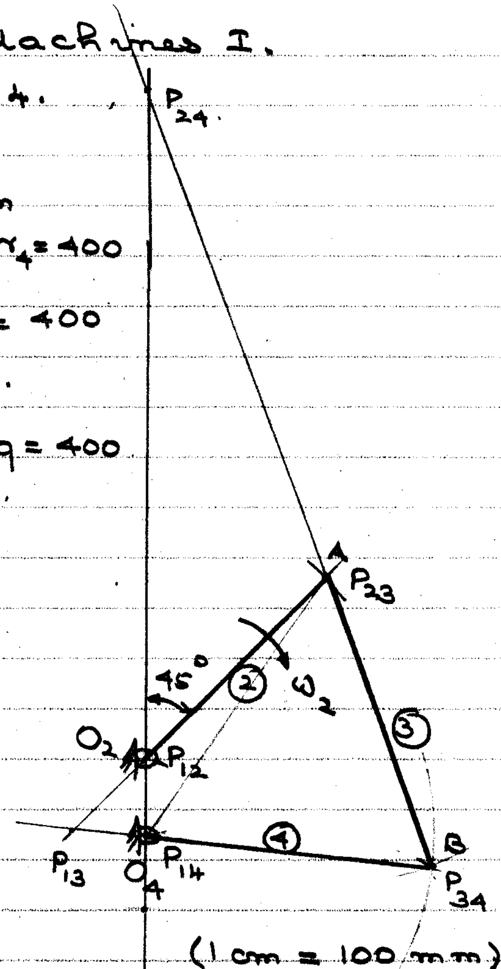
MECH 343/2 X : Theory of Machines I.
Solution to Assignment # 4.

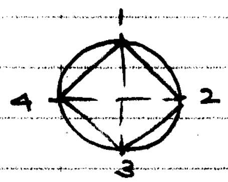


1. For the four-bar mechanism
   $r_1 = 100$, $r_2 = 350$, $r_3 = 425$, $r_4 = 400$

   O_2A: $s = 100$, $l = 425$, $p = 350$, $q = 400$
   $s + l < p + q$ crank.

   O_4B: $s = 100$, $l = 425$, $p = 350$, $q = 400$
   $s + l < p + q$ Crank.

   $\therefore$ It is double-crank or drag-link.



From pairing, $P_{12}, P_{23}, P_{34}, P_{14}$ can be marked.

P_{13} lies on $P_{12}P_{23}$ and $P_{14}P_{34}$

P_{24} lies on $P_{12}P_{14}$ and $P_{23}P_{34}$

ΔO_2AO_4: $O_4A^2 = 100^2 + 350^2 - 2 \times 100 \times 350 \times \cos 135^\circ$

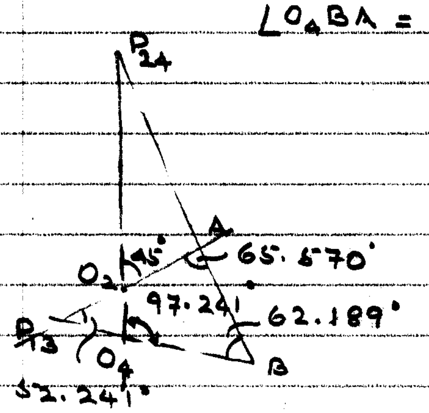
$O_4A = 426.61$ mm.

$\angle O_2O_4A = \sin^{-1} \dfrac{\sin 135^\circ}{O_4A} \times O_2A$

$= 35.459^\circ$

ΔAO_4B $\quad \angle AO_4B = \cos^{-1}\left(\dfrac{O_4A^2 + O_4B^2 - AB^2}{2 \cdot O_4A \cdot O_4B}\right) = 61.782^\circ$

$\angle O_4BA = \cos^{-1}\left(\dfrac{O_4B^2 + AB^2 - O_4A^2}{2 \cdot O_4B \cdot AB}\right) = 62.189^\circ$

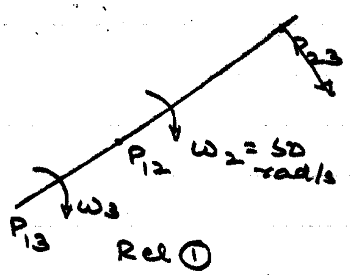


$\Delta P_{13}AB$: $AP_{13} = \dfrac{\sin 62.189}{\sin 52.241} \times 425$

$= 475.48$ mm

$BP_{13} = \dfrac{\sin 65.570}{\sin 52.241} \times 425$

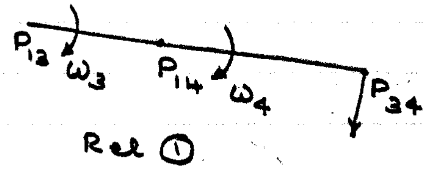
$= 489.44$ mm.



$$\omega_2 \cdot P_{12}P_{23} = \omega_3 \cdot P_{13}P_{23}$$

$$\therefore \omega_3 = \frac{50 \times 350}{475.48} = 36.805 \text{ rad/s}$$

(CW).



$$\omega_3 \cdot P_{13}P_{34} = \omega_4 \times P_{12}P_{34}$$

$$\therefore \omega_4 = \frac{36.805 \times 489.44}{400} = 45.034 \text{ rad/s}$$

(CW)

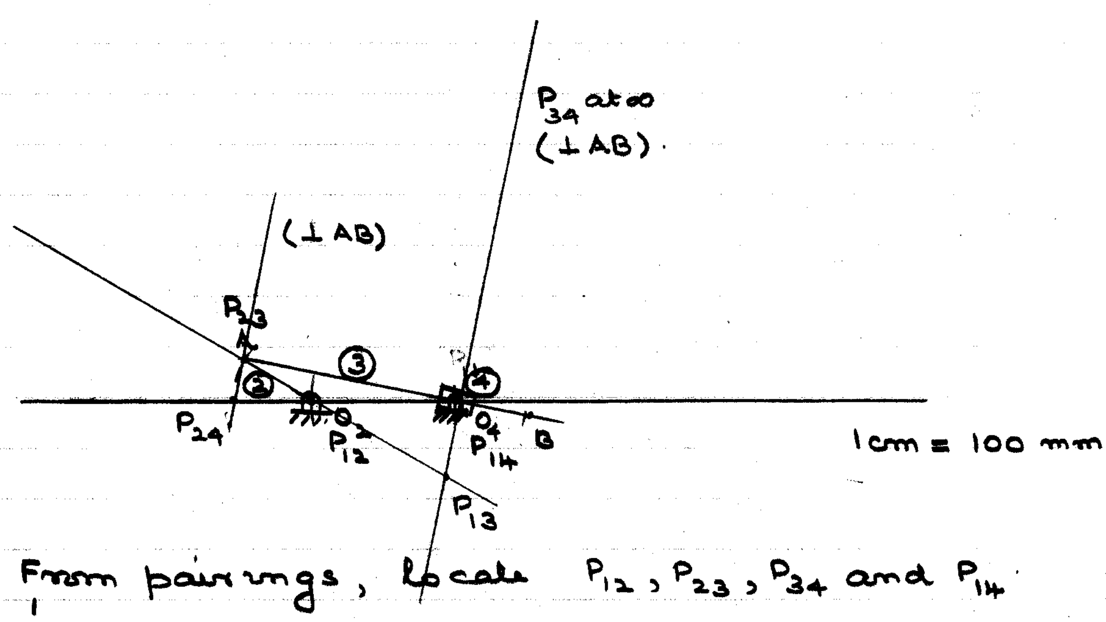
CW +: $\omega_{3|2} = 36.805 - 100 = -63.195$

$\therefore \omega_{3|2}$ is 63.195 rad/s in CCW sense

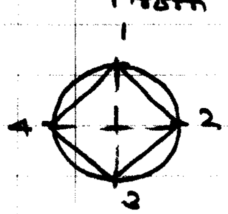
$\omega_{4|3} = 45.034 - 36.805 = 8.229$

$\therefore \omega_{4|3}$ is 8.229 rad/s in CW sense.

2.

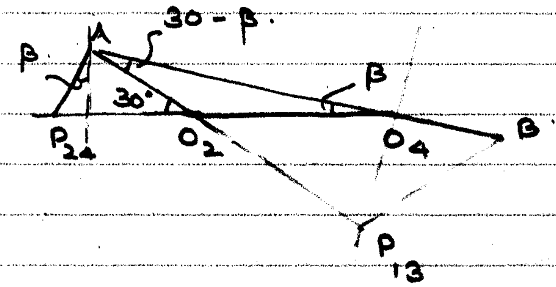


From pairings, locate P_{12}, P_{23}, P_{34} and P_{14}



P_{13} lies on $P_{12}P_{23}$ and $P_{14}P_{34}$

P_{24} lies on $P_{12}P_{14}$ and $P_{23}P_{34}$



$\Delta O_2 A O_4$.

$O_4A^2 = 100^2 + 200^2 - 2 \times 100 \times 200 \times \cos 150°$

$O_4A = 290.93$ mm.

$\sin\beta = O_2A \cdot \dfrac{\sin 30}{O_4A}$

$\beta = 9.896°$

$O_2\hat{A}O_4 = 30 - \beta$

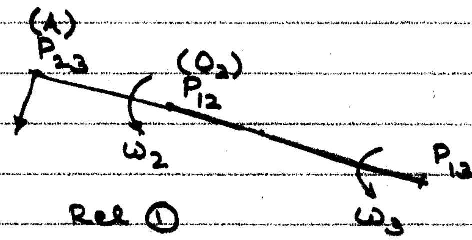
$= 20.104°$

$\Delta P_{24} A O_4$:

$O_4P_{24} = \dfrac{O_4A}{\cos\beta} = 295.32$ mm.

$\Delta P_{13} A O_4$:

$AP_{13} = \dfrac{O_4A}{\cos(30-\beta)} = 309.81$ mm.



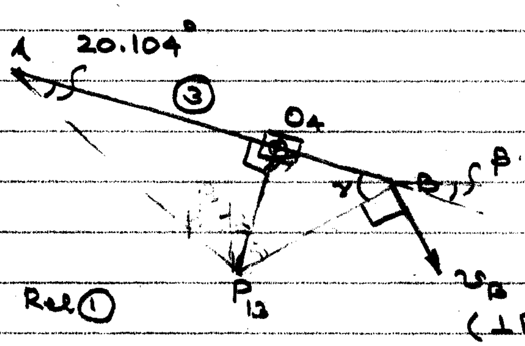
$\omega_2 \cdot P_{12}P_{23} = \omega_3 \cdot P_{13}P_{23}$

$\therefore \omega_3 = \dfrac{50 \times 100}{309.81}$

$= 16.139$ rad/s.
(CCW)

Note $\omega_4 = \omega_3$

$= 16.139$ rad/s
(CCW)



B is on 3

$O_4B = 400 - 290.93$

$= 109.07$ mm

$\Delta AP_{13}B$: $P_{13}B^2 = 309.81^2 + 400^2 - 2 \times 309.81 \times 400 \times \cos 20.104$

$\therefore P_{13}B = 152.43$ mm

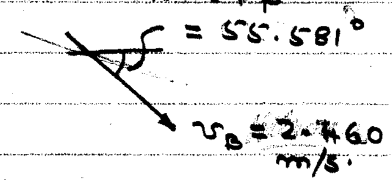
$\sin\gamma = \dfrac{309.81}{P_{13}P} \times \sin 20.104$

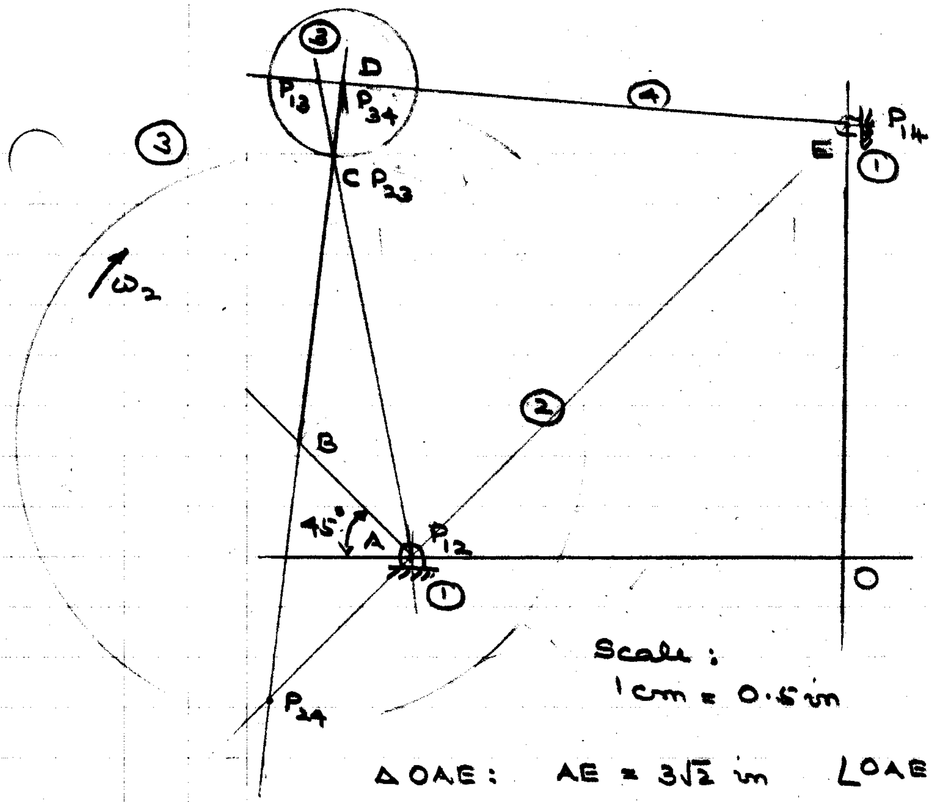
$\gamma = 44.315°$

$v_B = \omega_3 \times P_{13}B = 2.460$ m/s.

$90 - \gamma + \beta$

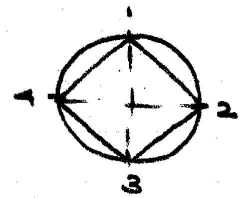
$= 55.581°$





Scale :
1 cm = 0.5 in

From pairings
P_{12}, P_{23}, P_{34}, P_{14}
are located.



P_{13} lies on $P_{12}P_{23}$ & $P_{14}P_{34}$
P_{24} lies on $P_{12}P_{14}$ & $P_{23}P_{34}$

ΔOAE: $AE = 3\sqrt{2}$ in $\quad \angle OAE = 45°$.

$\therefore \angle BAE = 90°$

ΔABE : $BE = \sqrt{1.25^2 + (3\sqrt{2})^2} = 4.4230$ in

$\angle AEB = \tan^{-1}\left(\frac{1.25}{3\sqrt{2}}\right) = 16.416°$

ΔBDE : $\cos \angle BED = \dfrac{3.5^2 + 4.423^2 - 2.5^2}{2(3.5)(4.423)}$

$\angle BED = 34.346°$

$\therefore \angle AED = 34.346 + 16.416 = 50.762°$

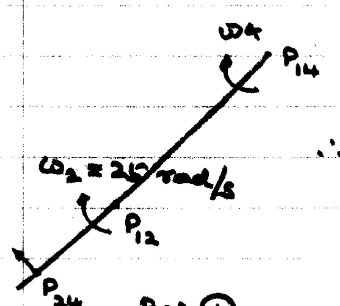
$\cos \angle BDE = \dfrac{2.5^2 + 3.5^2 - 4.423^2}{2(2.5)(3.5)}$

$\angle BDE = 93.481°$

ΔDEP_{24} $\quad \dfrac{EP_{24}}{\sin 93.481} = \dfrac{DP_{24}}{\sin 50.762} = \dfrac{3.5}{\sin 35.757}$

$\therefore EP_{24} = 5.9785$ in

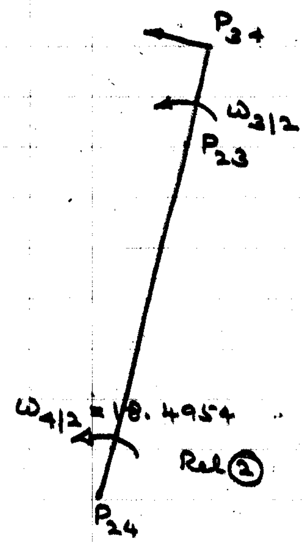
$DP_{24} = 4.6391$ in



$$\therefore \omega_4 = \frac{\omega_2 (5.9785 - 4.2426)}{5.9785}$$

$= 7.2589$ rad/s. (CW)

$\omega_{4/2} = 17.7411$ rad/s (CCW)

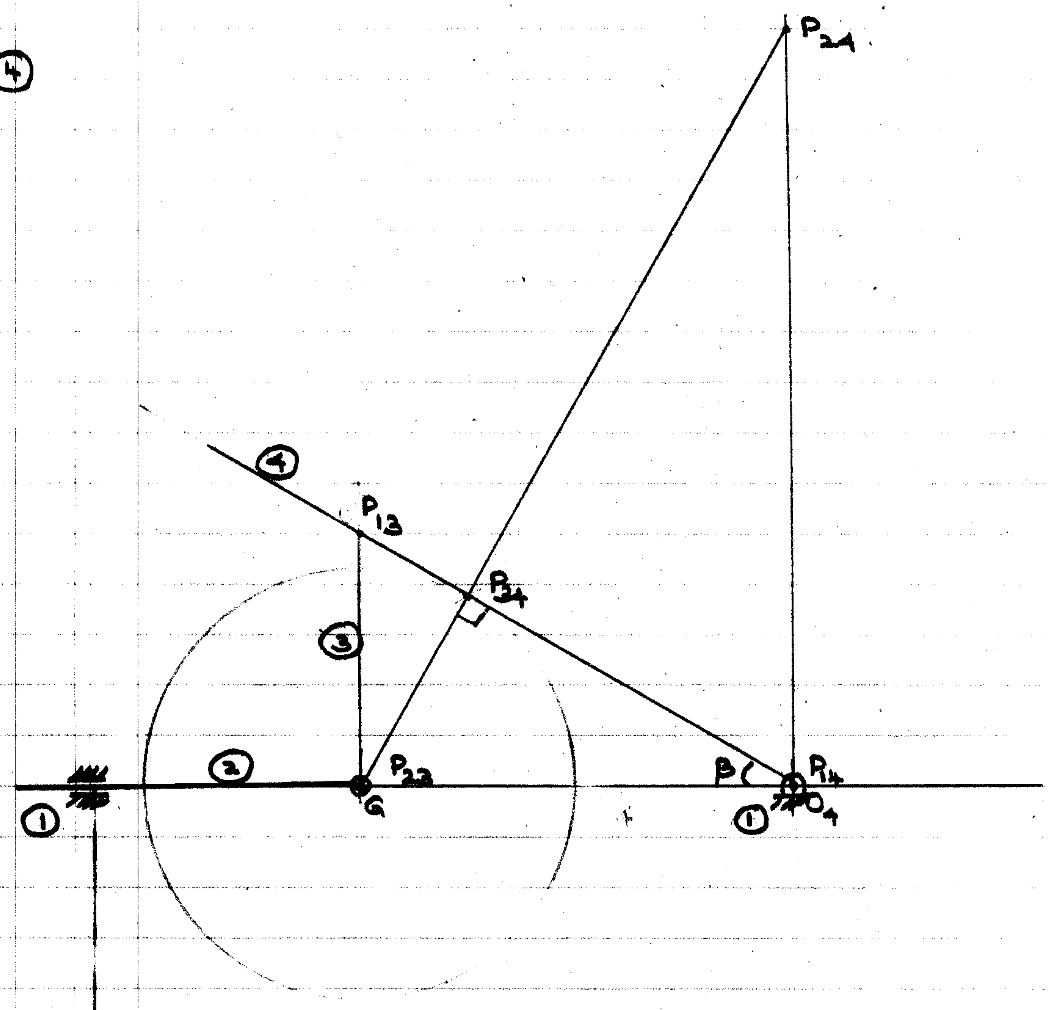


$$\omega_{3|2} = \frac{\omega_{4|2}(4.6391)}{(0.5)}$$
$$= 164.605 \text{ rad/s} \quad (\text{CCW})$$

$$\therefore \omega_3 = -25 + 164.605$$
$$= 139.605 \text{ rad/s} \quad (\text{CCW})$$

Note: Measuring distances from drawing will reduce drastically the time needed.

4

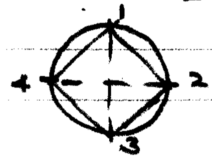


$\sin\beta = \frac{150}{300}$

$\therefore \beta = 30°$

P_{12} at ∞

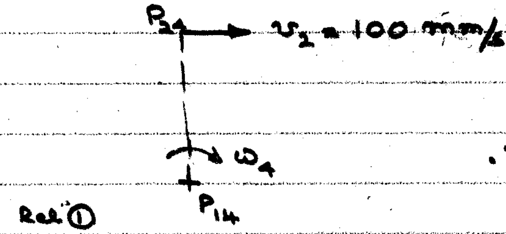
Mark P_{12}, P_{23}, P_{34} and P_{14}.



P_{13} lies on $P_{12}P_{23}$ and $P_{14}P_{34}$

P_{24} lies on $P_{12}P_{14}$ and $P_{23}P_{34}$;

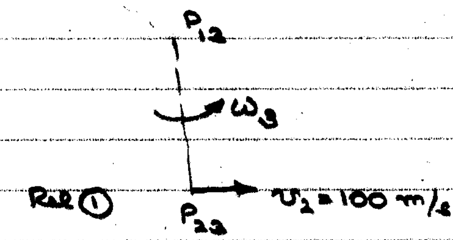
Here $\omega_1 = 0$ and $\omega_2 = 0$



$P_{14}P_{24} = O_4G \tan 60^\circ$
$= 519.62$ mm

$\therefore \omega_4 = \frac{v_2}{P_{14}P_{24}} = \frac{100}{519.62}$

$\therefore \omega_4 = 0.19245$ rad/s. CW

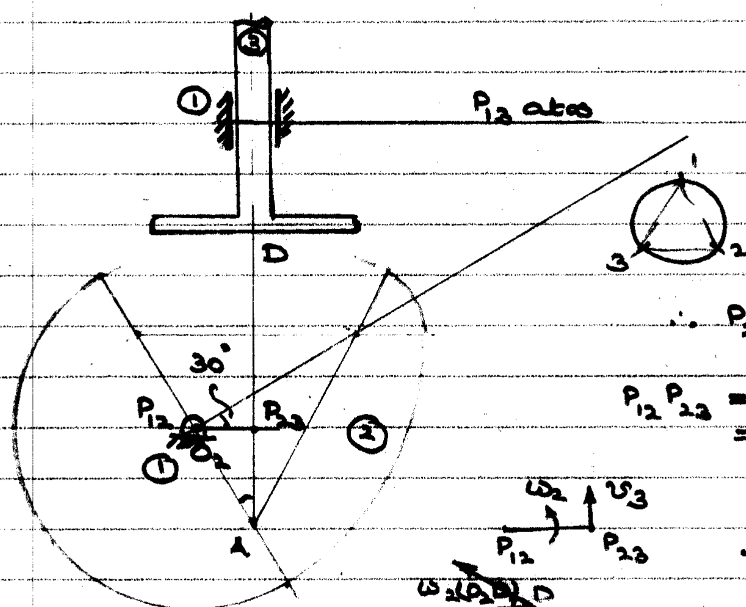


$P_{12}P_{23} = O_4G \tan 30^\circ$
$= 173.21$ mm.

$\therefore \omega_3 = \frac{v_2}{P_{13}P_{23}} = \frac{100}{173.21}$

$\therefore \omega_3 = 0.57735$ rad/s CCW.

5.



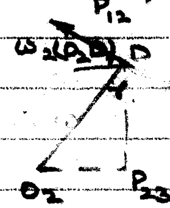
P_{12} is O_2.

P_{13} is at ∞

P_{23} lies on AD.

$\therefore P_{23}$ lies on AD and $P_{12}P_{13}$

$P_{12}P_{23} = O_2A \cdot \sin 30^\circ$
$= 7.5$ mm.



$\omega_2 = 50$ rad/s.

$\therefore v_3 = 0.0075 \times 50$
$= 0.375$ m/s.

velocity of rubbing $\Big\} = \omega_2 O_2D \cdot \cos\gamma$
$= \omega_2 \cdot P_{23}D$
$= (50)(0.040 + 0.015 \cos 30)$
$= 1.3505$ m/s.