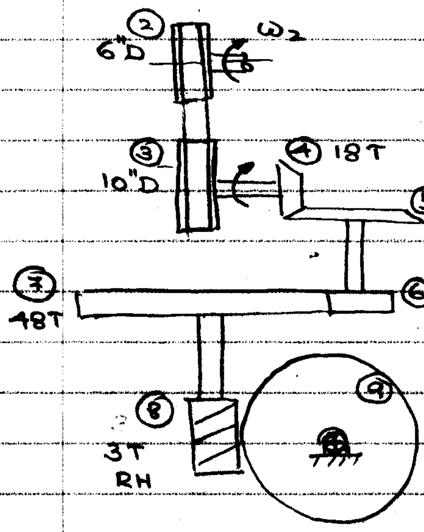
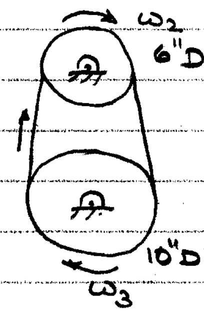
MECH 343/2 X : Theory of Machines ①

Solution to Assignment # 8.



Drive 2:3 : Direct V-belt

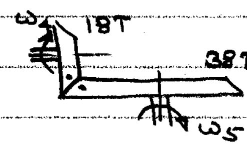


ω_3 is in the same sense as ω_2.

$$\frac{\omega_3}{\omega_2} = \frac{D_2}{D_3} = \frac{6}{10} \qquad (1)$$

3:4 Integral $\quad \frac{\omega_4}{\omega_3} = 1. \qquad (2)$

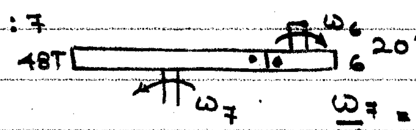
4:5



$$\frac{\omega_5}{\omega_4} = \frac{N_4}{N_5} = \frac{18}{38} \qquad (3)$$

5:6 Integral $\quad \frac{\omega_6}{\omega_5} = 1 \qquad (4)$

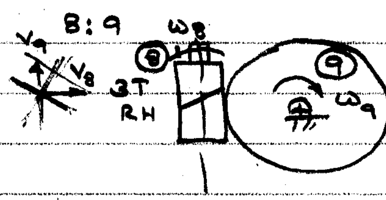
6:7



$$\frac{\omega_7}{\omega_6} = \frac{N_6}{N_7} = \frac{20}{48} \qquad (5)$$

7:8 Integral $\quad \frac{\omega_8}{\omega_7} = 1 \qquad (6)$

8:9



$$\frac{\omega_9}{\omega_8} = \frac{N_8}{N_9} = \frac{3}{36} \qquad (7)$$

$\therefore$ ω_9 is in CW sense when viewed from front.

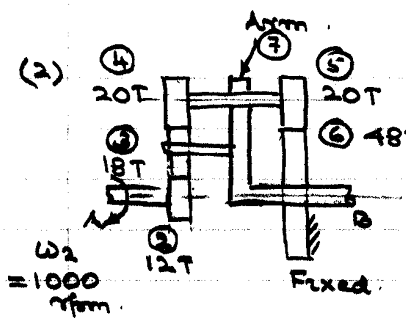
$$\frac{\omega_9}{\omega_2} = \left(\frac{3}{36}\right)(1)\left(\frac{20}{48}\right)(1)\left(\frac{18}{38}\right)(1)\left(\frac{6}{10}\right)$$

$$= \frac{3}{304}$$

$\omega_2 = 1200$ rpm.

$$\therefore \omega_9 = \frac{3}{304} \times 1200 = 11.84 \text{ rpm.}$$

(2)



Method (1) : Analytical Approach.

For this gear, 7 is the arm.

CW sense (when viewed from left) is chosen as the positive sense.

$\therefore \omega_2 = +1000$ rpm.

2 : 3 is external mesh ; 7 is the arm.

$$\therefore \frac{\omega_{3|7}}{\omega_{2|7}} = -\frac{N_2}{N_3} = -\frac{12}{18} \qquad (1)$$

3 : 4 is external mesh ; 7 is the arm

$$\therefore \frac{\omega_{4|7}}{\omega_{3|7}} = -\frac{N_3}{N_4} = -\frac{18}{20} \qquad (2).$$

4 : 5 integral

$$\therefore \omega_5 = \omega_4 \qquad \therefore \frac{\omega_{5|7}}{\omega_{4|7}} = 1 \qquad (3).$$

5 : 6 is external mesh ; 7 is the arm

$$\therefore \frac{\omega_{6|7}}{\omega_{5|7}} = -\frac{N_5}{N_6} = -\frac{20}{48} \qquad (4).$$

$$\therefore \frac{\omega_{6|7}}{\omega_{2|7}} = \left(-\frac{20}{48}\right)(1)\left(-\frac{18}{20}\right)\left(-\frac{12}{18}\right) = -\frac{1}{4}.$$

$$\therefore \frac{\omega_6 - \omega_7}{\omega_2 - \omega_7} = -\frac{1}{4}$$

$$\frac{0 - \omega_7}{\omega_2 - \omega_7} = -\frac{1}{4} \qquad \therefore \omega_7 = \frac{1}{5}\omega_2$$

$$= \frac{1}{5} \times 1000 = 200 \text{ rpm}.$$

$\therefore$ Sense of (7) is CW when viewed from left

But, when viewed from right,

B rotates in CCW sense at 200 rpm.

Method (2) : Tabulation Approach

	2	3	4 : 5	6	7
Fix 7	$\left(-\frac{N_3}{N_2}\right)\left(-\frac{N_4}{N_3}x\right)$	$-\frac{N_4}{N_3}x$	x	$-\frac{N_5}{N_6}x$	0
	$= \frac{5}{3}x$	$= -\frac{10}{9}x$	$= x$	$= -\frac{5}{12}x$	0
Lock & Rotate 7	y	y	y	y	y
	$y + \frac{5}{3}x$	$y - \frac{10}{9}x$	$y + x$	$y - \frac{5}{12}x$	y.

(6) is stationary $\quad \therefore y - \frac{5}{12}x = 0$

$$y = \frac{5}{12}x.$$

$$\omega_2 = y + \frac{5}{3}x = \frac{5}{12}x + \frac{5}{3}x = \frac{25}{12}x$$

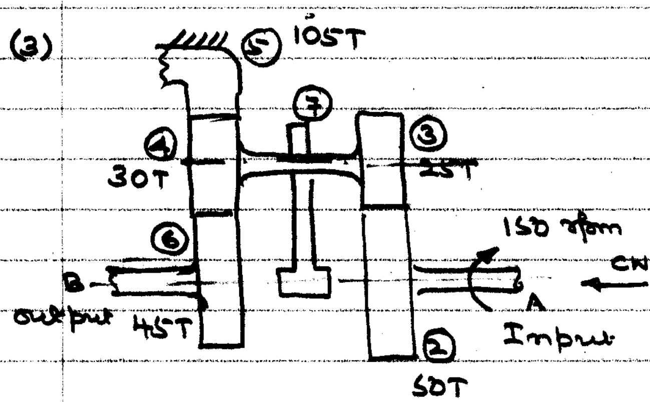
$$\frac{25}{12}x = 1000$$

$$x = 480 \qquad y = \frac{5}{12}x = 200$$

$\omega_7 = y = 200$ rpm.

(CW from left or CCW from right).

(3)



Analytical method:

Sign Convention:
CW sense from right is positive.

$\omega_2 = +150$ rpm.

2:3 Ext, 7 Arm

$$\frac{\omega_{3|7}}{\omega_{2|7}} = -\frac{N_2}{N_3} = -\frac{50}{25} \qquad (1)$$

3:4 Integral: $\omega_4 = \omega_3$

$$\therefore \frac{\omega_{4|7}}{\omega_{3|7}} = 1 \qquad (2)$$

4:5 Int, 7 Arm

$$\frac{\omega_{5|7}}{\omega_{4|7}} = +\frac{N_4}{N_5} = +\frac{30}{105} \qquad (3)$$

4:6 Ext, 7 Arm.

$$\frac{\omega_{6|7}}{\omega_{4|7}} = -\frac{N_4}{N_6} = -\frac{30}{45} \qquad (4)$$

(1) x (2) x (3)

$$\frac{\omega_{5|7}}{\omega_{2|7}} = \left(\frac{30}{105}\right)(1)\left(-\frac{50}{25}\right) = -\frac{4}{7}$$

$$\frac{\omega_5 - \omega_7}{\omega_2 - \omega_7} = -\frac{4}{7}$$

$$\frac{0 - \omega_7}{\omega_2 - \omega_7} = -\frac{4}{7}$$

$$\therefore 7\omega_7 = 4\omega_2 - 4\omega_7$$

$$\omega_7 = \frac{4}{11}\omega_2 = 54.55 \text{ rpm}.$$

(1) x (2) x (4) $\qquad \dfrac{\omega_{6/7}}{\omega_{2/7}} = \left(-\dfrac{30}{45}\right)(1)\left(-\dfrac{50}{25}\right) = \dfrac{4}{3}$

$$\frac{\omega_6 - \omega_7}{\omega_2 - \omega_7} = \frac{4}{3}$$

$$3\omega_6 - 3\left(\tfrac{4}{11}\omega_2\right) = 4\omega_2 - 4\left(\tfrac{4}{11}\omega_2\right)$$

$$\omega_6 = \frac{40}{33}\omega_2 = 181.8 \text{ rpm.}$$

When viewed from right B rotates in CW

∴ Viewed from left B rotates at 181.8 rpm in CCW sense.

Tabulation Method:

	2	3 : 4	5	6	7
Fix Arm	$-\frac{N_3}{N_2}x$	x	$+\frac{N_4}{N_5}x$	$-\frac{N_4}{N_6}x$	0
Rotate Arm	y	y	y	y	y
	$y - \frac{1}{2}x$	$y + x$	$y + \frac{2}{7}x$	$y - \frac{2}{3}x$	y.

5 is stationary: $\quad y + \frac{2}{7}x = 0$

$$y = -\frac{2}{7}x$$

$$\omega_2 = 150$$

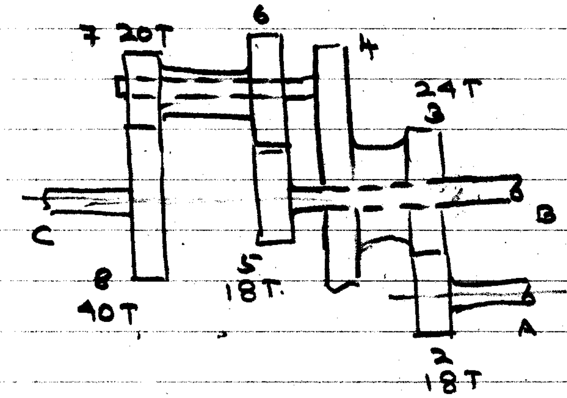
$$\therefore \; -\frac{2}{7}x - \frac{1}{2}x = 150$$

$$x = -190.91 \quad y = +54.55$$

$$\omega_6 = y - \frac{2}{3}x = 181.8 \text{ rpm.}$$

(As before)

4)



From geometry

$D_5 + D_6 = D_7 + D_8$

All teeth have same module.

$\therefore N_5 + N_6 = N_7 + N_8$

$\therefore N_6 = 20 + 40 - 18$
$= 42$

C is stationary $\quad \therefore \omega_8 = 0$

A : $\quad \omega_2 = +800$ rpm $\quad$ (CW sense from right) is chosen positive

Note : 2 is not a part of epicyclic gear train with arm 4.

Analytical method:

$$\frac{\omega_3}{\omega_2} = -\frac{N_2}{N_3} = -\frac{18}{24}$$

$$\therefore \omega_3 = -600 \text{ rpm} \quad (1)$$

3:4 Integral $\quad \omega_4 = \omega_3 = -600$ rpm $\quad$ (1a)

5:6 Ext, 4 Arm:

$$\frac{\omega_{6|4}}{\omega_{5|4}} = -\frac{N_5}{N_6} = -\frac{18}{42} \quad (2)$$

6:7 Integral $\quad \omega_7 = \omega_6 \quad \dfrac{\omega_{7|4}}{\omega_{6|4}} = 1. \quad (3)$

7:8 Ext, 4 Arm

$$\frac{\omega_{8|4}}{\omega_{7|4}} = -\frac{N_7}{N_8} = -\frac{20}{40} \quad (4)$$

(2) x (3) x (4)

$$\frac{\omega_{8|4}}{\omega_{5|4}} = \left(-\frac{20}{40}\right)(1)\left(-\frac{18}{42}\right) = \frac{3}{14}$$

$\omega_8 = 0 \qquad \dfrac{0 - (-600)}{\omega_5 - (-600)} = \dfrac{3}{14}$

$\omega_5 = 2200$ rpm.

$\therefore$ B rotates at 2200 rpm in CW sense when viewed from right.

Tabulation Method:

	2	3:4	5	6:7	8
Fix Arm		0	$-\frac{N_6}{N_5}x$	x	$-\frac{N_7}{N_8}x$
Rotate Arm		y	y	y	y
		y	$y-\frac{7}{3}x$	$y+x$	$y-\frac{1}{2}x$

2:3 Mesh $\left(-\frac{N_3}{N_2}\right)(y)$

$= -\frac{4}{3}y$

C is stationary: $y - \frac{1}{2}x = 0 \qquad x = 2y$.

$\omega_2 = 800$ rpm $\qquad \therefore -\frac{4}{3}y = 800$

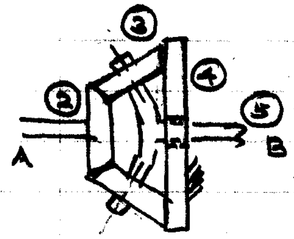
$\therefore y = -600, \quad x = -1200$

For shaft B:

$\omega_5 = y - \frac{7}{3}x = 2200$ rpm

(As before)

(5)



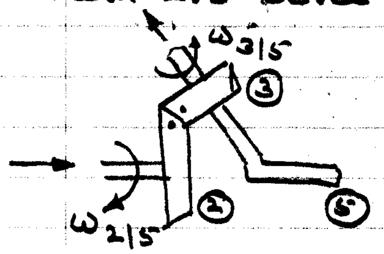
Here (2), (4) rotates about horizontal axis

CW sense from left is chosen positive

$\omega_2 = 800$ rpm.

Analytical Method:

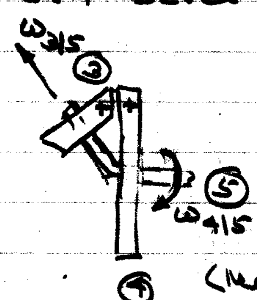
Mesh 2:3 Bevel, 5 is arm.



Rel Arm

$$\frac{\omega_{3|5}}{\omega_{2|5}} = + \frac{N_2}{N_3} = \frac{36}{21} \qquad (1)$$

Mesh 3:4 Bevel, 5 is arm.



(Marked in positive sense)

Actually $\omega_{4|5} < 0$.

$$\frac{\omega_{4|5}}{\omega_{3|5}} = - \frac{N_3}{N_4} = - \frac{21}{52} \qquad (2).$$

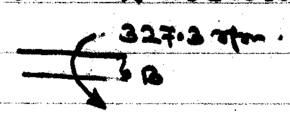
From (1) x (2) $\quad \dfrac{\omega_{4|5}}{\omega_{2|5}} = \left(-\dfrac{21}{52}\right)\left(\dfrac{36}{21}\right) = -\dfrac{9}{13}$

$\omega_4 = 0 \quad \therefore \dfrac{0 - \omega_5}{\omega_2 - \omega_5} = -\dfrac{9}{13}$

$$\omega_5 = \frac{9}{22}\,\omega_2 = 327.3 \text{ rpm}$$

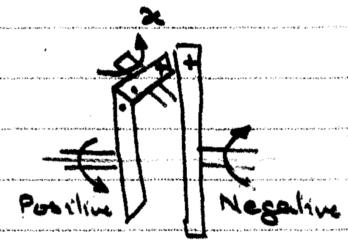
$\therefore$ B rotates at 327.3 rpm in CW sense when viewed from left.

or CCW sense when viewed from right.



Tabulation Method:

	2	3	4	5
Fix Arm	$+\frac{N_3}{N_2}x$	x	$-\frac{N_3}{N_4}x$	0
Rotate Arm	y	y	y	y
	$y+\frac{21}{36}x$	–	$y-\frac{21}{52}x$	y



CW from left is +

The rotations in different directions (Vector Addition)

$\omega_4 = 0$

$$\therefore \; y - \frac{21}{52}x = 0 \qquad x = \frac{52}{21}y$$

$\omega_2 = 800$

$$y + \frac{21}{36}\left(\frac{52}{21}y\right) = 800$$

$$\therefore y = 327.3 \text{ rpm.}$$

$\therefore$ 5 rotates at 327.3 rpm in CW sense from left.

As before.