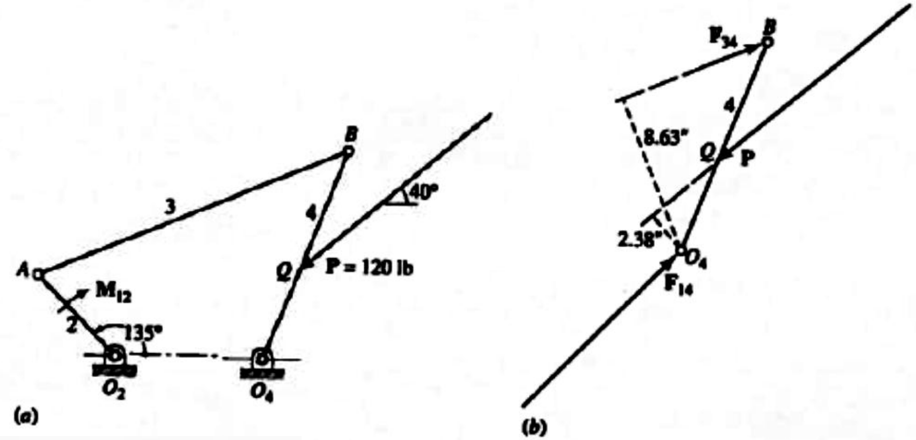
MECH 343 Theory of Machines I

Time: _ _ W _ _17:45 - 20:15

Lecture 11

Example

- In the four bar link crank 2 is riven by input torque M_{12} and external load P = 120lb @ 220° acts at point Q on link 4. For this position, find all the forces and reactions for the link to be in equilibrium



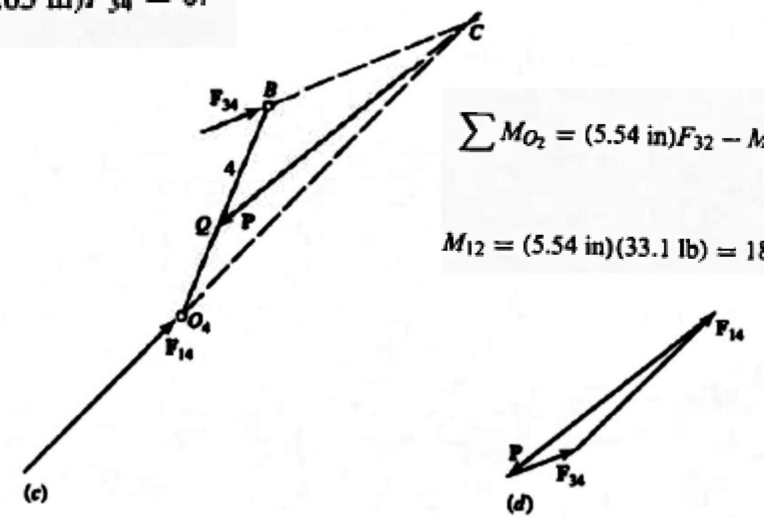
1. Draw FBD of link 3

$$\sum M_{O_4} = (2.38\text{ in})(120\text{ lb}) - (8.63\text{ in})F_{34} = 0.$$

2. Draw FBD of link 2

$$F_{34} = 33.1\text{ lb.}$$

3. Draw FBD of link 4, using P and direction of F_{34} (partially known from F_{43} in step 3) and moment along O_4, find a, b moment arms and then find F_{34} direction and mag



$$\sum M_{O_2} = (5.54\text{ in})F_{32} - M_{12} = 0.$$

$$M_{12} = (5.54\text{ in})(33.1\text{ lb}) = 183\text{ in}\cdot\text{lb},$$

4. Using F_{34}, and P, draw force polygon to find F_{14}, both direction and magnitude
5. Substitute this in steps 1 and 2 to find F_{43}, F_{23}, F_{32}, and F_{12}.
6. Using moment about O_2, find moment arm C and then find M_{12} in magnitude and direction

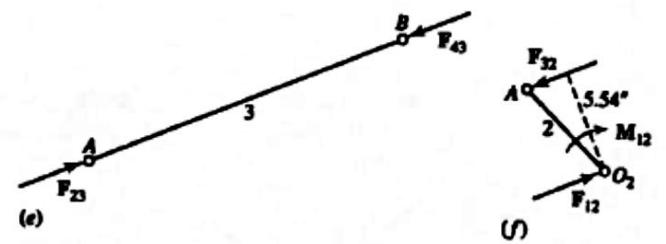


Figure 13.5 Graphic solution for Example 13.1: $R_{AO_2} = 6$ in, $R_{BA} = 18$ in, $R_{O_4O_2} = 8$ in, $R_{BO} = 12$ in, and $R_{QO_4} = 5$ in.

Example

EXAMPLE 13.1

The four-bar linkage of Fig. 13.5*a* has crank 2 driven by an input torque $\mathbf{M}_{12}$; an external load $\mathbf{P} = 120\ \text{lb}\angle 220°$ acts at point *Q* on link 4. For the position shown, find all the constraint forces and their reactions necessary for the linkage to be in equilibrium.

GRAPHIC SOLUTION

1. We draw the linkage and the given force or forces to scale, as illustrated in Fig. 13.5*a*. The size scale shown here is about 6.25 in/in, meaning that 1 in of the drawing represents 6.25 in of the linkage. The force scale shown is about 90 lb/in; that is, a vector shown 1 in long represents a force of 90 lb.
2. We begin drawings of the free-body diagrams of each of the moving links, as illustrated in Figs. 13.5*b*, 13.5*e*, and 13.5*f*. At this stage, all known information, such as the positions and the force **P**, should be drawn to scale. Unknown information, such as other applied forces on each link, are sketched lightly on the free-body diagrams, but will be redrawn to scale once they become known.
3. We ensure that all free-body diagrams are complete, that is, that all pertinent information is shown. For example, we must now either find the weights and centers of gravity of all links and the direction of gravity or, as done here, assume that the gravitational forces are small in comparison with the applied force **P** and that they can safely be ignored.
4. From the free-body diagram, we now observe that link 3 is a two-force member. Thus, according to the above discussion, we can now show the lines of action of $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ along the axis defined by points *A* and *B* in Fig. 13.5*e*. Because action and reaction forces must be equal and opposite, these also tell us the lines of action of $\mathbf{F}_{32}$ on link 2 in Fig. 13.5*f* and $\mathbf{F}_{34}$ on link 4 in Fig. 13.5*b*.

Example

5a. We proceed next to link 4, as illustrated in Fig. 13.5*b*, where neither the magnitude nor the direction of the frame reaction $\mathbf{F}_{14}$ is known as yet. One method for continuation is to measure the moment arms $\mathbf{P}$ and $\mathbf{F}_{34}$ about O_4, which are 2.38 in and 8.63 in, respectively. Then, taking counterclockwise moments as positive, Eqs. (13.8) gives

$$\sum M_{O_4} = (2.38 \text{ in})(120 \text{ lb}) - (8.63 \text{ in})F_{34} = 0.$$

The solution is $F_{34} = 33.1$ lb, where the fact that the result is positive confirms the sense shown for $\mathbf{F}_{34}$ in Fig. 13.5*b*.

5b. As an alternative approach to Step 5a, we could note that link 4 is a three-force member; therefore, when the lines of action of $\mathbf{P}$ and $\mathbf{F}_{34}$ are extended, they intersect at the point of concurrency C and define the line of action of $\mathbf{F}_{14}$, as illustrated in Fig. 13.5*c*.

6. No matter whether Step 5a or Step 5b is used, the force polygon illustrated in Fig. 13.5*d* can now be used to graphically solve the equation

$$\sum \mathbf{F} = \mathbf{P} + \mathbf{F}_{34} + \mathbf{F}_{14} = \mathbf{0},$$

giving $F_{34} = 33.1$ lb and $F_{14} = 89.0$ lb with directions as indicated in the force polygon. Note that this avoids the need for the moment equation of Step 5a; it is satisfied by the concurrency at point C.

7. From the free-body diagram of link 3, Fig. 13.5*e*, and the nature of action and reaction forces, we note that $\mathbf{F}_{23} = -\mathbf{F}_{43} = \mathbf{F}_{34}$. For a two-force member, it is important to note whether the link is in tension or in compression. In the case of tension, the link would fail as a result of yielding; however, in the case of compression, the link could fail by either yielding or buckling. Hence, buckling becomes an important topic in machine design and is addressed in Sections 13.14–13.17.

8. Because $\mathbf{F}_{32} = -\mathbf{F}_{23}$, this known force can be illustrated on the free-body diagram of link 2 in Fig. 13.5*f*. Because link 2 is subjected to two forces and a torque, $\mathbf{F}_{12}$ must be equal and opposite to $\mathbf{F}_{32}$. Alternatively, from the summation of forces on the free-body diagram of link 2 in Fig. 13.5*f*, we see that $\mathbf{F}_{12} = -\mathbf{F}_{32}$. When the moment arm of $\mathbf{F}_{32}$ about point O_2 is measured, it is

Example

found to be 5.54 in. Therefore, taking counterclockwise moments as positive, we obtain

$$\sum M_{O_2} = (5.54\ \text{in})F_{32} - M_{12} = 0,$$

which yields

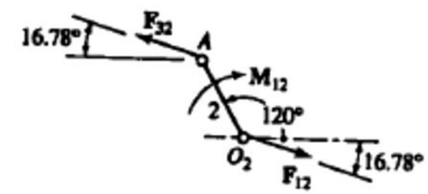
$$M_{12} = (5.54\ \text{in})(33.1\ \text{lb}) = 183\ \text{in} \cdot \text{lb}, \qquad \textit{Ans.}$$

where the positive sign confirms the clockwise sense illustrated in Fig. 13.5*f*.

Example

- In the slider crank, external load P = 100lb horizontal acts at point Q on link 4. block B is 8X3in and the pin is centrally located. Q is 1in above the center line. Determine the torque M_{12} to hold the mechanism in static equilibrium. Ignore gravity forces

1. Draw FBD of link 3 and since 2 force, show F_{23} and F_{43} (direction partially known) these tell us the LOA F_{34} on link 4 and F_{32} on link 2.
2. Draw FBD of link 2 with LOAs for F_{32} and F_{12} (direction partially known from step 1
3. Draw FBD of link 4, magnitude of F_{14} unknown but ⊥ to ground. Link 4 is 3 force member so, F_{34} and P LOAs should intersect and the point will be where F_{14} will be
4. Draw force polygon, with P and LOA of F_{34}, and F_{14}, to get their direction and magnitude
5. In steps 1 use F_{34} = - F_{43} = F_{23}, and in step 2 use F_{23} = - F_{32} = F_{12}.
6. Using moment about O_2, find moment arm h (4") and then find M_{12} in magnitude and direction



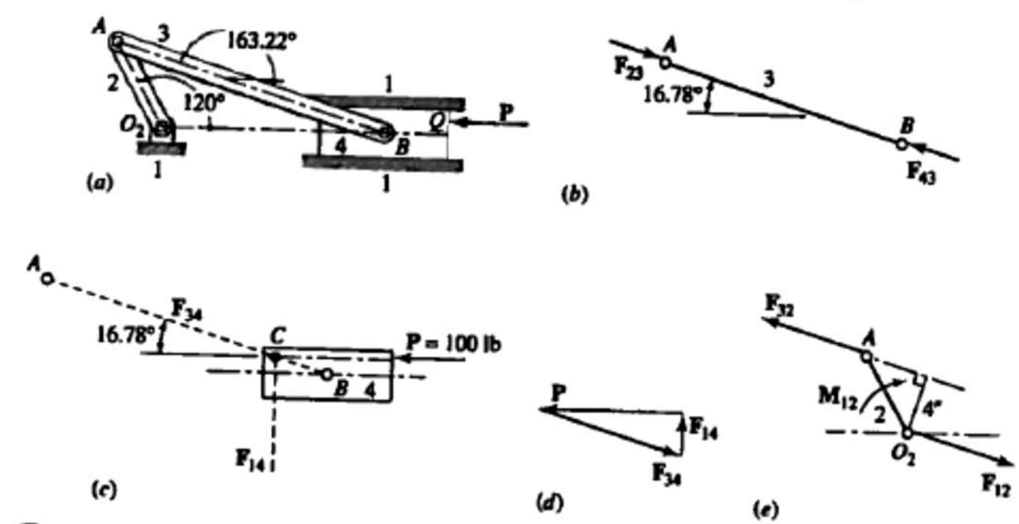


Figure 13.8 Graphic solution for Example 13.2: R_{AO_2} = 6 in and R_{BA} = 18 in.

Example

EXAMPLE 13.2

The slider–crank mechanism of Fig. 13.8*a* has an external load $P = 100$ lb acting horizontally at point Q on link 4. Block 4 is 8 in wide by 3 in. high with the pin centrally located. Point Q is 1 in above the centerline. Determine the torque $\mathbf{M}_{12}$ that must be applied to link 2 to hold the mechanism in static equilibrium at the position illustrated in Fig. 13.8*a*. Assume that gravitational and friction forces are small in comparison with the applied force **P** and can be ignored.

1. We draw the mechanism and the given force to scale, as illustrated in Fig. 13.8*a*. The size scale used here is 15 in/in and the force scale is 140 lb/in.
2. We draw the free-body diagrams of each moving link, as illustrated in Figs. 13.8*b*, 13.8*c*, and 13.8*e*.
3. We observe that link 3 is a two-force member. Thus, we can show the lines of action of $\mathbf{F}_{23}$ and $\mathbf{F}_{43}$ along the axis defined by points A and B in Fig. 13.8*b*. These also tell us the lines of action of $\mathbf{F}_{34}$ on link 4 in Fig. 13.8*c* and $\mathbf{F}_{32}$ on link 2 in Fig. 13.8*e*.
4. We proceed to link 4 where neither the magnitude nor the location of the frame reaction force $\mathbf{F}_{14}$ is known as yet. However, the direction of this force must be perpendicular to the ground because friction is neglected. Link 4 is a three-force member; therefore, when the lines of action of **P** and $\mathbf{F}_{34}$ are extended, they intersect at the point of concurrency C and define the line of action of $\mathbf{F}_{14}$, as illustrated in Fig. 13.8*c*.

Example

5. We use the force polygon illustrated in Fig. 13.8*d* to graphically solve the equation

$$\sum \mathbf{F} = \mathbf{P} + \mathbf{F}_{34} + \mathbf{F}_{14} = \mathbf{0},$$

giving

$$F_{34} = 105 \text{ lb} \quad \text{and} \quad F_{14} = 31 \text{ lb}.$$

6. From the free-body diagram of link 3, Fig. 13.8*b*, and the nature of action and reaction forces, we note that $\mathbf{F}_{23} = -\mathbf{F}_{43} = \mathbf{F}_{34}$. Link 3 is in compression; therefore, the problem of buckling should be investigated (see Sections 13.14–13.17).
7. From the free-body diagram of link 2, Fig. 13.8*e*, and the nature of action and reaction forces, we note that $\mathbf{F}_{32} = -\mathbf{F}_{23}$. From the summation of forces on the free-body diagram of link 2, we find $\mathbf{F}_{12} = -\mathbf{F}_{32}$. When the moment arm of $\mathbf{F}_{32}$ about point O_2 is measured, it is determined to be 4 in. Therefore, taking counterclockwise moments as positive, we obtain

$$\sum M_{O_2} = (4 \text{ in})F_{32} - M_{12} = 0,$$

which yields

$$M_{12} = (4 \text{ in})(105 \text{ lb}) = 420 \text{ in} \cdot \text{lb}, \qquad \textit{Ans.}$$

where the positive sign confirms the clockwise sense illustrated in Fig. 13.8*e*.