the main sources of dispersion in the fibers.

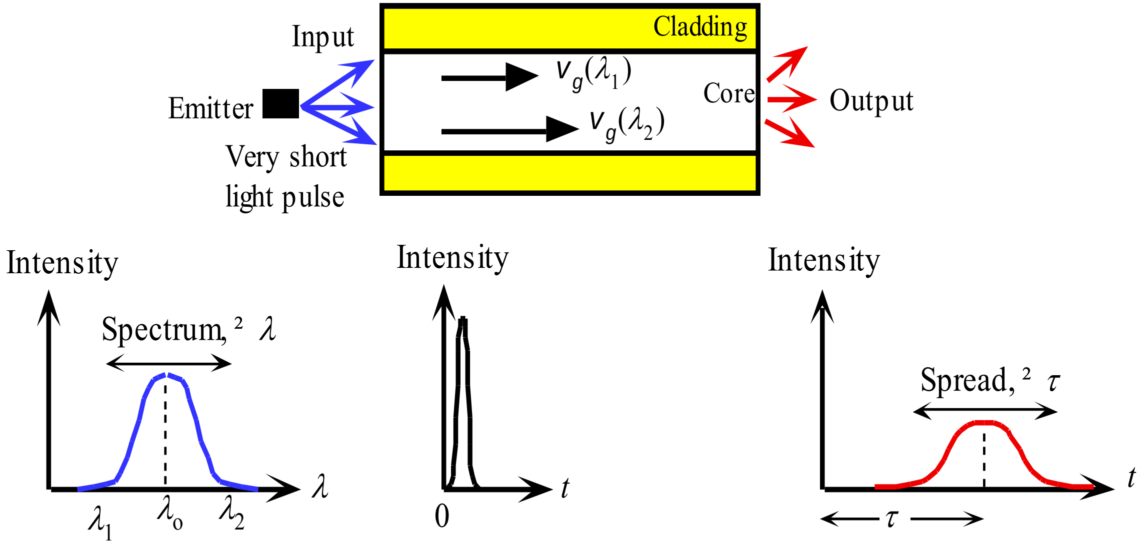
- Material dispersion: different wavelength of light travel at different velocities within a given medium.

Due to the variation of n_1 of the core wrt wavelength of the light.

- Waveguide dispersion: β depends on the wavelength, so even within a single mode different wavelengths will propagate at slightly different speeds.

Due to the variation of group velocity wrt V-number

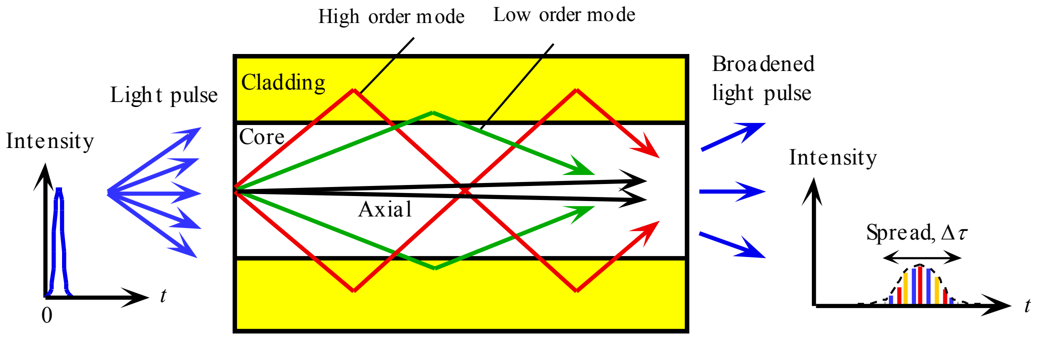
$$\boxed{\frac{\Delta\tau}{L} = |D|\Delta\lambda}$$



All excitation sources are inherently non-monochromatic and emit within a spectrum, $^{2}\lambda$, of wavelengths. Waves in the guide with different free space wavelengths travel at different group velocities due to the wavelength dependence of n_1. The waves arrive at the end of the fiber at different times and hence result i a broadened output pulse.

- Modal dispersion, in waveguides with more than one propagating mode. Modes travel with different group velocities.

Due to the number of modes traveling along the fiber with different group velocity and different path.



Schematic illustration of light propagation in a slab dielectric waveguide. Light pulse entering the waveguide breaks up into various modes which then propagate at differen group velocities down the guide. At the end of the guide, the modes combine to constitute the output light pulse which is broader than the input light pulse.

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$$\frac{\Delta\tau}{L} = |D_m|\Delta\lambda$$

$$D_m \approx -\frac{\lambda}{c}\left(\frac{d^2 n}{d\lambda^2}\right)$$

Material
Dispersion
Coefficient

$$\frac{\Delta\tau}{L} = |D_\omega|\Delta\lambda$$

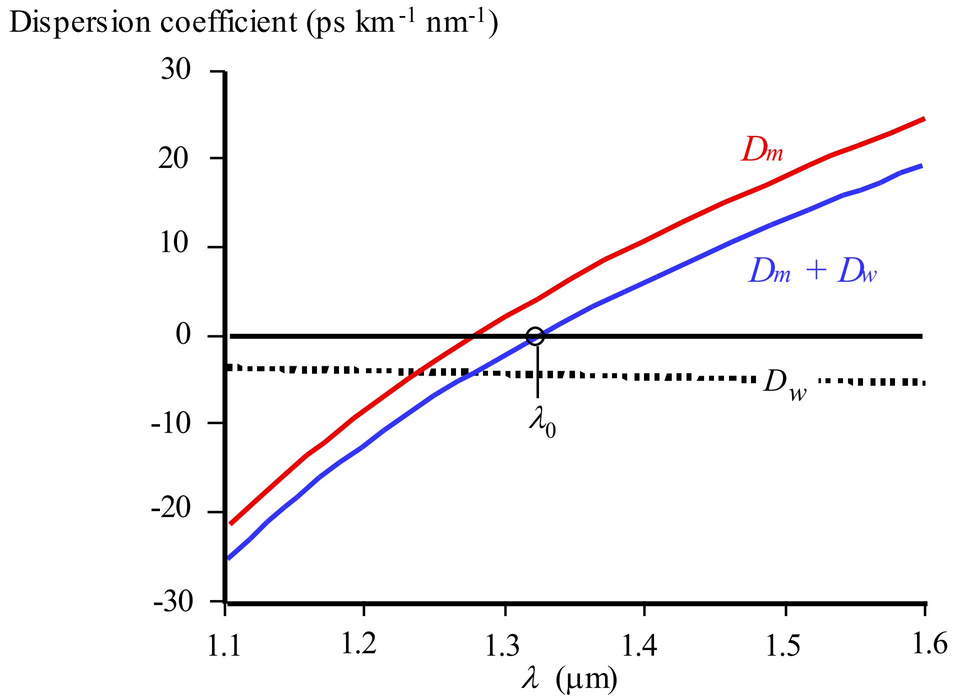
$$D_\omega \approx \frac{1.984 N_{g2}}{(2\pi a)^2 \, 2cn_2^2}$$

Waveguide
Dispersion
Coefficient

$$\frac{\Delta\tau}{L} = |D_P|\Delta\lambda$$

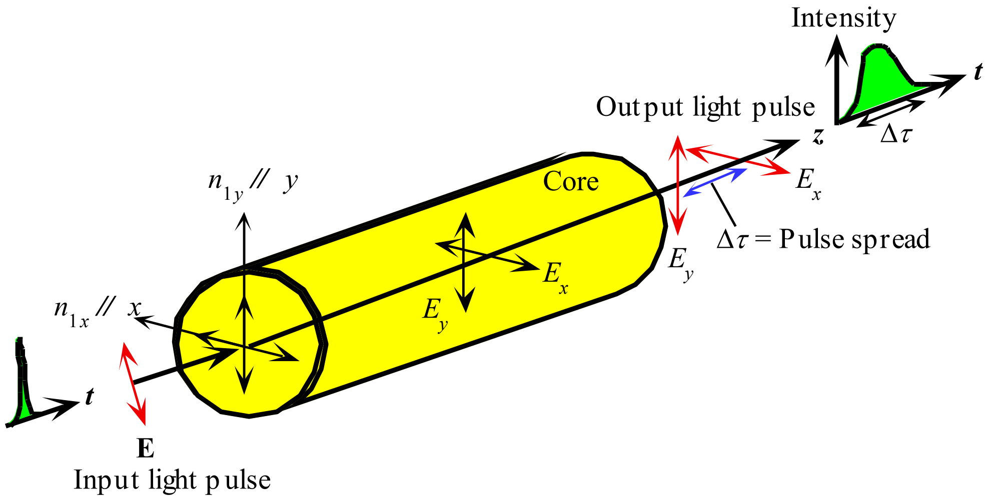
D_p is called profile dispersion; group velocity depends on Δ

$$\frac{\Delta\tau}{L} = |D_m + D_\omega + D_p|\Delta\lambda$$



Material dispersion coefficient (D_m) for the core material (taken as SiO_2), waveguide dispersion coefficient (D_w) (a = 4.2 µm) and the total or chromatic dispersion coefficient D_{ch} (= $D_m + D_w$) as a function of free space wavelength, λ.

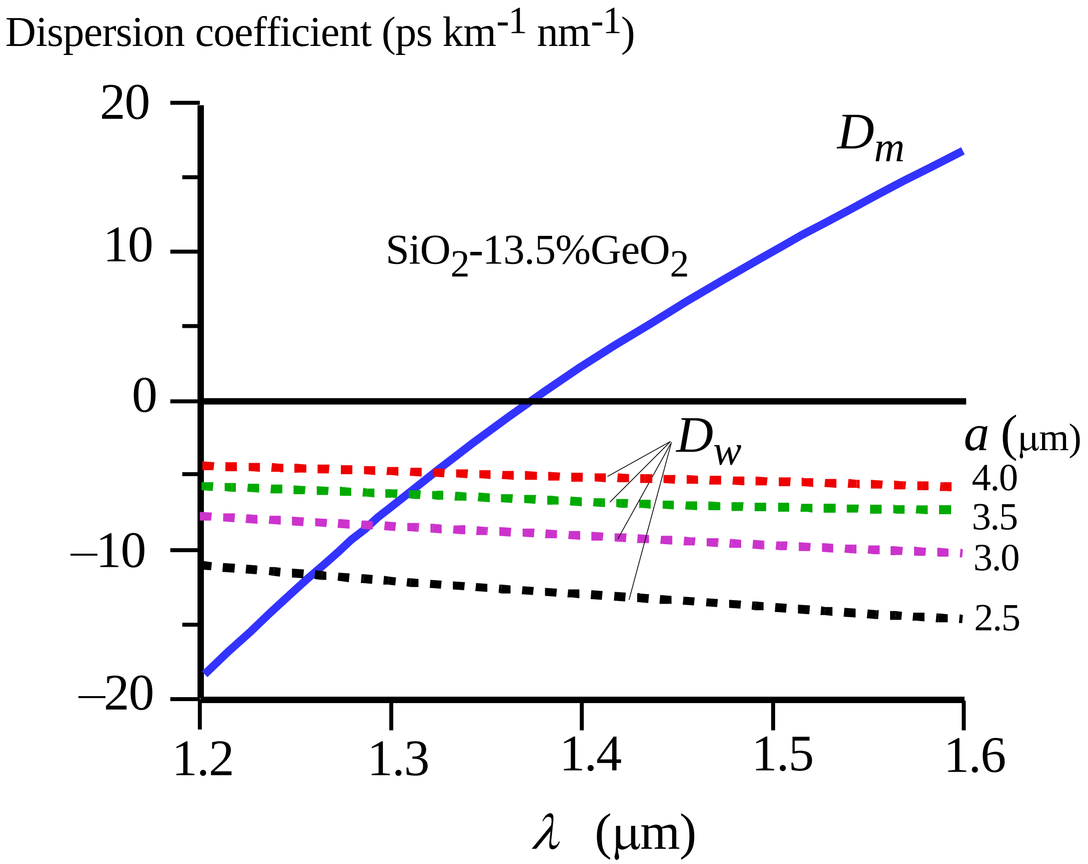
Polarization Dispersion



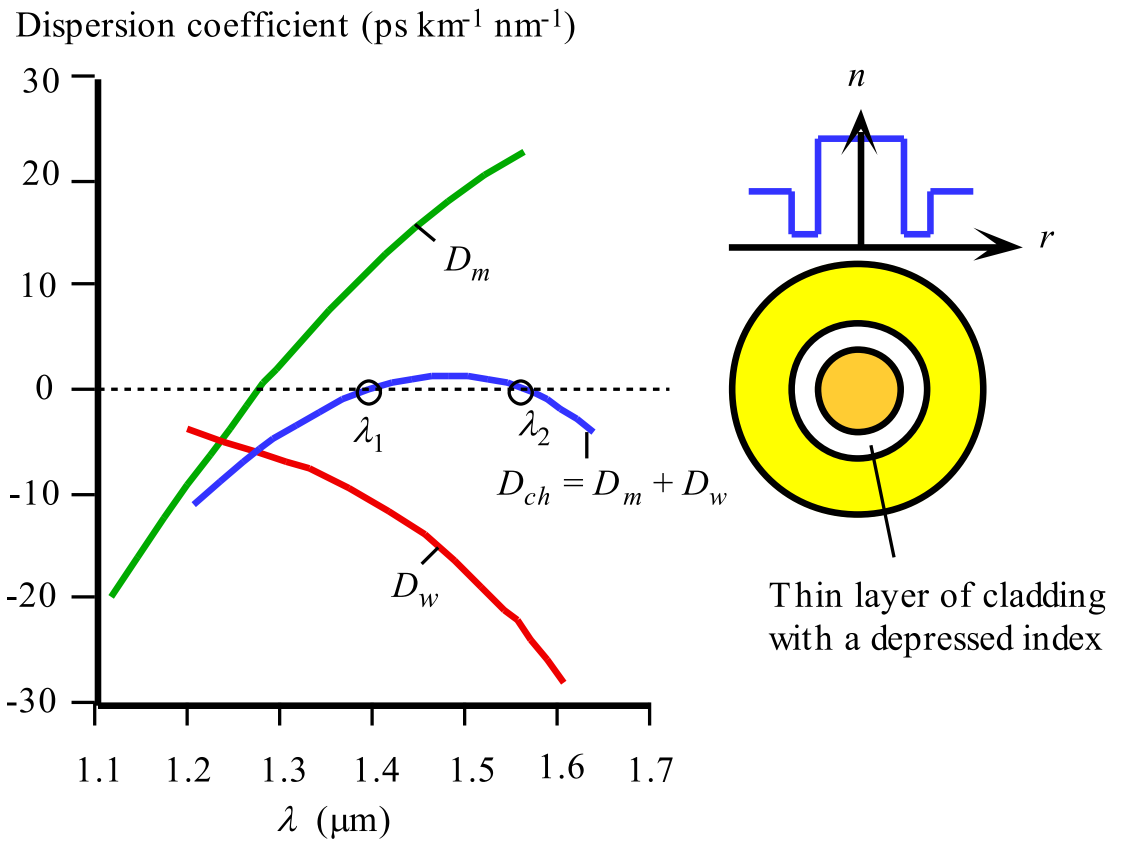
Suppose that the core refractive index has different values along two orthogonal directions corresponding to electric field oscillation direction (polarizations). We can take x and y axes along these directions. An input light will travel along the fiber with E_x and E_y polarizations having different group velocities and hence arrive at the output at different times e.g.

For λ =1.5, and 2a = 8μm
D_m=10 ps/km.nm and

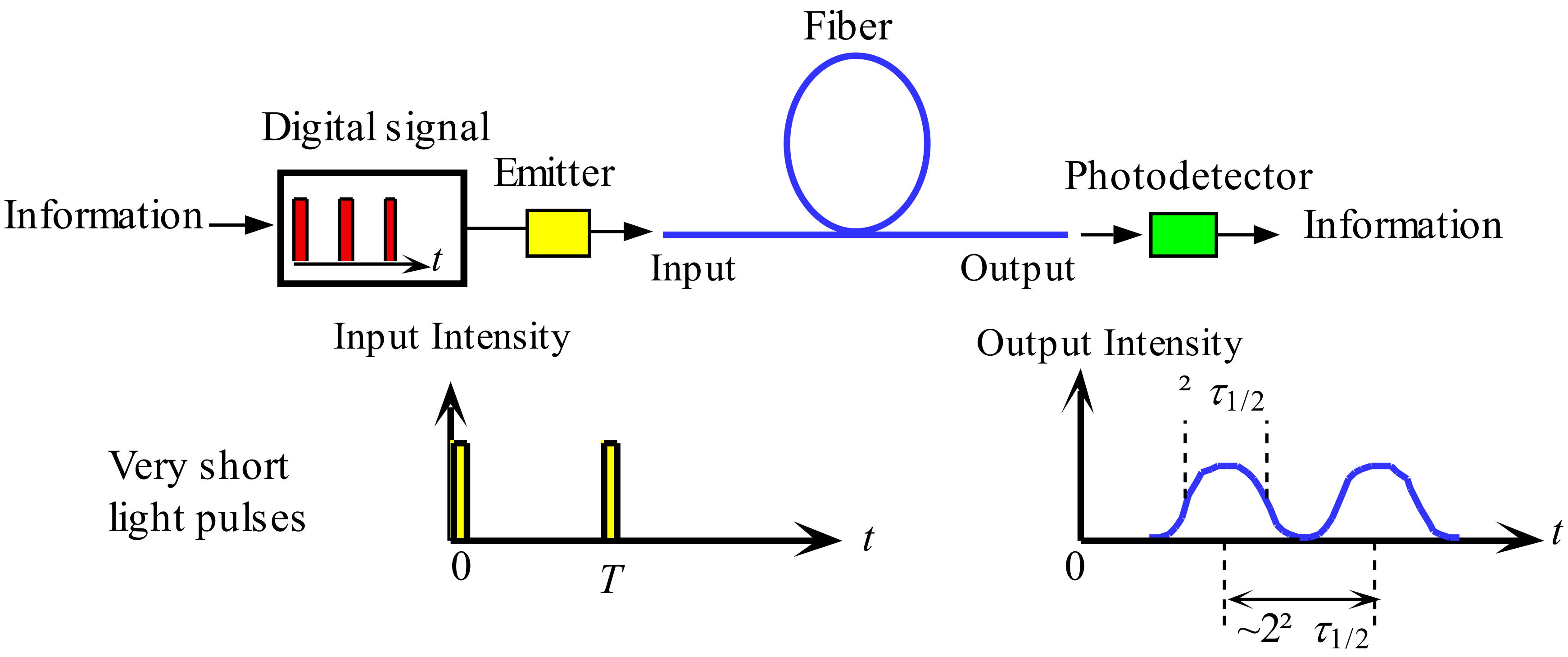
D_w=-6 ps/km.nm



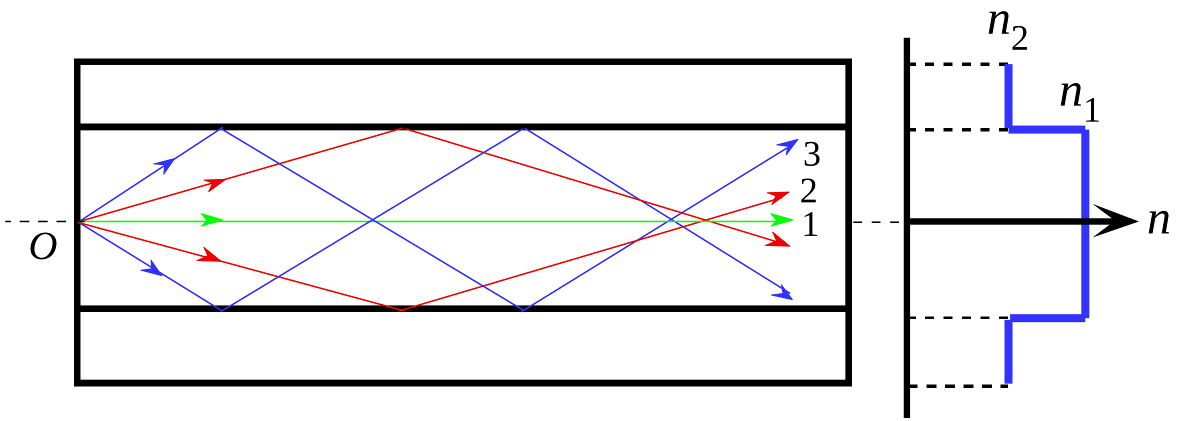
Material and waveguide dispersion coefficients in an optical fiber with a core SiO_2-13.5%GeO_2 for a = 2.5 to 4 μm.



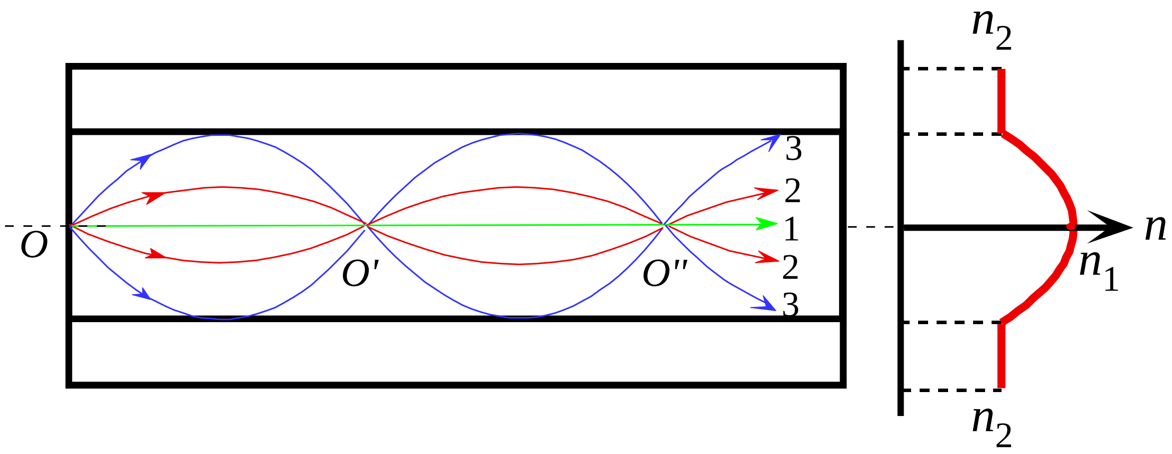
Dispersion flattened fiber example. The material dispersion coefficient (D_m) for the core material and waveguide dispersion coefficient (D_w) for the doubly clad fiber result in a flattened small chromatic dispersion between λ_1 and λ_2.



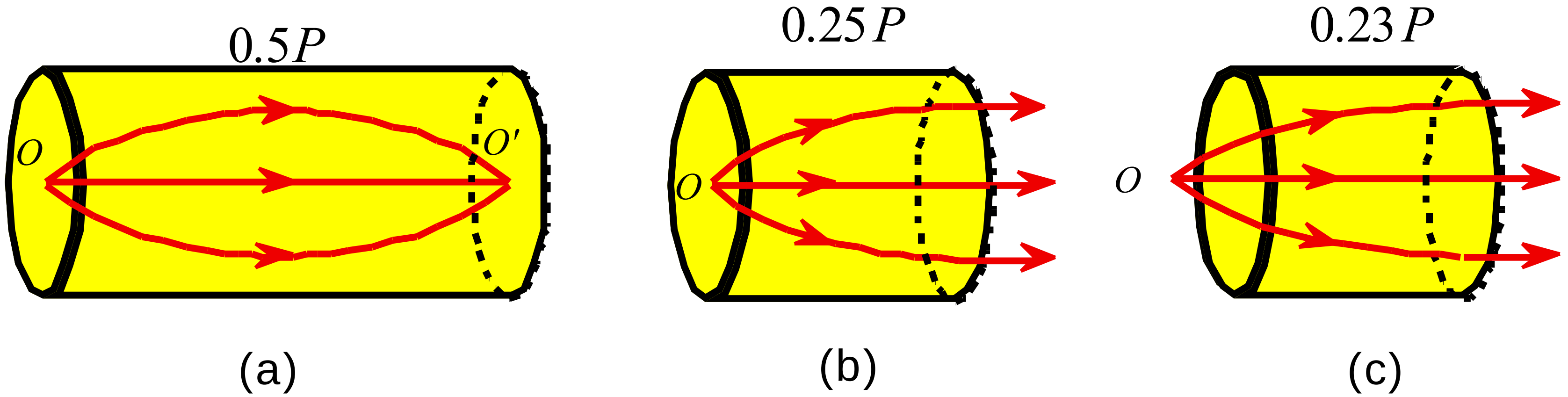
An optical fiber link for transmitting digital information and the effect of dispersion in the fiber on the output pulses.



(a) Multimode step index fiber. Ray paths are different so that rays arrive at different times.



(b) Graded index fiber. Ray paths are different but so are the velocities along the paths so that all the rays arrive at the same time.



Graded index (GRIN) rod lenses of different pitches. (a) Point O is on the rod face center and the lens focuses the rays onto O' on to the center of the opposite face. (b) The rays from O on the rod face center are collimated out. (c) O is slightly away fron the rod face and the rays are collimated out.

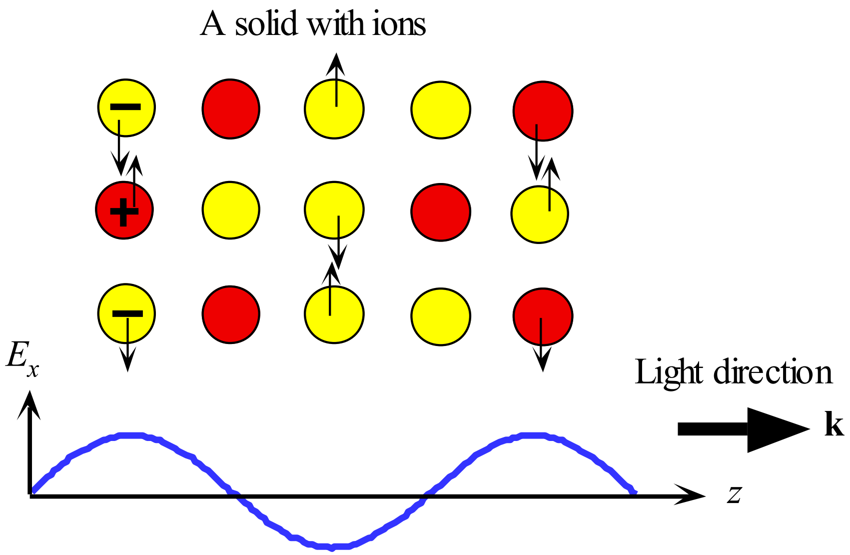
Sources of Loss and Attenuation in Fibers

Absorption depends on materials, amount of materials, wavelength, and the impurities in the substances.

It is cumulative and depends on the amount of materials, e.g. length of the fiber optics.

$$(1-\alpha)^d$$

α is the absorption per unit length and *d* is the distance that light travels



Lattice absorption through a crystal. The field in the wave oscillates the ions which consequently generate "mechanical" waves in the crystal; energy is thereby transferred from the wave to lattice vibrations.

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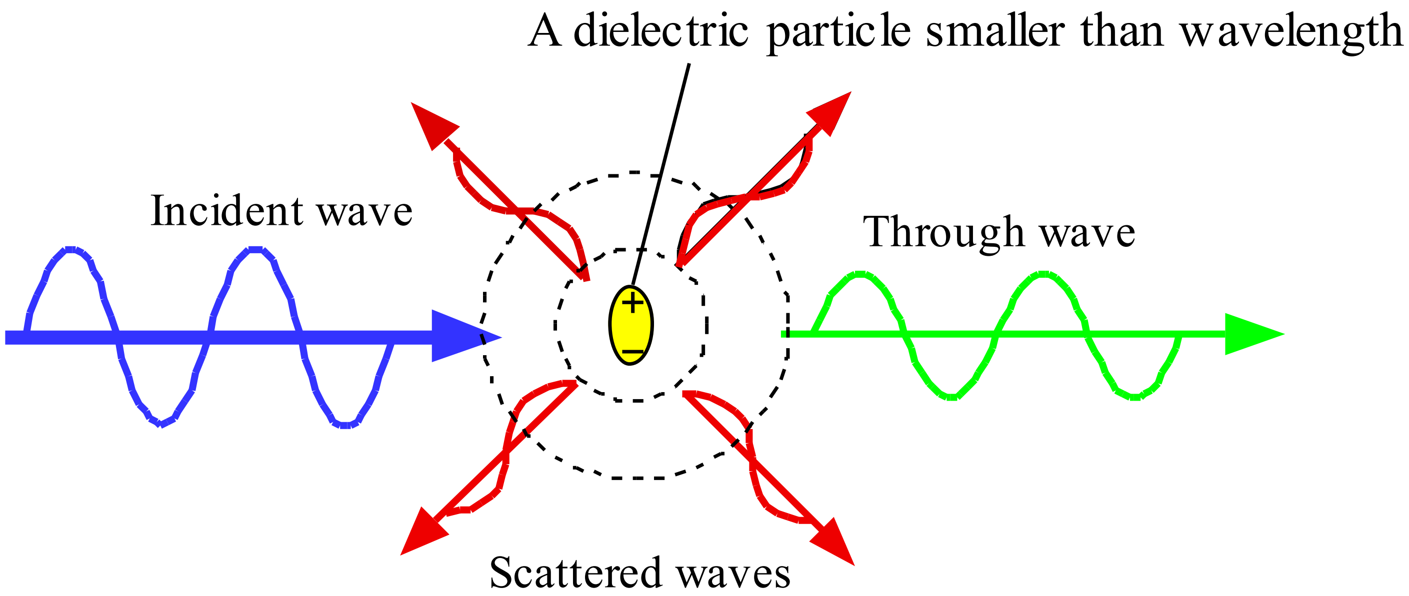
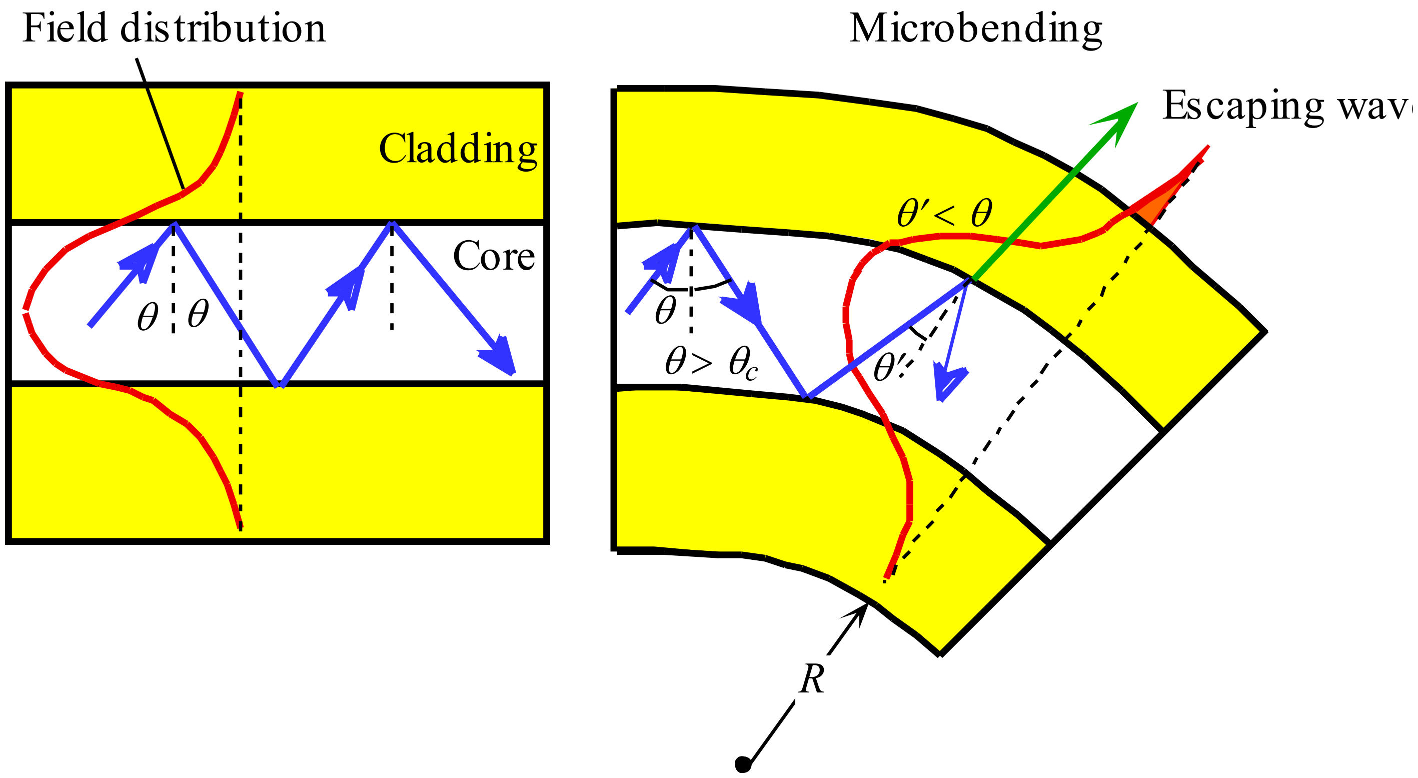


Rayleigh scattering involves the polarization of a small dielectric particle or a region that is much smaller than the light wavelength. The field forces dipole oscillations in the particle (by polarizing it) which leads to the emission of EM waves in "many" directions so that a portion of the light energy is directed away from the incident beam.



Sharp bends change the local waveguide geometry that can lead to waves escaping. The zigzagging ray suddenly finds itself with an incidence angle θ' that gives rise to either a transmitted wave, or to a greater cladding penetration; the field reaches the outside medium and some light energy is lost.

Attenuation in Optical Fiber

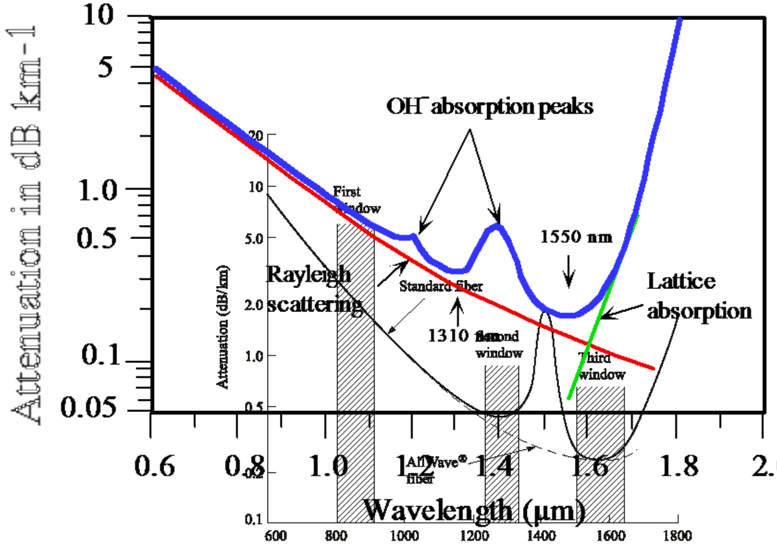


Illustration of a typical attenuation vs. wavelength characteristics of a silica based optical fiber. There are two communications channels at 1310 nm and 1550 nm.

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$$\alpha_{dB} = \frac{1}{L} 10 \log\left(\frac{P_{in}}{P_{out}}\right)$$

G. Keiser (Ref. 1)

$$P_{out} = P_{in}\left(e^{-\alpha L}\right)$$

$$\alpha_{dB} = 4.34\alpha$$