MECH 343 Theory of Machines I

Time: _ _ W _ _17:45 - 20:15

Lecture 6

Linear Velocities by Relative Velocities

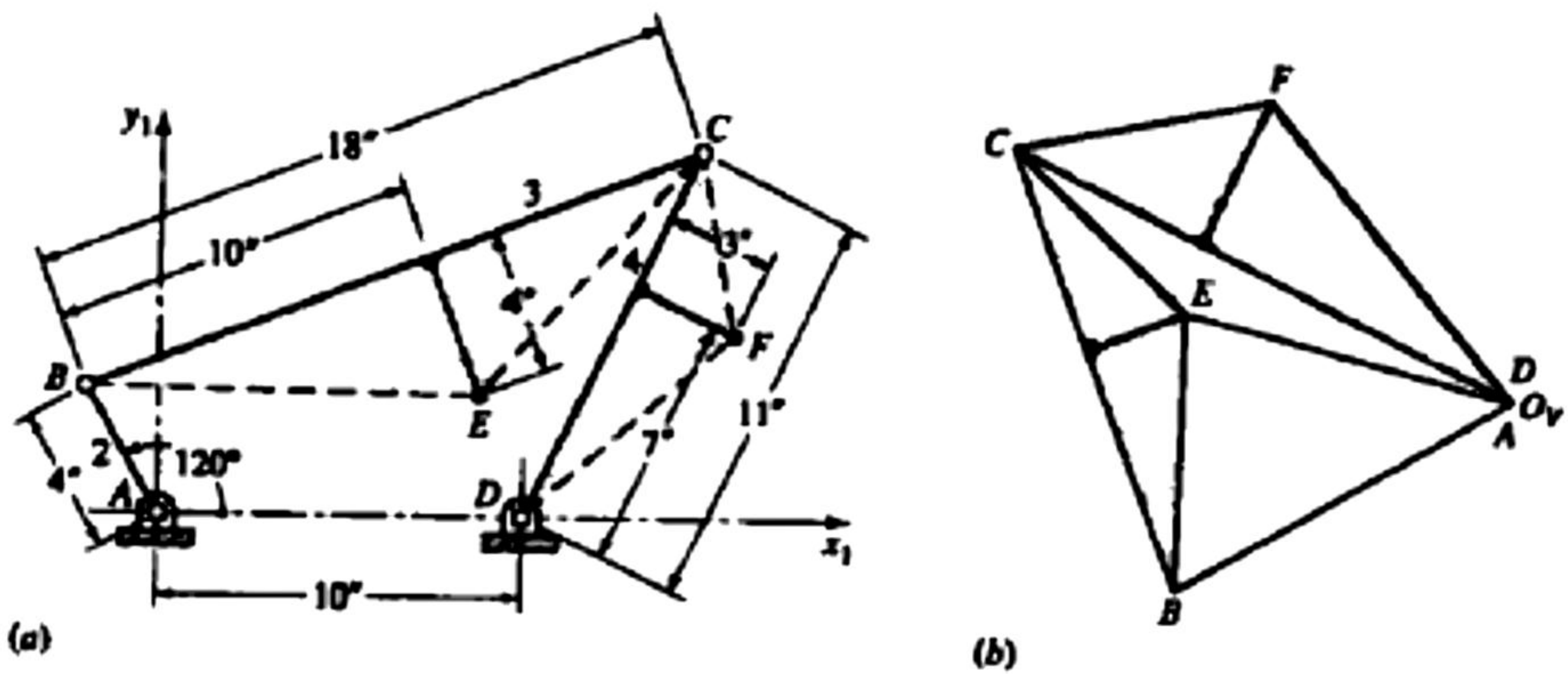
EXAMPLE 3.1

The four-bar linkage, drawn to scale in Fig. 3.7*a* with all the necessary dimensions, is driven by crank 2 at a constant angular velocity of $\omega_2 = 900$ rev/min ccw. Find the instantaneous velocities of point E in link 3 and point F in link 4 and the angular velocities of links 3 and 4 at the position shown.

SOLUTION

To obtain a graphical solution, we first calculate the angular velocity of link 2 in radians per second. This is

$$\omega_2 = \left(900\frac{\text{rev}}{\text{min}}\right)\left(2\pi\frac{\text{rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 94.2\text{ rad/s ccw.} \tag{1}$$



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Then we note that point A remains fixed and calculate the velocity of point B:

$$\mathbf{V}_B = \cancel{\mathbf{V}_A^0} + \mathbf{V}_{BA} = \omega_2 \times \mathbf{R}_{BA}$$

$$V_B = (94.2 \text{ rad/s})\left(\frac{4}{12}\text{ ft}\right) = 31.4 \text{ ft/s}. \tag{2}$$

We note that the form "$\omega \times \mathbf{R}$" was used for the velocity difference and not for the absolute velocity $\mathbf{V}_B$ directly. In Fig. 3.7b we choose the point O_V and a velocity scale factor. We note that the image point A is coincident with O_V and construct the line AB perpendicular to $\mathbf{R}_{BA}$ and toward the lower left because of the counterclockwise sense of ω_2; this line represents $\mathbf{V}_{BA}$.

If we attempt at this time to write an equation directly for the velocity of point E, we find by counting the unknowns that it cannot be solved yet. Therefore, we next write two equations for the velocity of point C. Because the velocities of points C_3 and C_4 must be equal (links 3 and 4 are pinned together at C), we have

$$\mathbf{V}_C = \overset{\surd\surd}{\mathbf{V}_B} + \overset{?\surd}{\mathbf{V}_{CB}} = \cancel{\mathbf{V}_D^0} + \overset{?\surd}{\mathbf{V}_{CD}}. \tag{3}$$

We now construct two lines in the velocity polygon: the line BC is drawn from B perpendicular to $\mathbf{R}_{CB}$, and the line DC is drawn from D (coincident with O_V because $\mathbf{V}_D = 0$) perpendicular to $\mathbf{R}_{CD}$. We label the point of intersection of these two lines point C. When the lengths of these two lines are scaled, we find that $V_{CB} = 38.4$ ft/s and $V_C = V_{CD} = 45.5$ ft/s. The angular velocities of links 3 and 4 can now be found,

$$\omega_3 = \frac{V_{CB}}{R_{CB}} = \frac{38.4 \text{ ft/s}}{18/12 \text{ ft}} = 25.6 \text{ rad/s ccw}, \qquad \textit{Ans.} \ (4)$$

$$\omega_4 = \frac{V_{CD}}{R_{CD}} = \frac{45.5 \text{ ft/s}}{11/12 \text{ ft}} = 49.6 \text{ rad/s ccw}, \qquad \textit{Ans.} \ (5)$$

where the directions of ω_3 and ω_4 were determined by the technique illustrated in Fig. 3.6c.

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There are now several methods of finding the velocity of point E, that is, $\mathbf{V}_E$. In one method we measure $\mathbf{R}_{EB}$ from the scale drawing of Fig. 3.7a and then, because points B and E are both attached to link 3, we can calculate*

$$V_{EB} = \omega_3 R_{EB} = (25.6 \text{ rad/s}) \left(\frac{10.8}{12} \text{ ft}\right) = 23.0 \text{ ft/s}. \tag{6}$$

We can now construct the line BE in the velocity polygon, drawn to the proper scale and perpendicular to $\mathbf{R}_{EB}$, thus solving† the velocity-difference equation,

$$\mathbf{V}_E = \mathbf{V}_B + \mathbf{V}_{EB}. \tag{7}$$

The result is

$$V_E = 27.6 \text{ ft/s}, \qquad \textit{Ans.}$$

as scaled from the velocity polygon.

Alternatively, the velocity of point E can be obtained from the velocity-difference equation,

$$\mathbf{V}_E = \mathbf{V}_C + \mathbf{V}_{EC}, \tag{8}$$

by a procedure identical to that used for Eq. (7). This solution would produce the triangle $O_V EC$ in the velocity polygon.

Suppose we wish to find $\mathbf{V}_E$ without the intermediate step of calculating ω_3. In this case, we write Eqs. (7) and (8) simultaneously, that is

$$\mathbf{V}_E = \overset{\surd\surd}{\mathbf{V}_B} + \overset{?\surd}{\mathbf{V}_{EB}} = \overset{\surd\surd}{\mathbf{V}_C} + \overset{?\surd}{\mathbf{V}_{EC}}. \tag{9}$$

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Drawing lines *EB* (perpendicular to $\mathbf{R}_{EB}$) and *EC* (perpendicular to $\mathbf{R}_{EC}$) in the velocity polygon, we find their intersection and thus solve Eq. (9).

Perhaps the easiest method of solving for $\mathbf{V}_E$, however, is to take advantage of the concept of the velocity image of link 3. Recognizing that the velocity-image points *B* and *C* have already been found, we can construct the triangle *BEC* in the velocity polygon, similar in shape to the triangle *BEC* in the scale diagram of link 3. This locates point *E* in the velocity polygon and, therefore, gives a solution for the velocity of point *E*.

The velocity of point *F* can also be found by any of the above methods using points *C*, *D*, and *F* of link 4. The result is

$$V_F = 31.8 \text{ ft/s.} \qquad \textit{Ans.}$$

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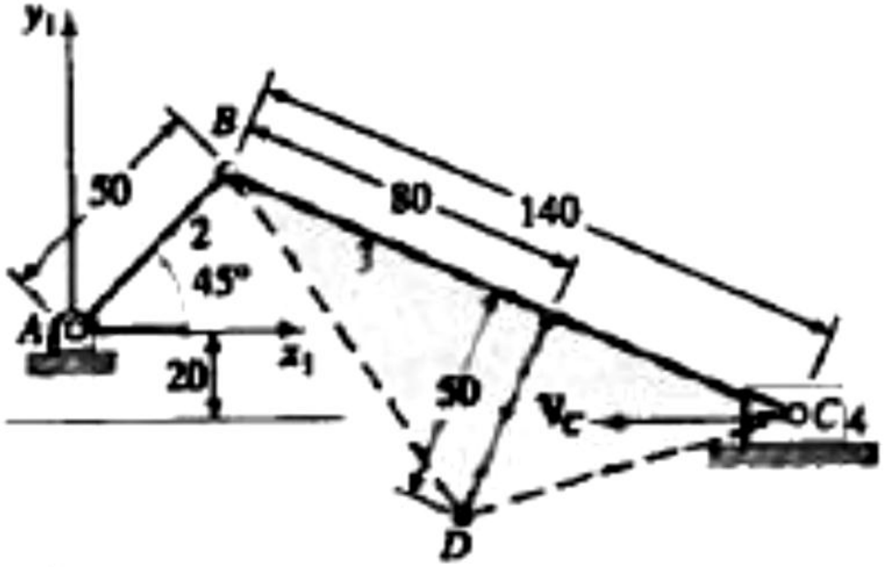
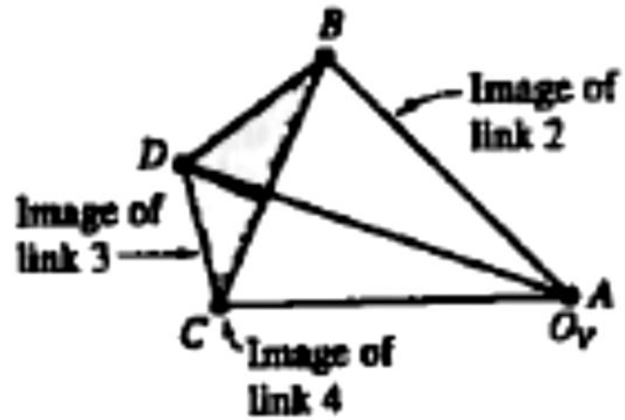
The offset slider-crank mechanism illustrated in Fig. 3.8*a* is driven by slider 4 at a velocity $\mathbf{V}_C = -10\hat{\mathbf{i}}$ m/s at the position shown. Determine the instantaneous velocity of point *D* and the angular velocities of links 2 and 3.



(*a*)



(*b*)

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SOLUTION

The velocity scale and pole O_V are chosen and $\mathbf{V}_C$ is drawn, thus locating point C as illustrated in Fig. 3.8*b*. Simultaneous equations are then written for the velocity of point B,

$$\mathbf{V}_B = \overset{\surd\surd}{\mathbf{V}_C} + \overset{?\surd}{\mathbf{V}_{BC}} = \cancel{\mathbf{V}}_A^0 + \overset{?\surd}{\mathbf{V}_{BA}}, \tag{10}$$

and solved for the location of point B in the velocity polygon.

Having found points B and C, we can construct the velocity image of link 3 as illustrated to locate point D; we then scale the line $O_V D$, which gives

$$V_D = 12.0 \text{ m/s} \qquad \textit{Ans.}$$

with the direction shown in the velocity polygon.

The angular velocities of links 2 and 3, respectively, are

$$\omega_2 = \frac{V_{BA}}{R_{BA}} = \frac{10.0 \text{ m/s}}{0.050 \text{ m}} = 200 \text{ rad/s ccw} \qquad \textit{Ans.} \tag{11}$$

$$\omega_3 = \frac{V_{BC}}{R_{BC}} = \frac{7.5 \text{ m/s}}{0.140 \text{ m}} = 53.6 \text{ rad/s cw.} \qquad \textit{Ans.} \tag{12}$$

In this example, Fig. 3.8*b*, the velocity image of each link is indicated in the polygon. Once the analysis of any problem is carried through completely, there will be a velocity

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image for each link of the mechanism. The following points are true, in general, and can be verified in the above two examples.

1. The velocity image of each link is a scale reproduction of the shape of the link in the velocity polygon.
2. The velocity image of each link is rotated 90° in the direction of the angular velocity of that link.
3. The letters identifying the vertices of each link are the same as those in the velocity polygon and progress around the velocity image in the same order and in the same angular direction as around the link.
4. The ratio of the size of the velocity image of a link to the size of the link itself is equal to the magnitude of the angular velocity of the link. In general, it is not the same for different links in the same mechanism.
5. The velocities of all points on a translating link are equal, and the angular velocity of the link is zero. Therefore, the velocity image of a link that is translating shrinks to a single point in the velocity polygon.
6. The point O_V in the velocity polygon is the image of all points with zero absolute velocity; it is the velocity image of the fixed link.
7. The absolute velocity of any point on any link is represented by the line from O_V to the image of the point. The velocity-difference vector between any two points, say P and Q, is represented by the line from image point P to image point Q.