Digital Signal Processing

ELEC 442/6601

Final Exam

Summer 2010

- Questions have equal weight
- Available time is 3 hours.
- Exam is closed book.
- You may use the provided formula sheet.
- You may use ENCS standard calculator.
- Show all the intermediate steps of your solution.
- Make reasonable assumptions if required.

Graduate Students:

Answer all 10 questions

Undergraduate Students:

Select 9 questions and answer them. If you answer 10 questions, they will be marked and extra bonus marks will be given.

Problem on Z-Transform

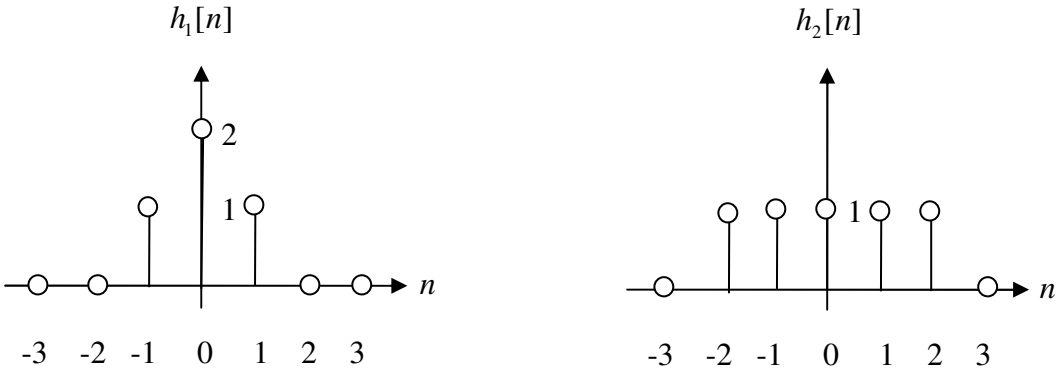
Consider the unstable LTI system with system function of

$$H(z) = \frac{1 - \frac{1}{3} z^{-1}}{1 + z^{-1} - 2z^{-2}}$$

1) Draw the pole-zero plot of the system function. Write down the difference equation describing the system.
2) Find the impulse response of the system. Is this system causal? Justify your answer.

Problem on Discrete Systems

Consider a system which is cascade of two systems with impulse responses of $h_1[n]$ and $h_2[n]$ as shown in the figure:



3) Determine the impulse response $h[n]$ and the frequency response $H\left(e^{j\omega}\right)$ of the overall system.
4) How can we make the overall system causal and suitable for implementation? Determine the impulse response of the new system and draw the signal flow graph of this causal system.

Problem on Filter Design

Consider the analog filter with transfer function of $H(s) = \frac{a}{s+a}$ where a is a constant.

5) Design a low pass digital filter with 3-dB cut-off frequency of 0.25π using above analog filter. Draw signal flow graph of the filter. Verify the cut-off frequency of the digital filter.
6) Use the low pass digital filter of part a and design a high pass digital filter with 3-dB cut-off frequency of 0.15π. Draw signal flow graph of the high pass filter. Verify the cut-off frequency of the digital high pass filter.

Problem on LTI Systems

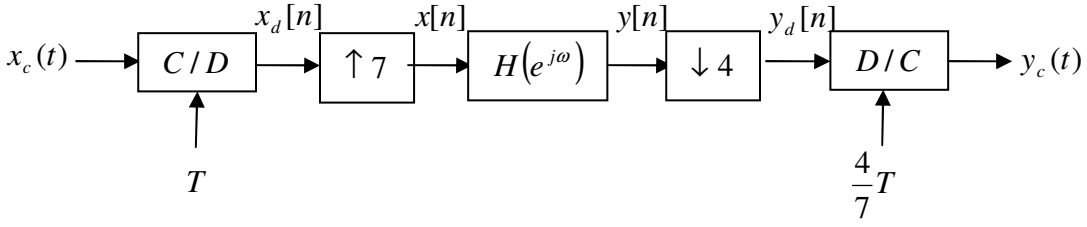
A discrete-time LTI system is determined by the difference equation:

$$y[n] = -\frac{3}{4}y[n-1] - \frac{1}{8}y[n-2] + \frac{1}{4}x[n] + x[n-1]$$

7) Show that this system is not a minimum phase system.
8) Find the impulse response of a minimum phase system which corresponds to the above system.

Problem on Sampling

Consider the system shown in the following figure:



In this system, the input $x_c(t)$ is a band-limited signal limited to $\Omega_m = \frac{\pi}{10T}$. This signal is sampled using an ideal C/D convertor at sampling period of *T* seconds. The up-sampled signal is passed through an ideal digital low pass filter with cut-off frequency of $\frac{\pi}{7}$. The output of the filter is down sampled. Then the discrete signal is converted to continuous by an ideal D/C convertor. Note that the D/C block has a low pass filter with cut-off frequency of π.

9) Write down the input-output relationship of all the blocks in time domain. Write down the input-output relationship of all the blocks in frequency domain as well.

10) Draw the frequency response at the output of each block. Find out the relation between input and output of the overall system in time-domain and frequency-domain.