ELEC442/6601 DSP: Final Exam

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Instructions:

1. ELEC442: Answer questions 1-5. ELEC6601: Answer all questions. Time given 3 hour.
2. Only **four** pages of two crib sheets and a basic calculator are allowed.
3. Return the question paper before you leave the exam room.

Q1 **(20 marks)**

DFT: Suppose we have two four-point sequences $x[n]$ and $h[n]$ as follows:
$x[n] = cos(\frac{\pi n}{2}), n = 0, 1, 2, 3$ and $h[n] = 2^n, n = 0, 1, 2, 3$.

a) Calculate the four-point DFT $X[k]$.
b) Calculate the four-point DFT $H[k]$.
c) Calculate the circular convolution $*^4$ of $x[n]$ and $h[n]$, $y[n] = x[n] *^4 h[n]$, in any way, e.g., directly or indirectly.

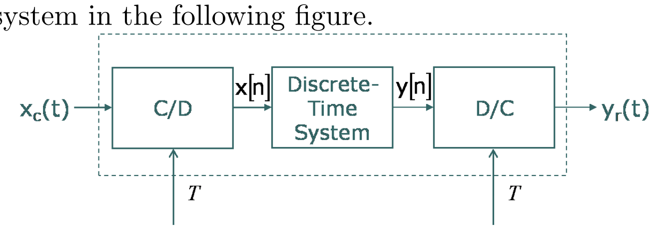
Q2 **(20 marks)**

LTI systems and frequency response: An LTI system is described by the input-output relation $y[n] = x[n] + 2x[n-1] + x[n-2]$

a) Determine the impulse response, $h[n]$.
b) Is this a stable system?
c) Determine the frequency response of the system $H(e^{jw})$. Obtain a simple expression for $H(e^{jw})$.
d) Plot the magnitude and phase of $H(e^{jw})$.
e) Now consider a new system whose frequency response is $H_1(e^{jw}) = H(e^{j(w+\pi)})$.
Determine $h_1[n]$, the impulse response of the new system.

Q3 **(20 marks)**

Sampling: Consider the system in the following figure.



Assume that i) the discrete-time system is LTI and ii) $X_c(j\Omega) = 0$ for $|\Omega| \geq 4000\pi$.

a) Determine the largest possible value for the sampling period T.
b) Determine the corresponding frequency response $H(e^{jw})$ for the discrete-time system such that

$$Y_c(j\Omega) == \begin{cases} |\Omega| X_c(j\Omega) & : \quad 1000\pi < |\Omega| < 2000\pi, \\ 0 & : \quad \text{otherwise.} \end{cases}$$

Q4 **(20 marks)**

LTI systems and Transform analysis: Consider a causal LTI system with the system function $H(z) = \frac{1-a^{-1}z^{-1}}{1-az^{-1}}$, where a is real.

a) Write the difference equation that relates the input and the output of this system.
b) For what range of values of a is the system stable?
c) For $a = 1/2$, plot the pole-zero diagram and shade the ROC.
d) Find the impulse response $h[n]$ for this system.
e) Determine the magnitude response of this system? What type of system is this system?
f) Give one important property of the group delay of this system?

Q5 **(20 marks)**

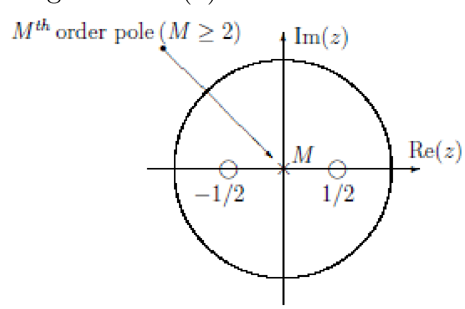
Filter design: Suppose that we are given a continuous-time low-pass filter with the frequency response $H_c(j\Omega)$ such that
$1 - \delta_1 \leq |H_c(j\Omega)| \leq 1 + \delta_1, \quad \text{when}|\Omega| \leq \Omega_p.$
$|H_c(j\Omega)| \leq \delta_2, quad \text{when}|\Omega| \geq \Omega_s.$

A set of discrete-time low-pass filters can be obtained from $H_c(s)$ by using the bi-linear transformation.

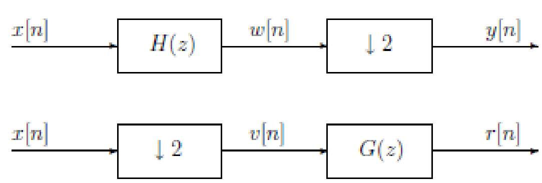
a) Assume that Ω_p is fixed, find the value of T_d such that the corresponding pass-band cutoff frequency for the discrete-time system is $\omega_p = \pi/2$.
b) With Ω_p fixed, sketch ω_p as a function of $0 < T_d < \infty$.

Q6 **(20 marks)**

Consider the following pole-zero diagram of $H(z)$:



Assume that $H(1) = 3/4$ and the ROC of $H(z)$ is $|z| > 0$:
a) Determine the system function $H(z)$.
b) Determine if the system $H(z)$ possesses the following properties:
stable, causal, all-pass, IIR, FIR, linear phase, generalized linear phase?
c) Consider the following system



Find $G(z)$ such that $y[n] = r[n]$.