MECH 343/2 X: Theory of Machines I.
Solution to Assignment #7.

1. $N_2 = 20$, $N_3 = 60$, $P = 6$ teeth/in.

(a) $P = \frac{N}{D}$. $D_2 = \frac{N_2}{P} = \frac{20}{6} = 3.3333''$

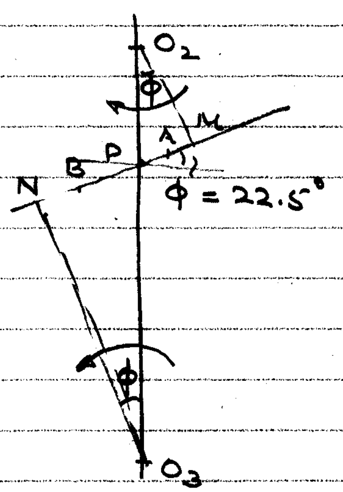
$D_3 = \frac{N_3}{P} = \frac{60}{6} = 10.0''$

$m = \frac{D}{N} = \frac{1}{6}$ in

$p = \frac{\pi D}{N} = \pi m = 0.5236''$

$C = \frac{1}{2}(D_2 + D_3) = 6.6667''$

(b)



$a_2 = a_3 = 1m = 0.1667''$

$O_2P = R_2 = \frac{1}{2} D_2 = 1.6667''$

$O_3P = R_3 = \frac{1}{2} D_3 = 5.0''$

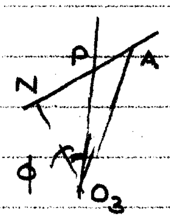
$O_3A = R_3 + a_3 = 5.1667''$

$O_2B = R_2 + a_2 = 1.8333''$

$PM = R_2 \sin\phi = 0.6378''$

$PN = R_3 \sin\phi = 1.9134''$

Contact begins at A



$AN = \sqrt{O_3A^2 - O_3N^2}$

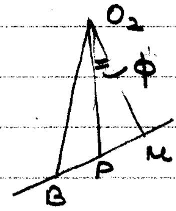
$= \sqrt{5.1667^2 - (5\cos 22.5)^2} = 2.3142''$

$AP = AN - PN$

$= 2.3142 - 1.9134 = 0.4008''$

$AP < PM$.

Contact ends at B



$BM = \sqrt{O_2B^2 - O_2M^2}$

$= \sqrt{1.8333^2 - (1.6667 \cos 22.5)^2}$

$= 0.9951$ in

$BP = BM - PM$

$= 0.9951 - 0.6378 = 0.3572''$

$BP < PN$

Since $AP < PM$ & $PP < PN$ No interference.

Path of contact $= AB$

$= AP + BP = 0.4008 + 0.3572$

$\therefore u = 0.7580$ in.

Arc of contact $= \dfrac{u}{\cos\phi} = 0.8205$ in

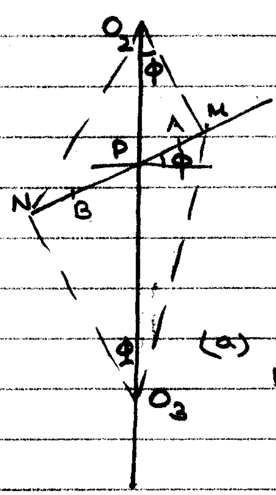
$\therefore q = 0.8205$ in

Average number of tooth pair in contact $\Big\} = \dfrac{q}{p} = \dfrac{0.8205}{0.5236} = 1.567$.

$m_c = 1.567 > 1$

$\therefore$ Continuous contact is ensured.

2.



For no interference:

$R_2 + a_2 < O_2N$.

$R_3 + a_3 < O_3M$.

$R_2 = \frac{1}{2}D_2 = 50$ mm

$R_3 = \frac{1}{2}D_3 = 90$ mm

$\therefore a_2 < \sqrt{O_2M^2 + NM^2} - R_2$.

$a_3 < \sqrt{O_3N^2 + NM^2} - R_3$.

Now $O_2M = R_2\cos\phi = 46.194$ mm

$PM = R_2 \sin\phi = 19.134$ mm.

$O_3N = R_3\cos\phi = 83.149$ mm

$PN = R_3\sin\phi = 34.442$ mm.

$NM = PN + PM = 53.576$ mm.

$$\therefore a_2 < \sqrt{46.194^2 + 53.576^2} - 50 = 20.741 \text{ mm}$$

$$a_3 < \sqrt{83.149^2 + 53.576^2} - 90 = 8.915 \text{ mm}.$$

$$\therefore a < 20.741, \quad a < 8.915$$

$$\therefore a_{max} = 8.915.$$ (This corresponds to wheel)

(b). $a = 1\,m$

$= \frac{D}{N}$. $\quad \therefore \frac{D}{N} < a_{max}$

$$\therefore N_2 > \frac{D_2}{a_{max}} = 11.22.$$

$$N_3 > \frac{D_3}{a_{max}} = 20.19$$

$\therefore N_2$ must be atleast 12.

N_3 must be atleast 21.

In order to maintain $\frac{N_3}{N_2} = \frac{D_3}{D} = 2.$

$N_2 > 12$

$N_3 > 22.$

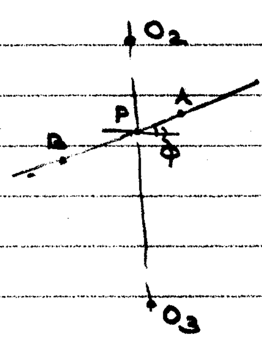
(c) $a = \frac{D_2}{N_2} = \frac{D_3}{N_3}$

$$\therefore a = \frac{100}{12} = 8.333 \text{ mm}.$$

(For design, the largest available standard addendum which is smaller than 8.333 mm must be chosen. $\therefore a = 8$ mm)

For this problem:

$a = 8.333$ mm.



$$AP = \sqrt{98.333^2 - (90\cos 22.5°)^2} - 90\sin 22.5°$$
$$= 18.052 \text{ mm}.$$

$$BP = \sqrt{58.333^2 - (50\cos 22.5°)^2} - 50\sin 22.5°$$
$$= 16.487 \text{ mm}.$$

$$x = 18.052 + 16.487 = 35.539 \text{ mm}.$$

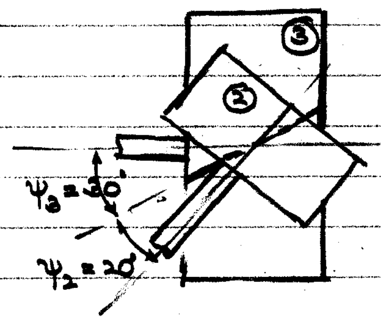
$$q = \frac{x}{\cos\phi} = 37.385 \text{ mm}.$$

$$p = \pi m = 26.179 \text{ mm}$$

$\therefore m_c = \frac{q}{p} = 1.428 > 1.$

$\therefore$ contact is ensured.

3.



Note: The pinion and wheel are left handed.

$\Sigma = \psi_2 + \psi_3 = 50°$

(a) $p_n = \frac{\pi D}{N}\cos\psi$ = normal pitch.

$\therefore$ normal module $= \frac{D\cos\psi}{N} = m_n$

normal diametral pitch $= \frac{N}{D\cos\psi} = P_n$.

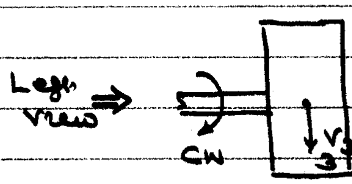
$\therefore D = \frac{N}{P_n\cos\psi}$

$D_2 = \frac{N_2}{P_n\cos\psi_2} = \frac{30}{5\cos 20°} = 6.3851''$

$D_3 = \frac{N_3}{P_n\cos\psi_3} = \frac{60}{5\cos 30°} = 13.8564''$

(b) $C = \frac{1}{2}(D_2 + D_3) = 10.1208''$

(c) Wheel

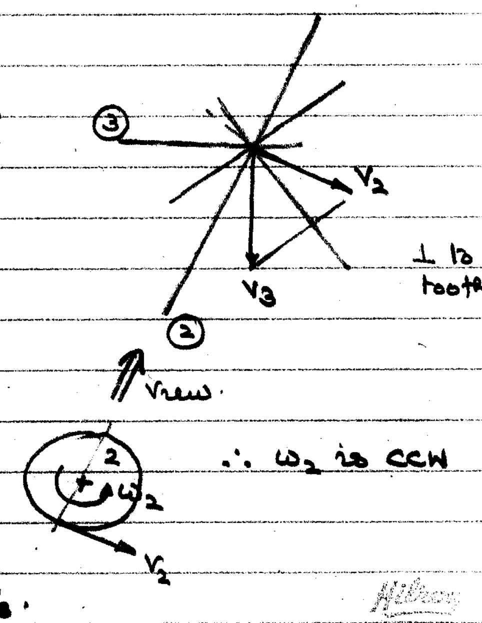


$V_3 = \frac{1}{2}D_3\omega_3$

$= 181.38''/s.$

$\omega_3 = \frac{250\times 2\pi}{60}$ rad/s

$= 26.180$ rad/s.



$V_2\cos\psi_2 = V_3\cos\psi_3.$

$\therefore V_2 = \frac{V_3\cos\psi_3}{\cos\psi_2}$

$= 167.16''/s.$

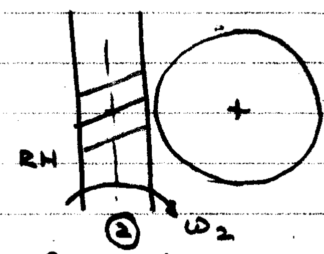
$\omega_2 = \frac{V_2}{\frac{1}{2}D_2} = 52.360$ rad/s.

$\therefore \omega_2$ is CCW

Note: $\frac{\omega_2}{\omega_3} = \frac{N_3}{N_2} = \frac{60}{30} = 2$

$$V_s = V_2 \sin\psi_2 + V_3 \sin\psi_3$$
$$= 167.16 \sin 20 + 181.38 \sin 30$$
$$= 147.86\,''/s = 12.321\ ft/s.$$

4.



$\lambda = 30°$

$\therefore \psi_2 = 90 - 30° = 60°$

$\Sigma = \psi_2 + \psi_3$

$\therefore \psi_3 = 30°$

(b) For worm:

Axial pitch $= \frac{1}{2}''$ (This must be given)

$\therefore$ Lead $= Np_a = 3(\frac{1}{2}) = 1.5''$

$\tan\lambda = \frac{\text{Lead}}{\pi D}$

$\therefore D_2 = \frac{1.5}{\pi \tan 30°} = 0.8270''$

For wheel:

$\frac{\omega_2}{\omega_3} = \frac{N_3}{N_2}$

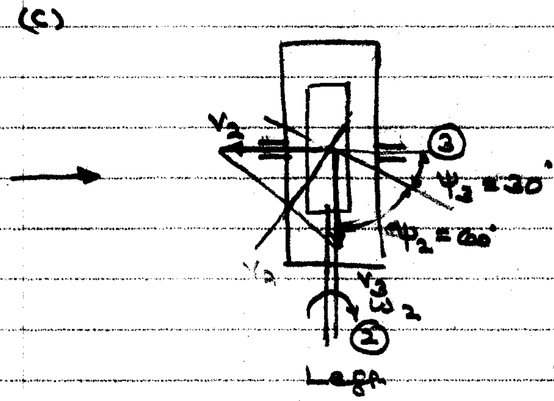
$\therefore 15 = \frac{N_3}{3}$

$\therefore N_3 = 45$

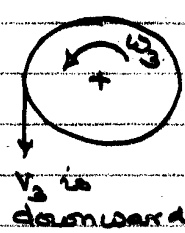
$\therefore \frac{D_3 \cos\psi_3}{N_3} = \frac{D_2 \cos\psi_2}{N_2}$

$\therefore D_3 = \frac{D_2 \cos\psi_2}{\cos\psi_3} \cdot \frac{N_3}{N_2}$
$= 7.1620''$

(c)



Left view

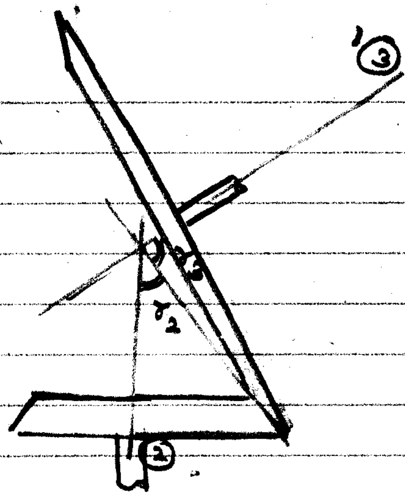


V_3 is downward.

$\therefore$ In the front view ③ rotates in CCW sense.

$\frac{\omega_3}{\omega_2} = \frac{N_2}{N_3}$ $\therefore \omega_3 = \frac{\omega_2}{15}$
$= 100$ RPM.

5.



For bevel gears.

$$\frac{R_3}{R_2} = \frac{\sin\gamma_3}{\sin\gamma_2}$$

and

$$\frac{\omega_2}{\omega_3} = \frac{R_3}{R_2} = \frac{N_3}{N_2}$$

$N_2 = 15 \quad N_3 = 33.$

$$\therefore \frac{N_3}{N_2} = \frac{33}{15} = 2.2$$ Reduction ratio.

(a)

$$\therefore \frac{\sin\gamma_3}{\sin\gamma_2} = 2.2.$$

$$\therefore \sin(120 - \gamma_2) = (2.2)(\sin\gamma_2).$$

$$(\sin 120)(\cos\gamma_2) - (\cos 120)(\sin\gamma_2) = (2.2)(\sin\gamma_2)$$

$$\therefore \tan\gamma_2 = \frac{\sin 120^\circ}{2.2 + \cos 120^\circ} = 0.5094$$

$$\gamma_2 = 26.996^\circ$$

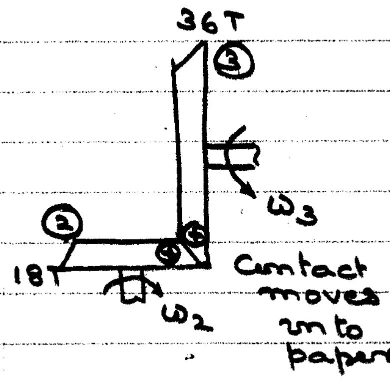
$$\gamma_3 = 120 - \gamma_2 = 93.004^\circ.$$

(b) $$p = \frac{\pi D_2}{N_2} = \frac{\pi D_3}{N_3}$$ " p = circumferential pitch.

$$D_2 = \frac{pN_2}{\pi} = \frac{(15)(15)}{\pi} = 71.620 \text{ mm}$$

$$D_3 = \frac{pN_3}{\pi} = \frac{(15)(33)}{\pi} = 157.563 \text{ mm}.$$

6. Bevel Gears:

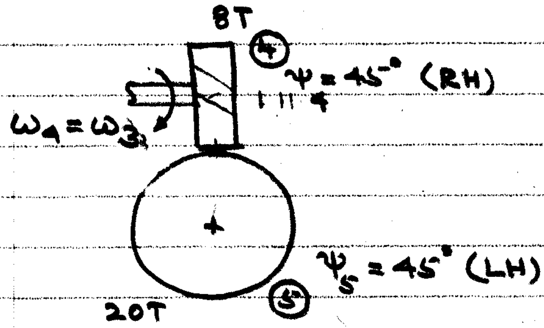


$$\frac{\omega_3}{\omega_2} = \frac{N_2}{N_3}$$

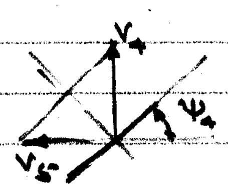
$$\omega_3 = \frac{18}{36} \times 1050$$

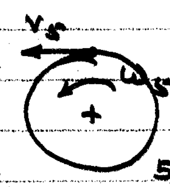
$$= 525 \text{ RPM}.$$

Worm & Wheel:



Top View





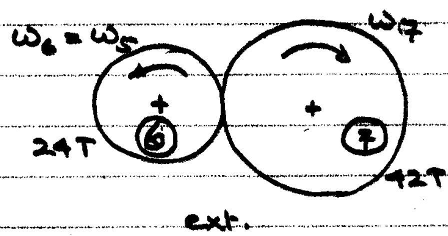
$(\psi_4 + \psi_5 = 90^\circ)$.

$$\frac{\omega_5}{\omega_4} = \frac{N_4}{N_5}$$

$$\omega_5 = \frac{8}{20} \times 525$$

$$= 210 \text{ RPM}.$$

Spur gear:



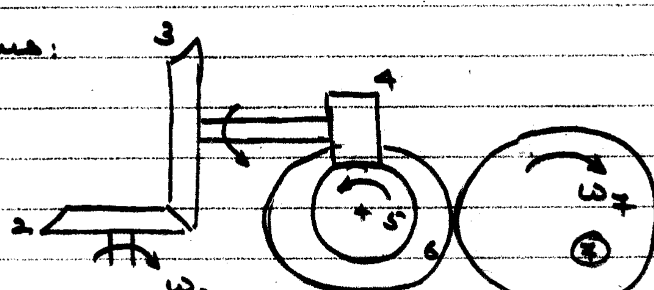
ext.

$$\frac{\omega_7}{\omega_6} = \frac{N_6}{N_7}$$

$$\therefore \omega_7 = \frac{24}{42} \times 210$$

$$= 120 \text{ RPM}.$$

Thus:



$$\text{Gear ratio} = \frac{\omega_7}{\omega_2} = \frac{120}{1050} = 0.1143 < 1$$

$\therefore$ Reduction Gear.

$$\therefore \left.\begin{array}{c}\text{Reduction}\\ \text{ratio}\end{array}\right\} = \frac{\omega_2}{\omega_7} = \frac{1050}{120} = 8.75$$