MECH 344/M
Machine Element Design

Time: M _ _ _ _14:45 - 17:30

Lecture 11

Contents of today's lecture

15 Spur Gears

SAMPLE PROBLEM 15.1D Meshing Spur Gear and Pinion

Two parallel shafts with 4-in. center distance are to be connected by 6-pitch, 20° spur gears providing a velocity ratio of −3.0. (a) Determine the pitch diameters and numbers of teeth in the pinion and gear. (b) Determine whether there will be interference when standard full-depth teeth are used. (c) Determine the contact ratio. (See Figure 15.16.)

SOLUTION

Known: Spur gears of known pitch size, pressure angle, and center distance mesh to provide a known velocity ratio.

Find:

a. Determine pitch diameters (d_p, d_g) and the numbers of teeth (N_p, N_g).

b. Determine the possibility of interference with standard full-depth teeth.

c. Calculate the contact ratio (CR).

Schematic and Given Data:

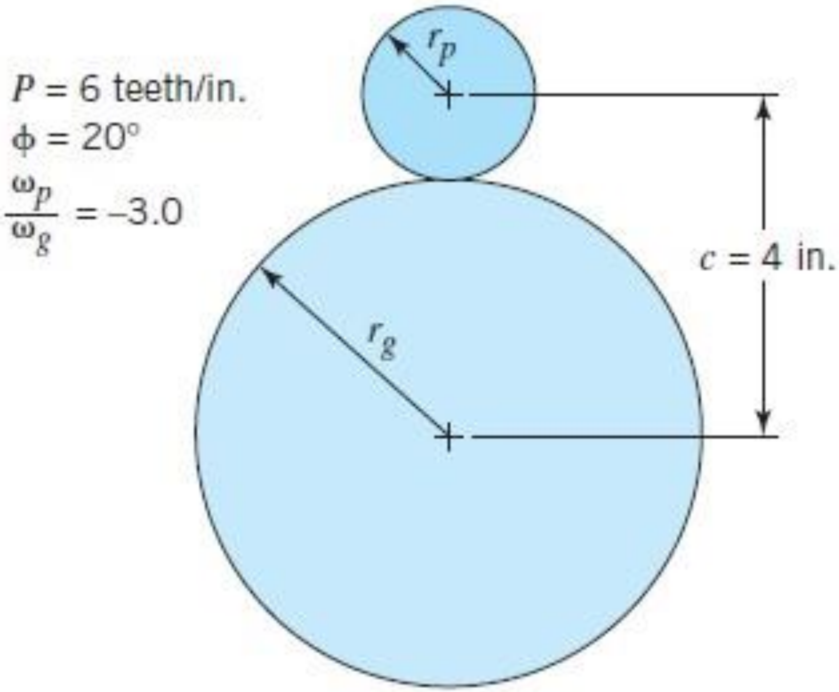


FIGURE 15.16

Spur gears for Sample Problem 15.1D.

Decisions and Assumptions:

1. If interference results from the use of standard full-depth gear teeth, unequal addenda gears will be selected.
2. The gear teeth will have standard involute tooth profiles.
3. The two gears will be located at their theoretical center distance, $c = (d_p + d_g)/2$ where $d_p = N_p/P$, $d_g = N_g/P$; that is, the gears will mesh at their pitch circles.

Design Analysis:

1. We have $r_p + r_g = c = 4$ in.; $r_g/r_p = -$velocity ratio $= 3$; hence, $r_p = 1$ in., $r_g = 3$ in., or $d_p = 2$ in., $d_g = 6$ in.
2. The term "6-pitch gears" means that $P = 6$ teeth per inch of pitch diameter; hence, $N_p = 12$, $N_g = 36$.
3. In order to use Eq. 15.8 to check for interference, we first determine the base circle radii of pinion and gear. From Eq. 15.11, $r_{\mathrm{bp}} = 1$ in.(cos 20°), and $r_{\mathrm{bg}} = 3$ in.(cos 20°). Substitution in Eq. 15.8 gives $r_{a(\max)} = 1.660$ in. for the pinion and 3.133 in. for the gear.

$$d_b = d\cos\phi, \qquad r_b = r\cos\phi, \qquad \text{and} \quad p_b = p\cos\phi \qquad \textbf{(15.11)}$$

$$r_{a(\max)} = \sqrt{r_b^2 + c^2\sin^2\phi}$$

4. The limiting outer gear radius is equivalent to an addendum of only 0.133 in., whereas a standard full-depth tooth has an addendum of $1/P = 0.167$ in. Clearly, the use of standard teeth would cause interference.
5. Let us use unequal addenda gears (nonstandard), with somewhat arbitrarily chosen addenda of $a_g = 0.060$ in. for the gear and $a_p = 0.290$ in. for the pinion. (The reasoning is to select maximum addenda for greatest contact ratio, while at the same time limiting the gear addendum to stay safely away from interference, and limiting the pinion addendum to maintain adequate width of top land. The latter is shown as t_0 in Figure 15.9, and its minimum acceptable value is sometimes taken as $0.25/P$.)

Name	Symbol	Units	Relationship	Example
Number of teeth	N_p N_g			$N_p = 12$ $N_g = 36$
Nominal diametral pitch	P	per in.		$P = 6$
Nominal pressure angle	ϕ	degrees		$\phi = 20°$
Gear ratio	η		$\eta = N_g/N_p$	$\eta = 3$
Base width	b	in.	$9/P < b < 14/P$	$1.5 < b < 2.33$
Nominal pitch radius	r_p r_g	in.	$r_p = N_p/(2P)$ $r_g = N_g/(2P)$	$r_p = 1$ $r_g = 3$
Nominal base radius	r_{bp} r_{bg}	in.	$r_{bp} = r_p \cos \phi$ $r_{bg} = r_g \cos \phi$	$r_{bp} = 0.939$ $r_{bg} = 2.82$
Nominal center distance	c	in.	$c = \dfrac{(N_p + N_g)}{(2P)}$	$c = 4$
Maximum addendum radius to avoid interference	r_{ap}^{max} r_{ag}^{max}	in.	$r_{ap}^{max} = \sqrt{r_{bp}^2 + c^2 \sin^2 \varphi}$ $r_{ag}^{max} = \sqrt{r_{bg}^2 + c^2 \sin^2 \varphi}$	$r_{ap}^{max} = 1.66$ $r_{ag}^{max} = 3.13$

Name	Symbol	Units	Relationship	Example
Standard addendum radius	r_{ap} r_{ag}	in.	$r_{ap} = (N_p + 2)/(2P)$ $r_{ag} = (N_g + 2)/(2P)$	$r_{ap} = 1.17$ $r_{ag} = 3.17$ *(Interference)*
Addendum radius	r_{ap} r_{ag}	in.	*(Non-standard)*	$r_{ap} = 1.29$ $r_{ag} = 3.06$
Standard dedendum radius	r_{dp} r_{dg}	in.	$r_{dp} = (N_p - 2.5)/(2P)$ $r_{dg} = (N_g - 2.5)/(2P)$ *(Standard)*	$r_{dp} = 0.792$ $r_{dg} = 2.792$
Nominal module	m	mm	$m = 25.4/P$	$m = 4.23$
Nominal circular pitch	p	in.	$p = \pi/P$	$p = 0.523$
Nominal tooth thickness	t	in.	$t = \pi/(2P)$	$t = 0.262$
Nominal contact ratio	CR		$\Delta_p = \sqrt{r_{ap}^2 - r_{bp}^2}$ $\Delta_g = \sqrt{r_{ag}^2 - r_{bg}^2}$ $CR = \dfrac{\Delta_p + \Delta_p - c \sin\phi}{p \cos\phi}$	$CR = 1.43$

6. Substitution in Eq. 15.11 gives $p_b = (\pi/6)\cos 20° = 0.492$ in. Substitution in Eq. 15.9 [with $r_{ap} = 1.290$ in., $r_{bp} = 1$ in.$(\cos 20°)$, $r_{ag} = 3.060$ in., $r_{bg} = 3$ in.$(\cos 20°)$] gives CR $= 1.43$, which should be a suitable value.

Comments:

1. If after the gears are mounted, the center distance is found to be slightly greater than the theoretical (calculated) center distance of 4.0 in., this would mean that the calculated diameters, d_p and d_g, are smaller than the actual gear and pinion pitch diameters and that the backlash is greater than initially calculated.
2. Had we wished to use standard tooth proportions in solving this sample problem, we could have (a) increased the diametral pitch (thereby giving more teeth on the pinion—and this outweighs the influence of giving more teeth to the gear) or (b) increased the pressure angle to 25° (which would be more than enough to eliminate interference).
3. This problem may also be solved using the worksheet in Appendix J.

15.4 Gear Force Analysis

- In Figures - line ab is normal to the contacting tooth surfaces, and that (neglecting sliding friction) it was the line of action of the forces between mating teeth.

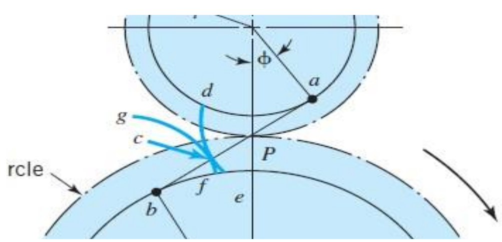
- The force between mating teeth can be resolved at the pitch point (P) into two components.

1. Tangential component F_t, which, when multiplied by the pitch line velocity, accounts for the power transmitted.

2. Radial component F_r, which does no work but tends to push the gears apart.

- From figure the relationship between F_t & F_r is

$$F_r = F_t \tan\phi \tag{15.12}$$



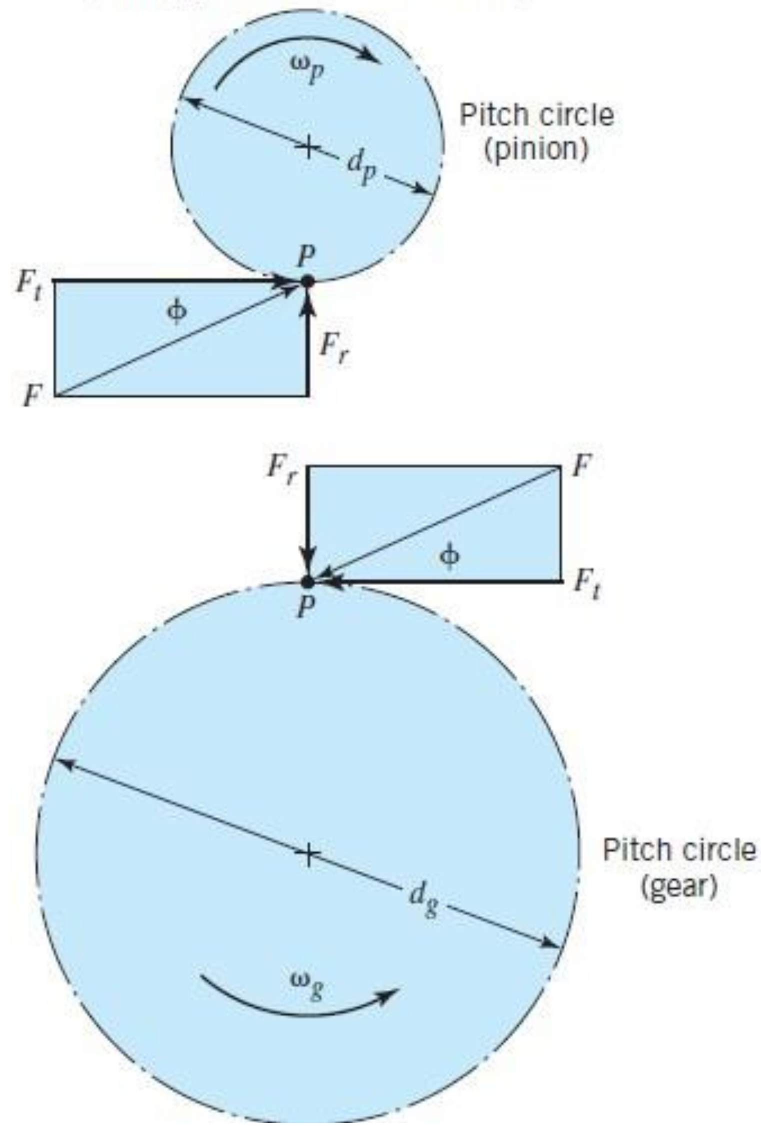


FIGURE 15.17
Gear-tooth force F, shown resolved at pitch point. The driving pinion and driven gear are shown separately.

15.4 Gear Force Analysis

- To analyze the relationships between the gear force components and the associated shaft power and rotating speed, the gear pitch line velocity V, in feet per minute, is

$$V = \pi dn/12 \qquad (15.13)$$

- where d is the pitch diameter in inches of the gear rotating n rpm. The transmitted power in horsepower (hp) is

$$\dot{W} = F_t V/33{,}000 \qquad (15.14)$$

- where F_t is in pounds and V in feet per minute.
- In SI units $V = \pi dn/60{,}000$ (15.13a)
- where d is the pitch diameter in mm of the gear rotating n rpm and V is in m/s. The transmitted power in watts (W) is

$$\dot{W} = F_t V \qquad (15.14a)$$

- Where F_t is in newtons

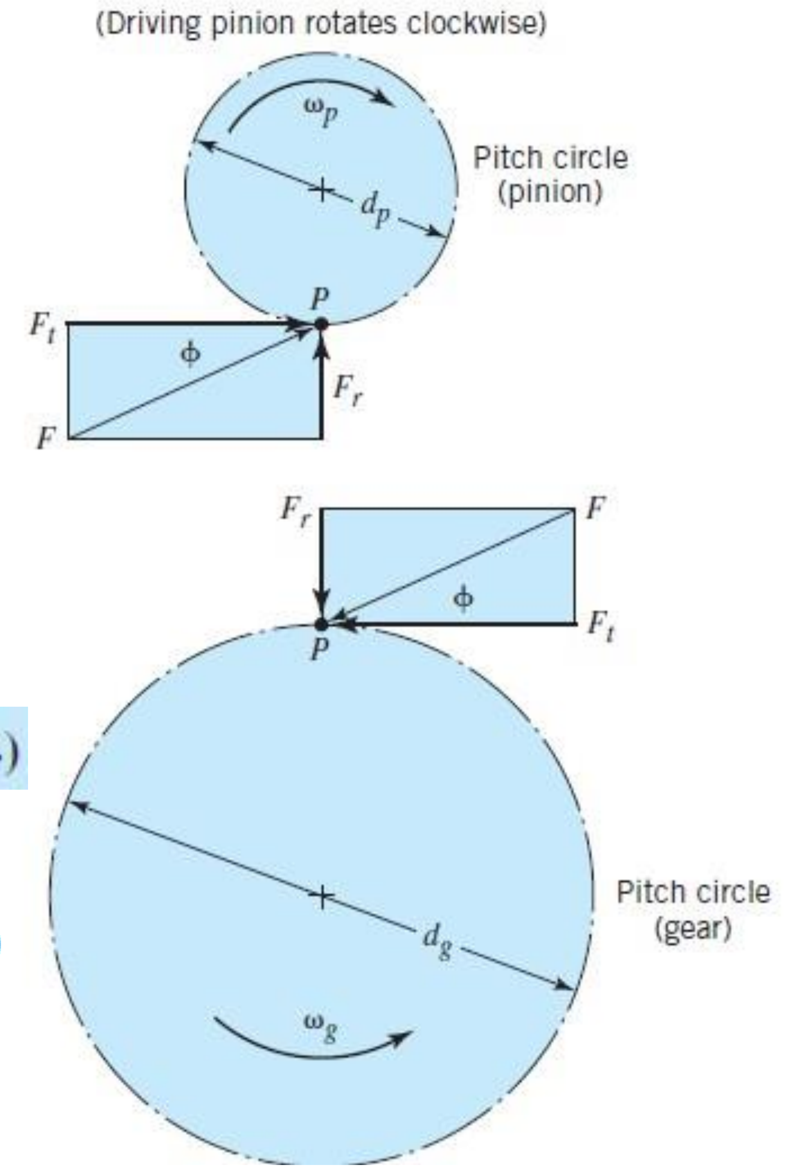


FIGURE 15.17
Gear-tooth force F, shown resolved at pitch point. The driving pinion and driven gear are shown separately.

SAMPLE PROBLEM 15.2 Forces on Spur Gears

Figure 15.18*a* shows three gears of $P = 3$, $\phi = 20°$. Gear *a* is the driving, or input, pinion. It rotates counterclockwise at 600 rpm and transmits 25 hp to idler gear *b*. Output gear *c* is attached to a shaft that drives a machine. Nothing is attached to the idler shaft, and friction losses in the bearings and gears can be neglected. Determine the resultant load applied by the idler to its shaft.

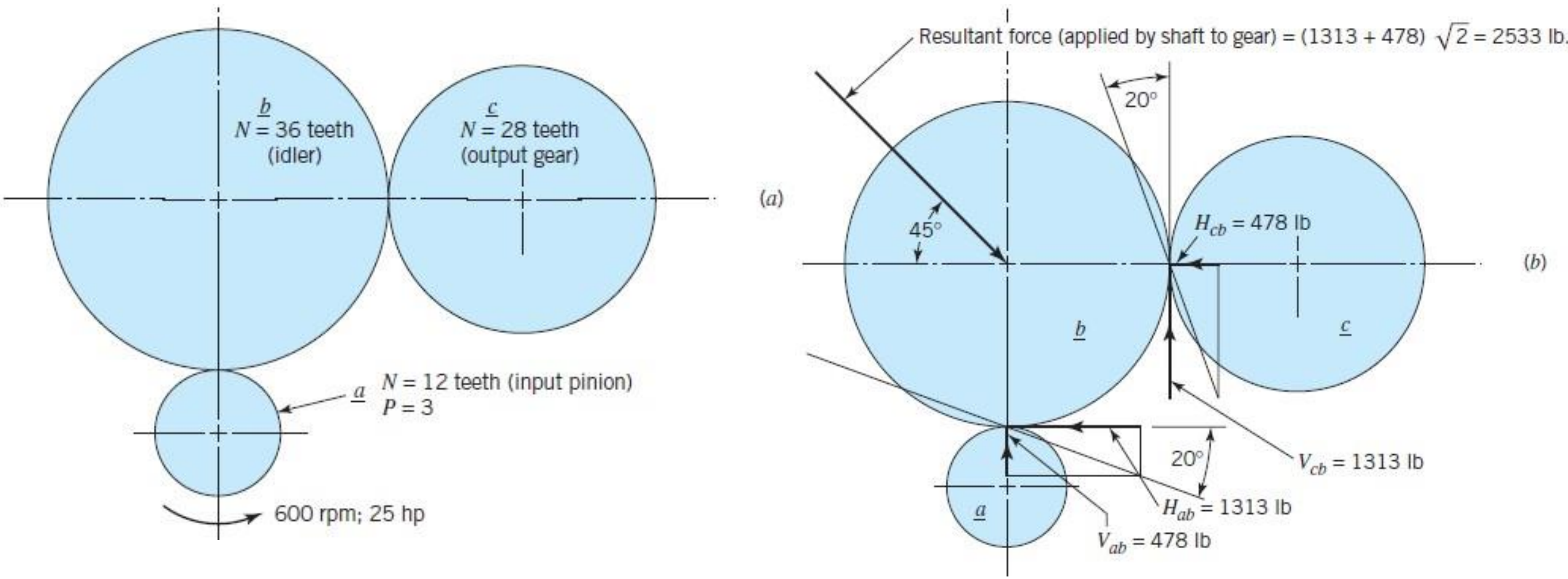


FIGURE 15.18
Gear forces in Sample Problem 15.2. (*a*) Gear layout. (*b*) Forces acting on idler *b*.

SOLUTION

Known: Three spur gears of specified diametral pitch, numbers of teeth, and pressure angle mesh to transmit 25 hp from input gear to output gear through an idler gear. The input gear rotation speed and direction are given.

Find: Determine the resultant load of the idler gear on its shaft.

Schematic and Given Data: See Figure 15.18.

Assumptions:

1. The idler gear and shaft serve the function of transmitting power from the input gear to the output gear. No idler shaft torque is applied to the idler gear.
2. Friction losses in the bearings and gears are negligible.
3. The gears mesh at the pitch circles.
4. The gear teeth have standard involute tooth profiles.
5. The shafts for gears a, b, and c are parallel.

Analysis:

1. Applying Eq. 15.3 to gear a gives

$$d_a = N_a/P = (12 \text{ teeth})/(3 \text{ teeth per inch}) = 4 \text{ in.}$$

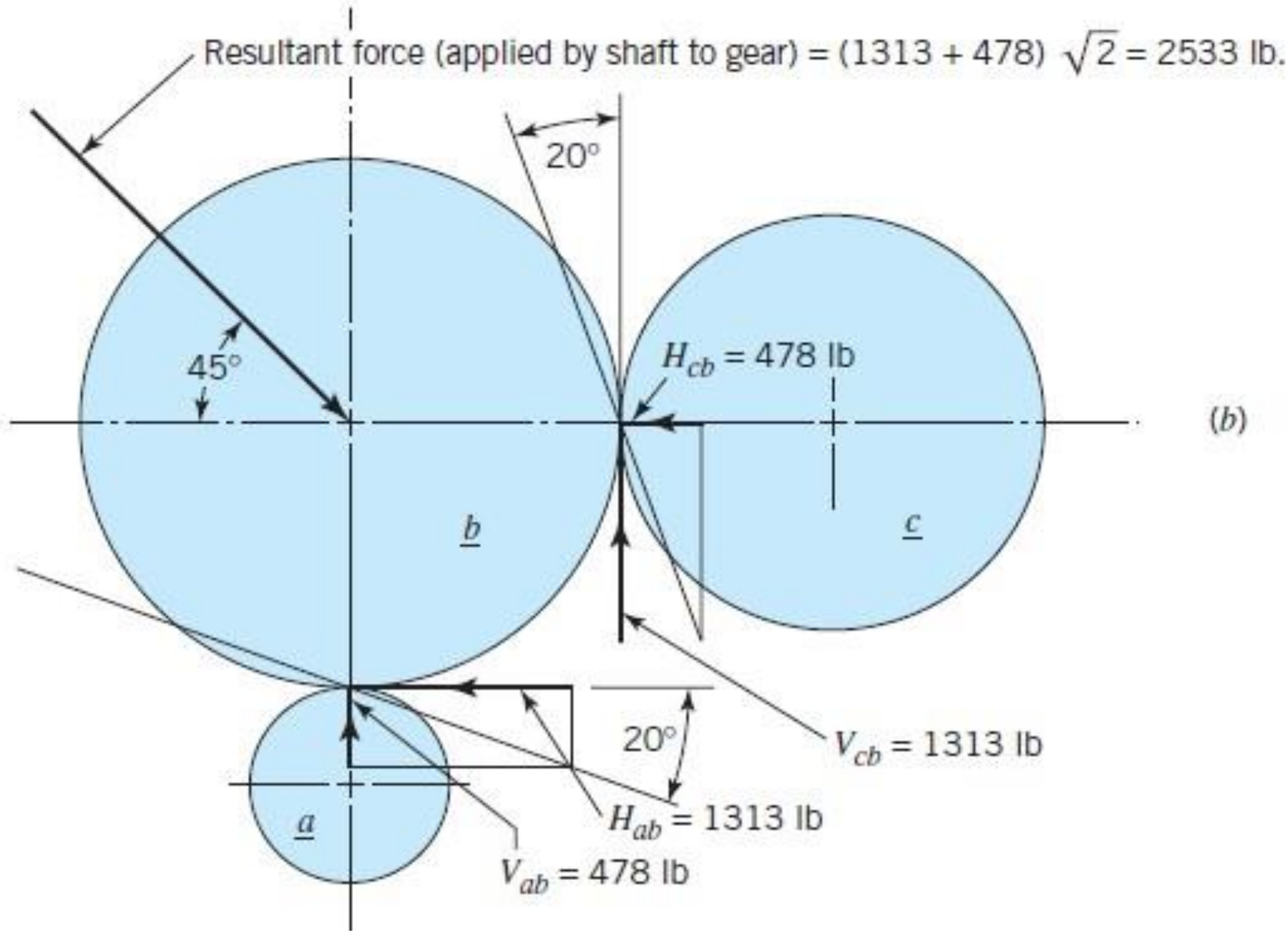
2. All three gears have the same pitch line velocity. Applying Eq. 15.13 to gear a, we have

$$V = \frac{\pi d_a n_a}{12} = \frac{\pi(4 \text{ in.})(600 \text{ rpm})}{12} = 628.28 \text{ ft/min}$$

3. Applying Eq. 15.14 to gear a and solving for F_t gives

$$F_t = \frac{33{,}000(25 \text{ hp})}{628.28 \text{ fpm}} = 1313 \text{ lb}$$

This is the horizontal force of gear b applied *to* gear a, directed to the right. Figure 15.18b shows the equal and opposite horizontal force of a applied to b, labeled H_{ab}, and acting to the left.



Resultant force (applied by shaft to gear) = (1313 + 478) $\sqrt{2}$ = 2533 lb.
20°
45°
H_{cb} = 478 lb
(b)
b
c
20°
V_{cb} = 1313 lb
H_{ab} = 1313 lb
a
V_{ab} = 478 lb

4. From Eq. 15.12, the corresponding radial gear-tooth force is $F_r = V_{ab} = (1313)(\tan 20°) = 478$ lb.
5. Forces H_{cb} and V_{cb} are shown in proper direction in Figure 15.18*b*. (Remember, these are forces applied *by c to b.*) Since the shaft supporting idler *b* carries no torque, equilibrium of moments about its axis of rotation requires that $V_{cb} = 1313$ lb. From Eq. 15.12, $H_{cb} = (1313)(\tan 20°)$, or 478 lb.
6. Total gear-tooth forces acting on *b* are $1313 + 478 = 1791$ lb both vertically and horizontally, for a vector sum of $1791\sqrt{2} = 2533$ lb acting at 45°. This is the resultant load applied *by* the idler *to* its shaft.

Comment: The equal and opposite force applied *by* the shaft *to* the idler gear is shown in Figure 15.18*b*, where the idler is shown as a free body in equilibrium.

15.5 Gear-Tooth Strength

- After gear geometry and force analysis, looking into how much power a gear pair will transmit without tooth failure.
- Figure shows photoelastic pattern of stresses in gear-tooth. Highest stresses exist where the lines are closest together.

1. the point of contact with the mating gear, where force F is acting,
2. in the fillet at the base of the tooth.

- Designing gears will deal with bending fatigue at the base of the tooth and with surface durability
- As will be seen, the load capacity and failure mode of a pair of gears are affected by their rotating speed.

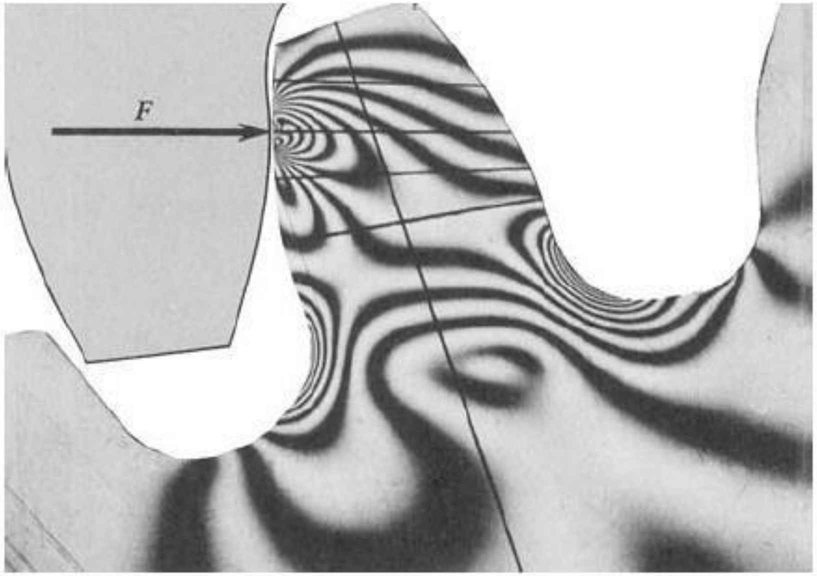


FIGURE 15.19 Photoelastic pattern of stresses in a spur gear tooth. (From T. J. Dolan and E. L. Broghammer, "A Study of Stresses in Gear Tooth Fillets," Proc. 14th Eastern Photoelasticity Conf., PE December 1941.)

15.6 Basic Analysis of Gear-Tooth-Bending Stress (Lewis Equation)

- The first analysis of gear-tooth stresses was presented by Lewis in 1892 - still used for gear-tooth bending stress analysis.
- Figure shows a gear tooth loaded as a cantilever beam, with resultant force F applied to the tip. Mr. Lewis made the following simplifying assumptions.
- The full load is applied to the tip of a single tooth. This is obviously the most severe condition and is appropriate for gears of "ordinary" accuracy.
- For high precision gears, however, the full load is never applied to a single tooth tip.
- With a CR > 1, each new pair of teeth comes into contact while the previous pair is still engaged.

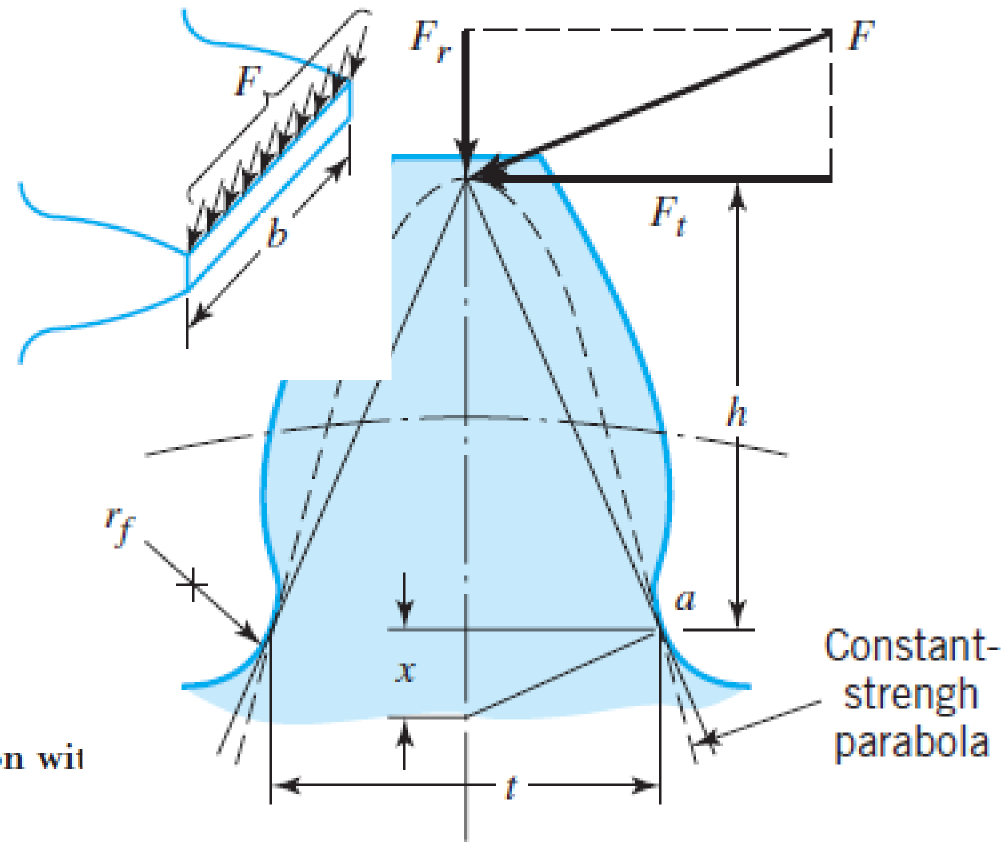
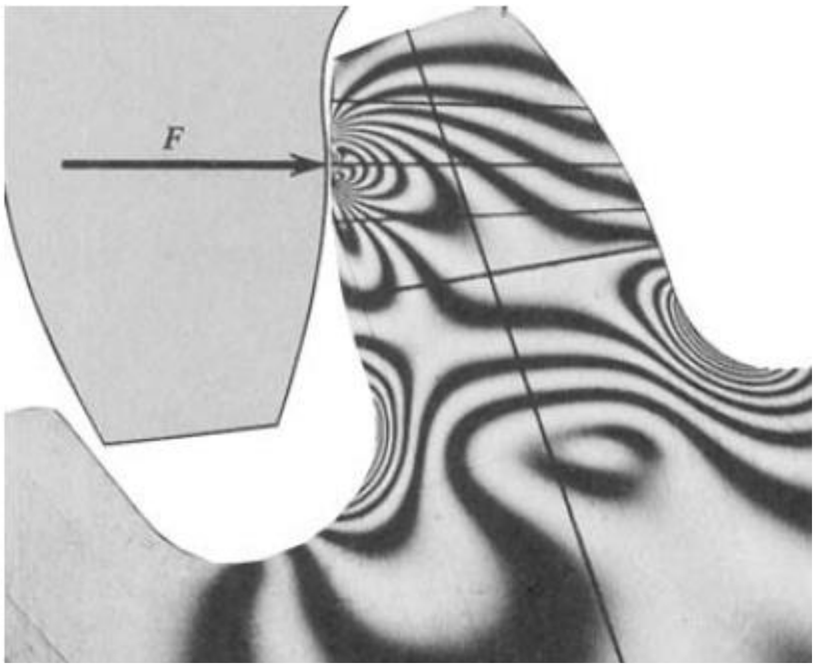


FIGURE 15.20

Bending stresses in a spur gear tooth (comparison wit constant-stress parabola).

15.6 Basic Analysis of Gear-Tooth-Bending Stress (Lewis Equation)

- After the contact point moves down some distance from the tip, the previous teeth go out of engagement and the new pair carries the full load (unless, CR>2).
- This is the situation depicted in stress Figure. Thus, with precision gears, the tooth should be regarded as carrying only part of the load at its tip, and the full load at a point on the tooth face where the bending moment arm is shorter.



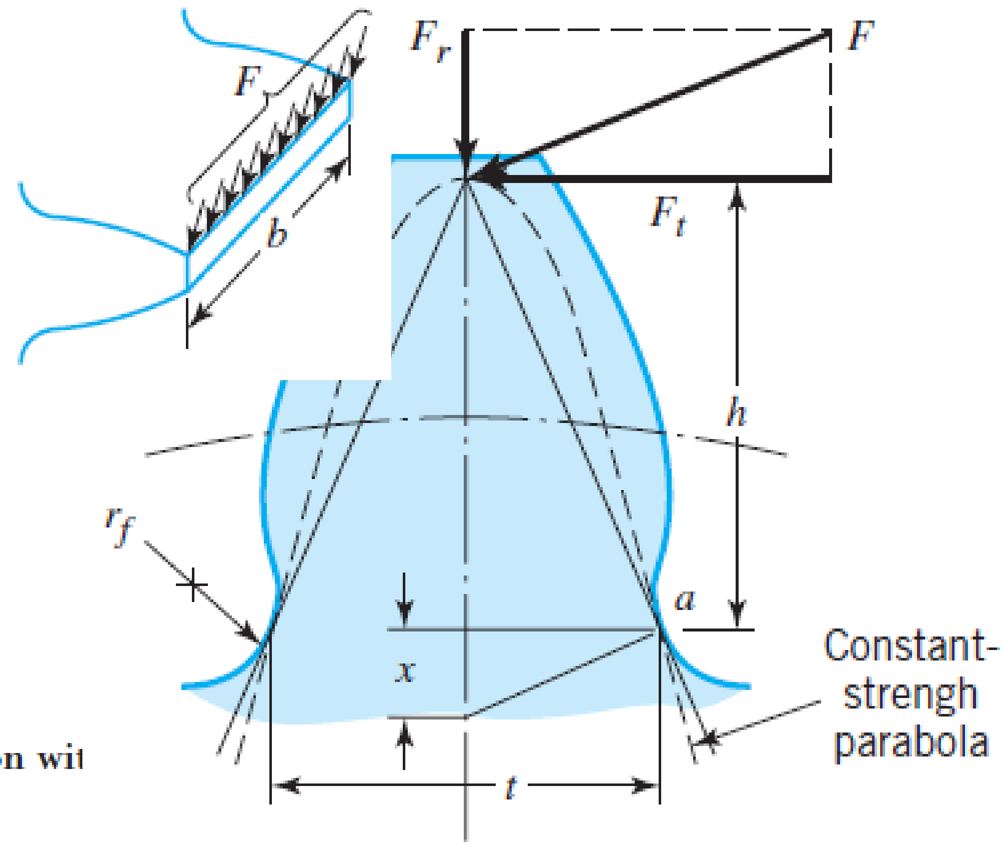
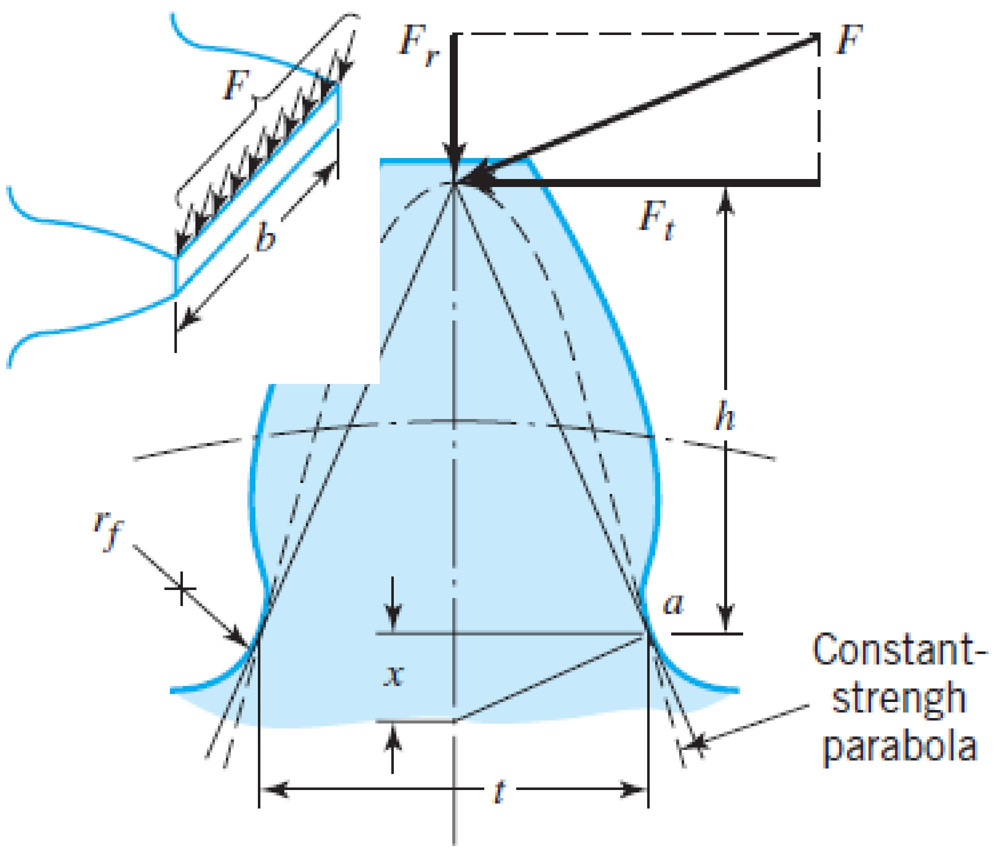


FIGURE 15.20
Bending stresses in a spur gear tooth (comparison wit constant-stress parabola).

15.6 Basic Analysis of Gear-Tooth-Bending Stress (Lewis Equation)

- The radial component, F_r , is negligible. This is a conservative assumption, as F_r produces a compressive stress that subtracts from the bending tension at point a of Figure. (The fact that it adds to the bending compression in the opposite fillet is unimportant because fatigue failures always start on the tensile side.)
- The load is distributed uniformly across the full face width. This is a non-conservative assumption and can be instrumental in gear failures involving wide teeth and misaligned or deflecting shafts.
- Forces which are due to tooth sliding friction are negligible.
- Stress concentration in the tooth fillet is negligible. K factors were unknown in Mr. Lewis's time but are now known to be important.



15.6 Basic Analysis of Gear-Tooth-Bending Stress (Lewis Equation)

- Proceeding with the development of the Lewis equation, from Figure, the gear tooth is everywhere stronger than the inscribed constant strength parabola, except for the section at a where the parabola and tooth profile are tangent. At point a

$$\sigma = \frac{Mc}{I} = \frac{6F_t h}{bt^2} \qquad \sigma = M/Z \text{ and } Z = bt^2/6 \qquad \text{(c)}$$

- For similar triangles

$$\frac{t/2}{x} = \frac{h}{t/2}, \qquad \text{or} \qquad \frac{t^2}{h} = 4x \qquad \text{(d)}$$

- Substituting d into c gives

$$\sigma = \frac{6F_t}{4bx} \qquad \text{(e)}$$

- if lewis factor y is 2x/3p then

$$\sigma = \frac{F_t}{bpy}$$

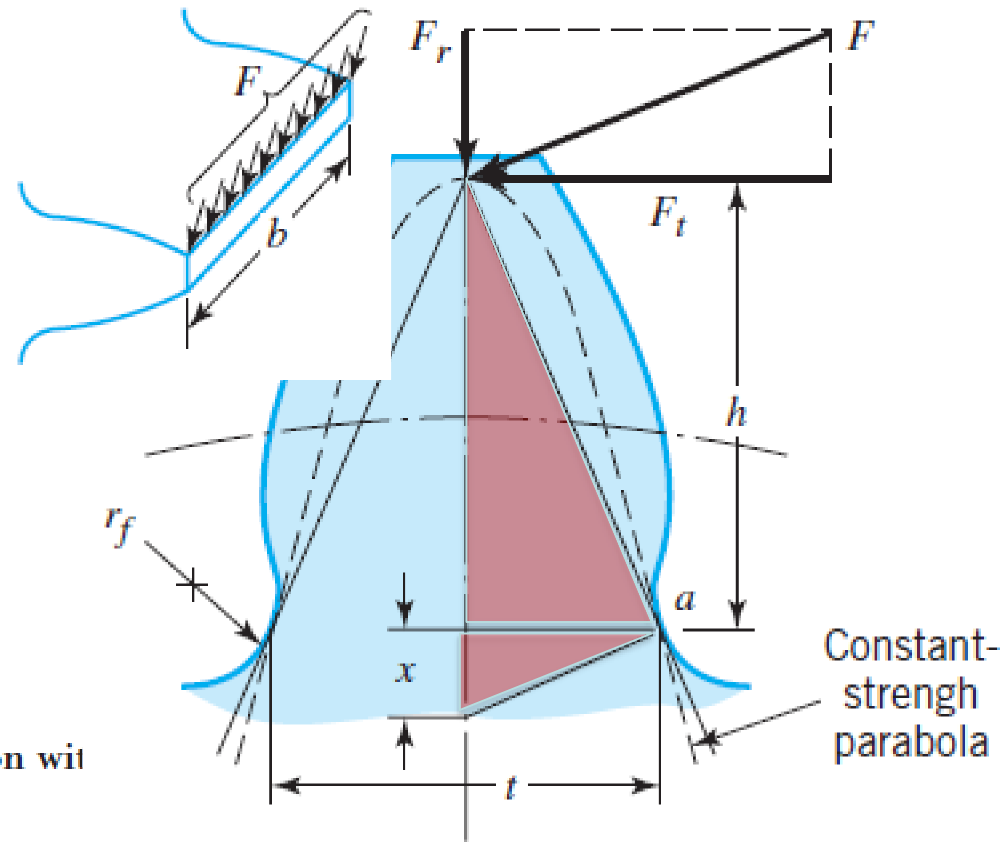
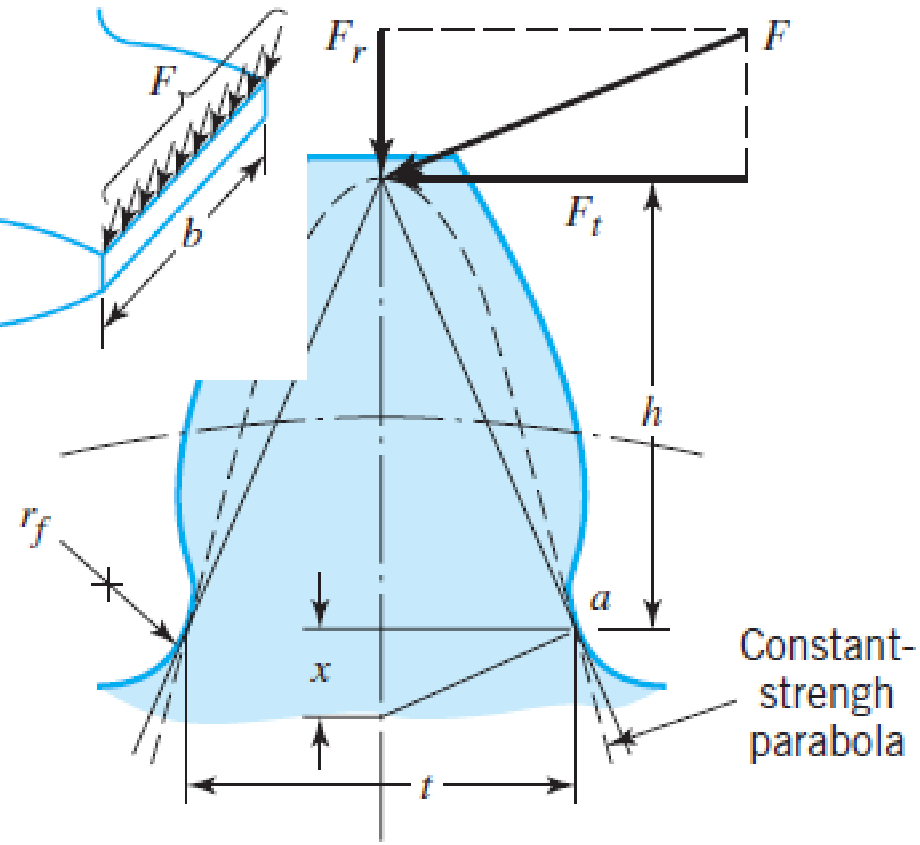


FIGURE 15.20
Bending stresses in a spur gear tooth (comparison wit constant-stress parabola).

15.6 Basic Analysis of Gear-Tooth-Bending Stress (Lewis Equation)

$$\sigma = \frac{F_t P}{bY} \quad \text{or} \quad \sigma = \frac{F_t}{mbY} \qquad (15.16)$$

- Gears are made to standard diametral pitch P, which is p = π/P and then y = Y/ π
- where Y is the Lewis form factor based on DP, Por Module, m. Both Y & y are functions of tooth shape (not size) and therefore vary with the number of teeth in the gear.
- Values of Y for standard gear systems are given in Figure 15.21.
- For nonstandard gears, the factor can be obtained by graphical layout of the tooth or by digital computation.
- Lewis equation indicates that tooth-bending stresses vary (1) directly with load F_t, (2) inversely with tooth width b, (3) inversely with tooth size p, 1/P, or m and (4) inversely with tooth shape factor Y or y.



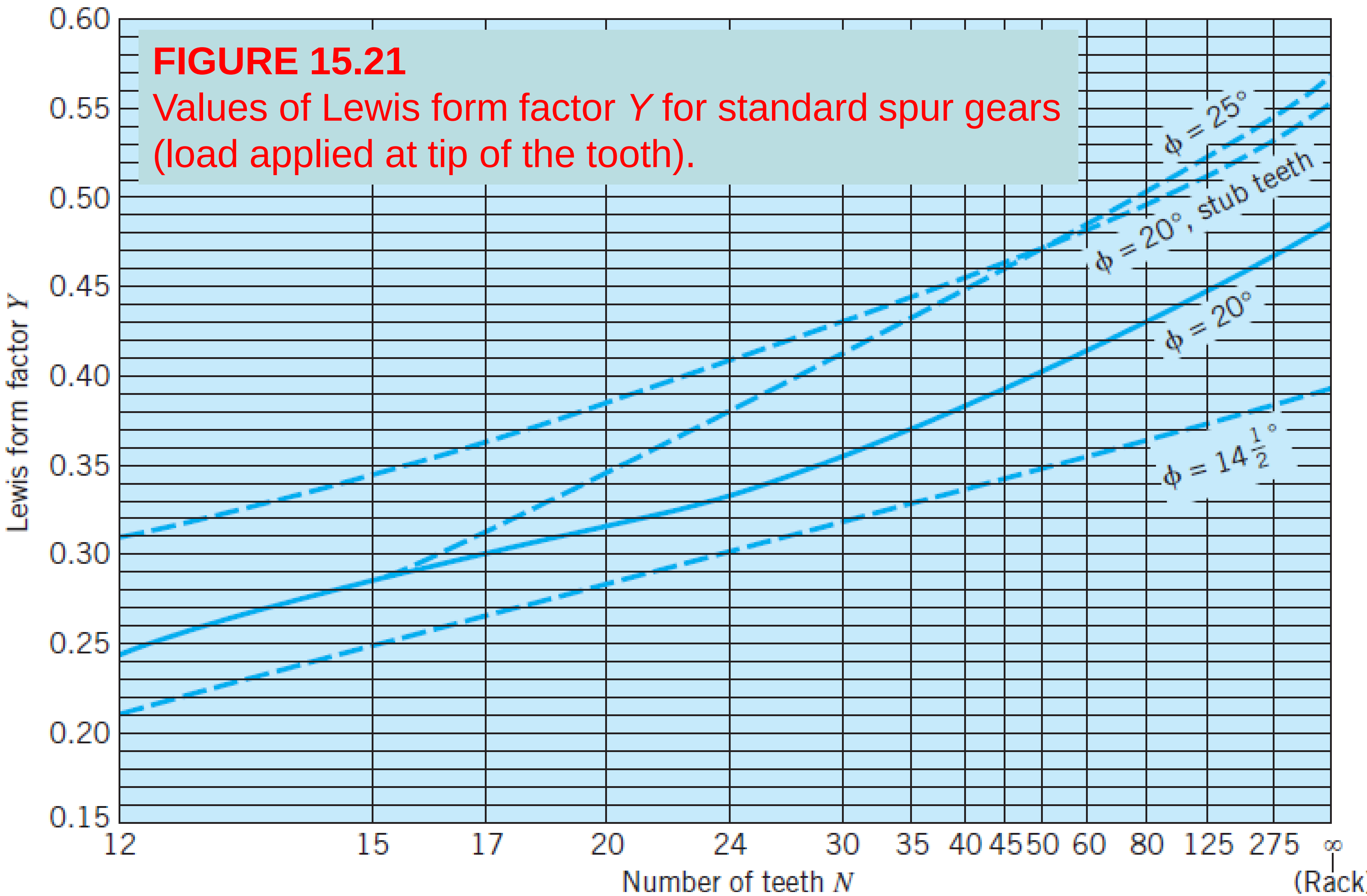


FIGURE 15.21
Values of Lewis form factor Y for standard spur gears (load applied at tip of the tooth).

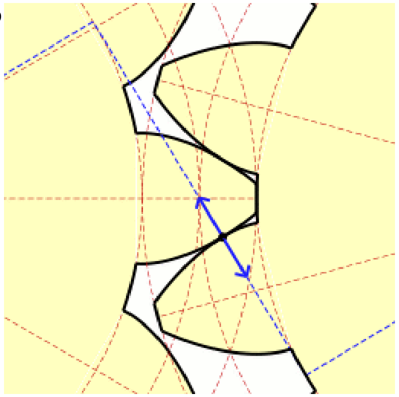
15.7 Refined Analysis of Gear-Tooth-Bending Strength: Basic Concepts

- In addition to 4 factors in Lewis equation, modern gear design procedures take into account several additional factors that influence geartooth- bending stresses.

1. Pitch line velocity. The > the linear velocity of the gear teeth, the > the impact of successive teeth as they come into contact. These impacts happen because the tooth profiles can never be made with absolute perfection; and even if they were, deflections are inevitable, for operating loads cause a slight impact as each new pair of teeth come into engagement.

2. Manufacturing accuracy. An important factor influencing impact loading. Furthermore, manufacturing accuracy is the factor determining whether or not teeth share the load when two or more pairs of teeth are theoretically in contact.

3. Contact ratio. For precision gears 1<CR<2, the transmitted load is divided among two pairs of teeth whenever a new tooth comes into contact at its tip. As the contact point moves down the new tooth, the meshing teeth ahead go out of contact. So two loading conditions : (a) carrying part of the load (assumed to be 1/2) at the tooth tip and (b) carrying the full load at the point of highest single-tooth contact. For gears of (2 < CR < 3), we should consider a three-way division of load at tooth tip contact, and a two-way division at the highest point of double-tooth contact.



4. Stress concentration at the base of the tooth
5. Degree of shock loading involved in the application
6. Accuracy and rigidity of mounting
7. Moment of inertia of the gears and attached rotating members. Slight tooth inaccuracies tend to cause momentary angular accelerations and decelerations of the rotating members. If the rotating inertias are small, the members easily accelerate without imposing high momentary tooth loads. With large inertias, the rotating members tend strongly to resist acceleration, thereby causing large momentary tooth loads. Significant torsional elasticity between the gear teeth and the major sources of inertia may tend to isolate the gear teeth from the harmful inertial effect.

- The problem of gear-tooth-bending fatigue requires an evaluation of (a) the fluctuating stresses in the tooth fillet and (b) the fatigue strength of the material at this same highly localized location.
- So far only stresses have been considered; now consider the strength aspect.

15.7 Refined Analysis of Gear-Tooth-Bending Strength: Basic Concepts

- The important strength property is usually the bending fatigue strength, as represented by the endurance limit. $S_n = S'_n C_L C_G C_S C_T C_R$
- which, for steel members, is usually $S = (0.5S_u)C_L C_G C_S C_T C_R$
- Gear teeth generally loaded in one direction. Idler teeth and planet pinions are loaded in both directions.
- ideally make a σ_m-σ_a diagram for each case, figure shows the basis for the common generalization:
- For infinite life, peak stresses must be < the reversed bending endurance limit for an idler gear, but peak stresses can be 40% > for a driving or driven gear.
- For a reliability of other than 50%, gear-bending strength calculations are commonly based on the assumption that the tooth-bending fatigue strength has a normal distribution with one standard deviation being about 8 % of the nominal endurance limit.
- If gear teeth operate at elevated temperatures, the fatigue properties of the material at the temperatures involved must be used.

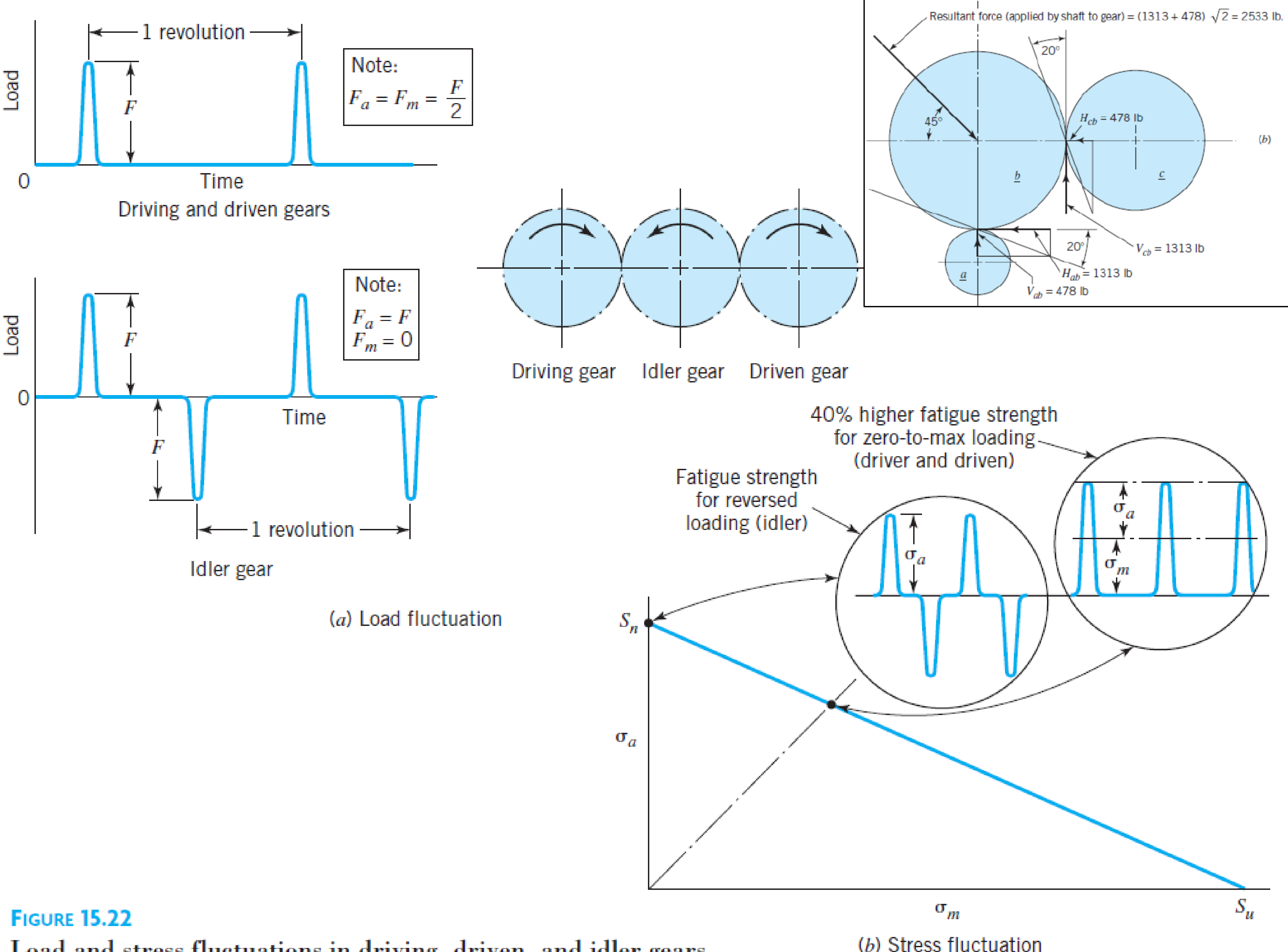


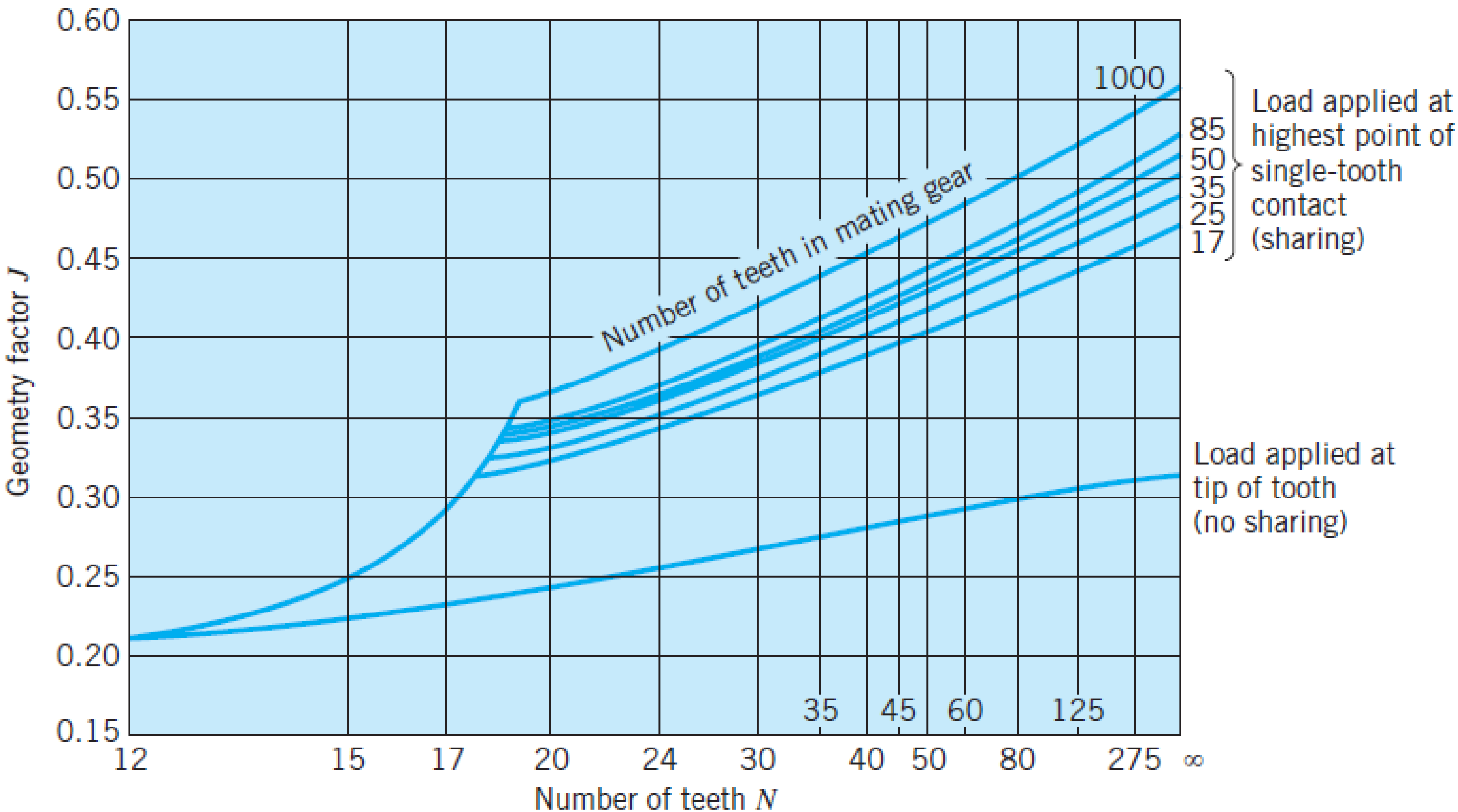
FIGURE 15.22
Load and stress fluctuations in driving, driven, and idler gears.

15.8 Refined Analysis of Gear-Tooth-Bending Strength: Recommended Procedure

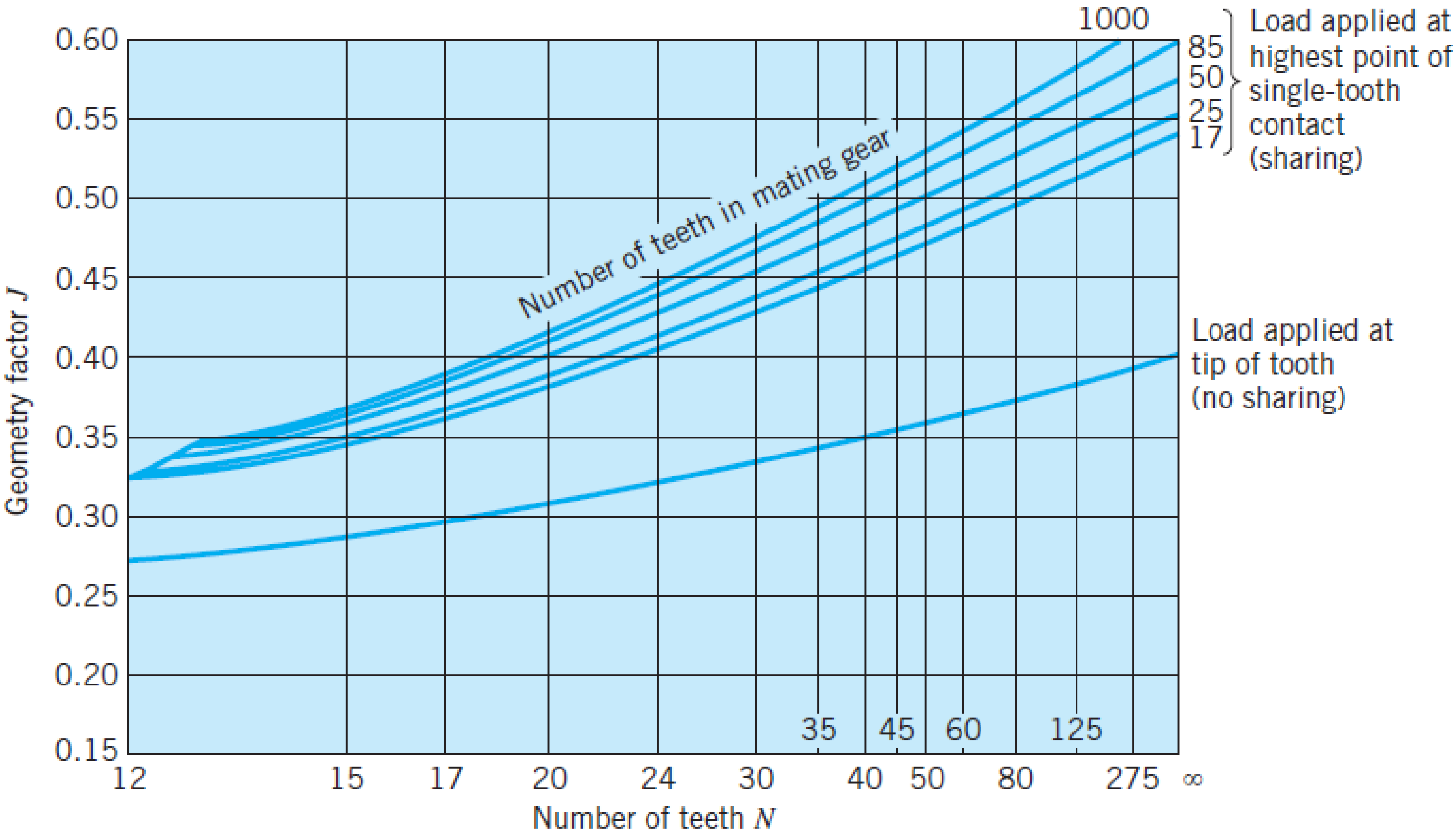
- Gear design and analysis should be done with latest AGMA stds and the procedure is representation of common practice
- In the absence of more specific information, the factors affecting gear-tooth bending stress can be taken into account by adding factors to the Lewis equation

$$\sigma = \frac{F_t P}{bJ} K_v K_o K_m \qquad \textbf{(15.17)}$$

- where
- J = spur gear geometry factor from Figure 15.23. This factor includes the Lewis form factor Y and also a stress concentration factor based on a tooth fillet radius of 0.35/P. Values are given for no load sharing (nonprecision gears) and also for load sharing (high-precision gears). In load sharing the J factor depends on the number of teeth in the mating gear, (which determines CR), which determines the highest point of single-tooth contact.



(*a*) 20° full-depth teeth



(*b*) 25° full-depth teeth

FIGURE 15.23

Geometry factor J for standard spur gears (based on tooth fillet radius of $0.35/P$). (From AGMA Information Sheet 225.01; also see AGMA 908-B89.)

15.8 Refined Analysis of Gear-Tooth-Bending Strength: Recommended Procedure

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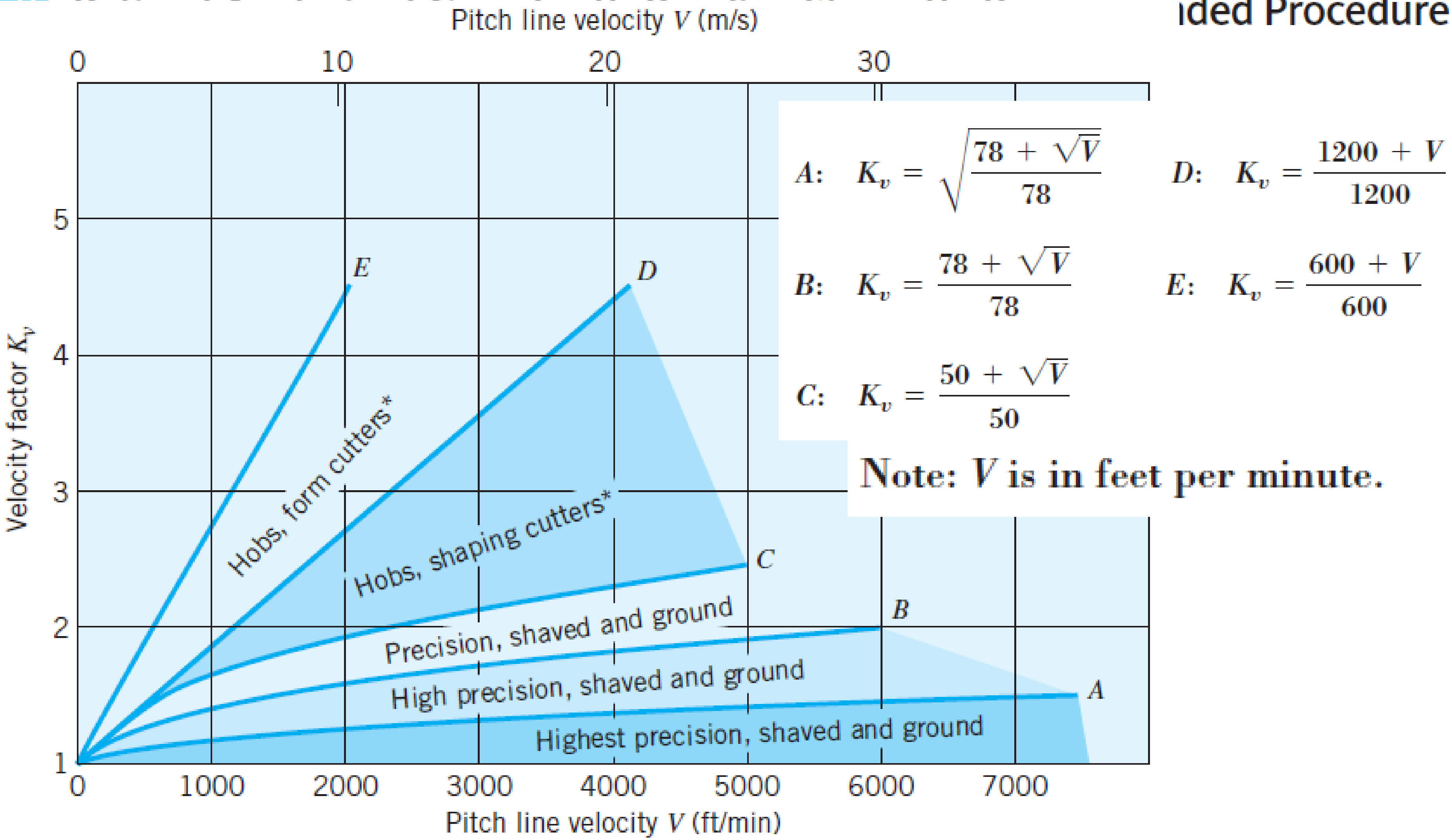
- K_v = velocity or dynamic factor, indicating the severity of impact as successive pairs of teeth engage. This is a function of pitch line velocity and manufacturing accuracy. Figure 15.24 gives guidelines pertaining to representative gear manufacturing processes. For reference, curve A is for AGMA quality control number (class), Q_v = 9, curve B for Q_v = 6 and curve C for Q_v = 4.



* Limited to about 350 Bhn

FIGURE 15.24

Velocity factor K_v. (Note: This figure, in a very rough way, is intended to account for the effects of tooth spacing and profile errors, tooth stiffness, and the velocity, inertia, and stiffness of the rotating parts.)

15.8 Refined Analysis of Gear-Tooth-Bending Strength: Recommended Procedure

- K_o = overload factor, reflecting the degree of nonuniformity of driving and load torques. Table 15.1 have long been used as a basis for rough estimates.
- Km = mounting factor, reflecting the accuracy of mating gear alignment. Table 15.2 is used as a basis for rough estimates.

TABLE 15.1 Overload Correction Factor K_o

	Driven Machinery		
Source of Power	**Uniform**	**Moderate Shock**	**Heavy Shock**
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

15.8 Refined Analysis of Gear-Tooth-Bending Strength: Recommended Procedure

- K_o = overload factor, reflecting the degree of nonuniformity of driving and load torques. Table 15.1 have long been used as a basis for rough estimates.
- Km = mounting factor, reflecting the accuracy of mating gear alignment. Table 15.2 is used as a basis for rough estimates.

TABLE 15.2 Mounting Correction Factor K_m

	Face Width (in.)			
Characteristics of Support	**0 to 2**	**6**	**9**	**16 up**
Accurate mountings, small bearing clearances, minimum deflection, precision gears	1.3	1.4	1.5	1.8
Less rigid mountings, less accurate gears, contact across the full face	1.6	1.7	1.8	2.2
Accuracy and mounting such that less than full-face contact exists		Over 2.2		

15.8 Refined Analysis of Gear-Tooth-Bending Strength: Recommended Procedure

- The effective fatigue stress from Eq. 15.17 must be compared with the corresponding fatigue strength. For infinite life the appropriate endurance limit is

$$S_n = S'_n C_L C_G C_S k_r k_t k_{ms} \quad (15.18)$$

where

S'_n = standard R. R. Moore endurance limit

C_L = load factor = 1.0 for bending loads

C_G = gradient factor = 1.0 for $P > 5$, and 0.85 for $P \leq 5$

C_S = surface factor from Figure 8.13. Be sure that this pertains to the surface *in the fillet*, where a fatigue crack would likely start. (In the absence of specific information, assume this to be equivalent to a machined surface.)

k_r = reliability factor, C_R, determined from Figure 6.19. For convenience, values corresponding to an endurance limit standard deviation of 8 percent are given in Table 15.3

TABLE 15.3 Reliability Correction Factor k_r, from Figure 6.19 with Assumed Standard Deviation of 8 Percent

Reliability (%)	50	90	99	99.9	99.99	99.999
Factor k_r	1.000	0.897	0.814	0.753	0.702	0.659

15.8 Refined Analysis of Gear-Tooth-Bending Strength: Recommended Procedure

- k_t = temperature factor, C_T . For steel gears use $k_t = 1.0$ if the T< 160°F. If not,

$$k_t = \frac{620}{460 + T} \quad (\text{for } T > 160°\text{F}) \tag{15.19}$$

- k_{ms} = mean stress factor. use 1.0 for idler gears (subjected to two-way bending) and 1.4 for input and output gears (one-way bending).
- The safety factor for bending fatigue can be taken as the ratio of fatigue strength (Eq. 15.18) to fatigue stress (Eq. 15.17).
- Its numerical value should be chosen in accordance with Section 6.12. Since factors K_o, K_m, and k_r have been taken into account separately, the "safety factor" need not be large
- Typically, a safety factor of 1.5 might be selected, together with a reliability factor corresponding to 99.9 percent reliability.

$$\sigma = \frac{F_t P}{bJ} K_v K_o K_m \tag{15.17}$$

$$S_n = S'_n C_L C_G C_S k_r k_t k_{ms} \tag{15.18}$$

SAMPLE PROBLEM 15.3 Gear Horsepower Capacity for Tooth-Bending Fatigue Failure

Figure 15.25 shows a specific application of a pair of spur gears, each with face width $b = 1.25$ in. Estimate the maximum horsepower that the gears can transmit continuously with only a 1 percent chance of encountering tooth-bending fatigue failure.

SOLUTION

Known: A steel pinion gear with specified hardness, diametral pitch, number of teeth, face width, rotational speed, and 20° full-depth teeth drives a steel gear of 290 Bhn hardness at 860 rpm with only a 1 percent chance of tooth-bending fatigue failure.

Find: Determine the maximum horsepower that the gears can transmit continuously.

Schematic and Given Data:

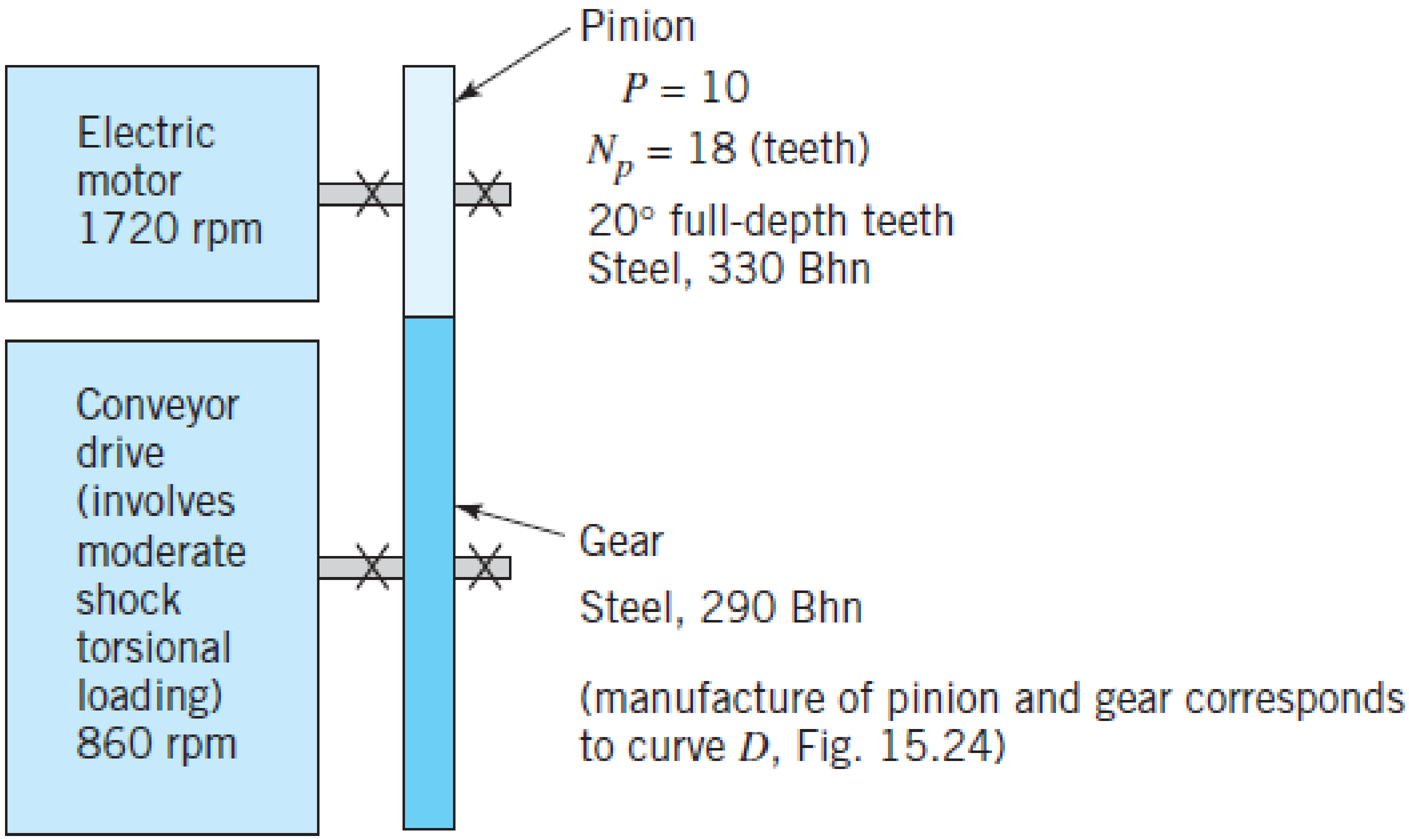


FIGURE 15.25

Data for Sample Problem 15.3.

Assumptions:

1. The gear teeth have a machined surface.
2. The gear-tooth fillet area temperature is less than 160°F.
3. The gears rotate in one direction (and hence experience one-way bending).
4. The transmitted load is applied at the tip of the gear tooth (no load sharing).
5. The manufacturing quality of the pinion and gear corresponds to curve D, Figure 15.24.
6. The output conveyor drive involves moderate torsional shock.
7. The characteristics of support include less rigid mountings, less accurate gears, and contact across the full face.
8. The gears fail solely by tooth-bending fatigue (no surface fatigue failure occurs).
9. No factor of safety will be necessary. Accounted for separately are the overload factor K_o, mounting factor K_m, and the reliability factor k_r.
10. The gears are mounted to mesh along the pitch circles.
11. The gear teeth are of equal face width.
12. The material endurance limit can be approximated by 250 (Bhn) psi.
13. The modified Lewis equation assumptions are reasonable. The J-factor data is accurate. The data plots and tables for obtaining C_a, C_s, and k_t can be relied on. The velocity factor K_v, the overload factor K_o, and the mounting factor K_m from available data are reasonably accurate.
14. The gear material is homogeneous, isotropic, and completely elastic.
15. Thermal and residual stresses are negligible.

Analysis:

1. The bending endurance strength is estimated from Eq. 15.18 as

$$S_n = S'_n C_L C_G C_S k_r k_t k_{ms}$$

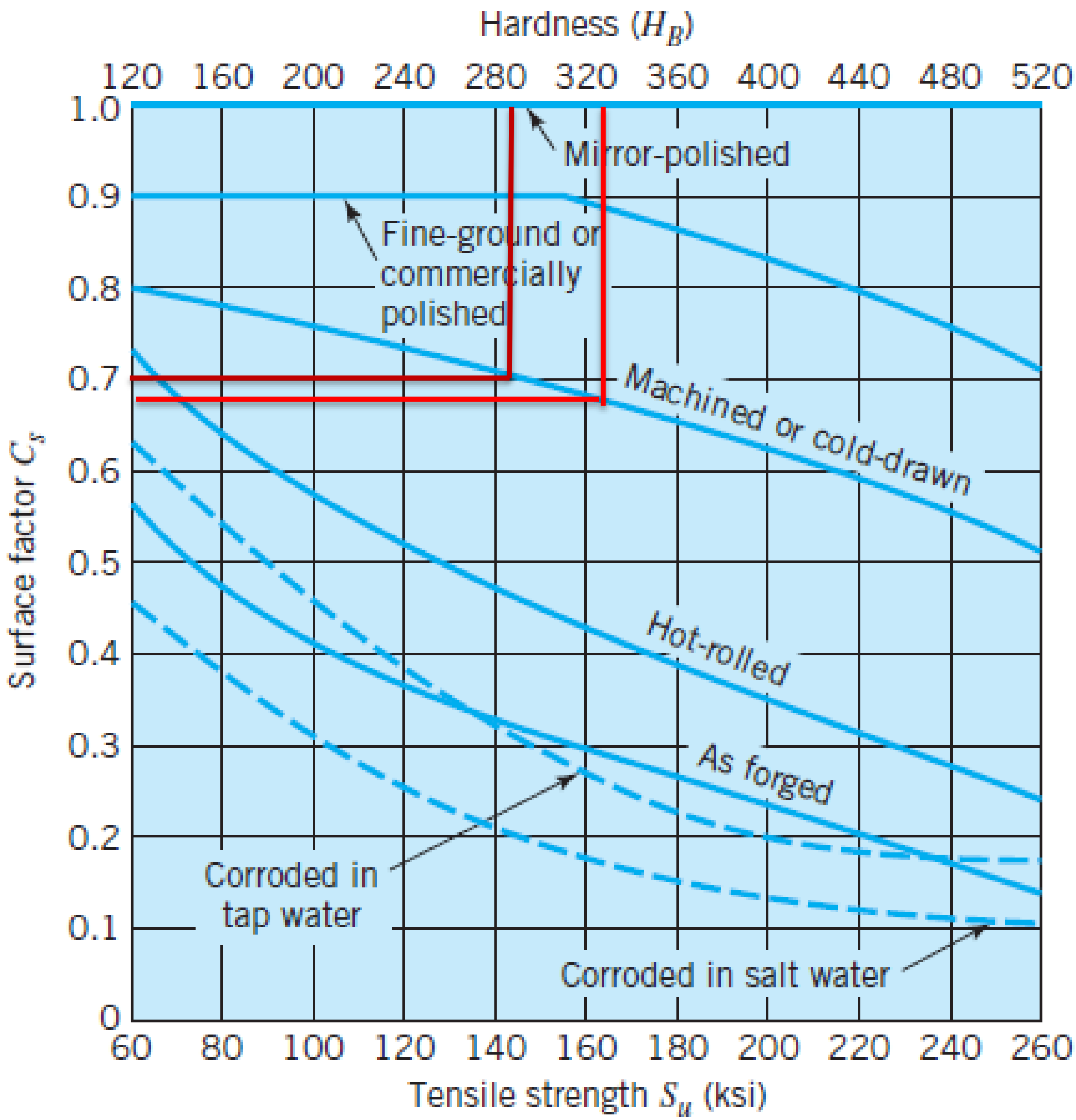
where

$$
\begin{aligned}
S'_n &= 290/4 = 72.5 \text{ ksi (gear)} \\
&= 330/4 = 82.5 \text{ ksi (pinion)} \\
C_L &= 1 \text{ (for bending loads)} \\
C_G &= 1 \text{ (since } P > 5) \\
C_S &= 0.68 \text{ (pinion)} \quad \text{(from Figure 8.13, machined surfaces)} \\
&= 0.70 \text{ (gear)} \\
k_r &= 0.814 \text{ (from Table 15.3; 99 percent reliability)} \\
k_t &= 1.0 \text{ (temperature should be } <160°\text{F)} \\
k_{ms} &= 1.4 \text{ (for one-way bending)} \\
S_n &= 63.9 \text{ ksi (pinion); } S_n = 57.8 \text{ ksi (gear)}
\end{aligned}
$$

From chap 8 S'_n in Ksi is .25Bhn

2. The bending fatigue stress is estimated from Eq. 15.17 as

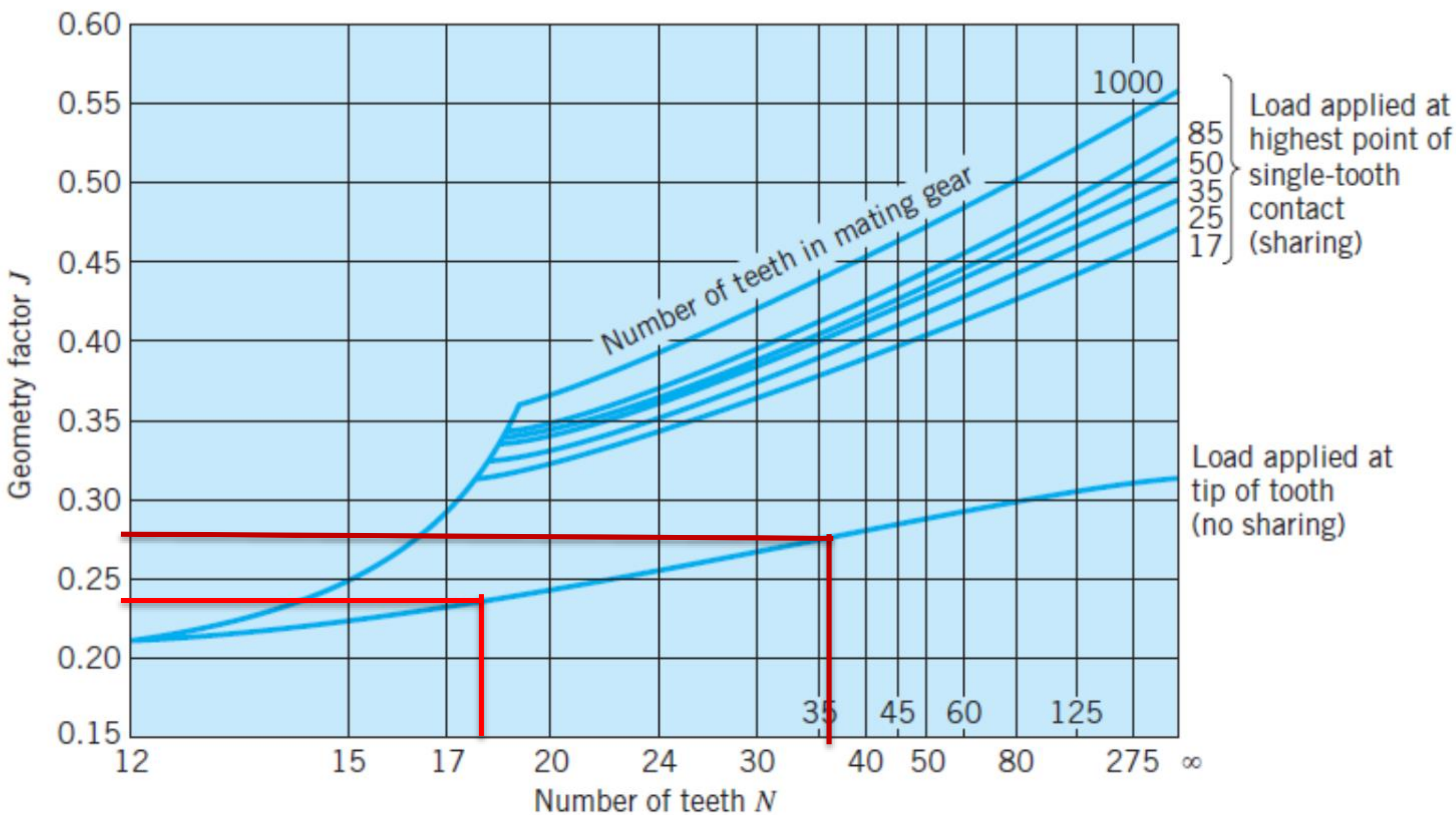
$$\sigma = \frac{F_t P}{bJ} K_v K_o K_m$$



Hardness (H_B)
120 160 200 240 280 320 360 400 440 480 520
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Surface factor C_s
Mirror-polished
Fine-ground or commercially polished
Machined or cold-drawn
Hot-rolled
As forged
Corroded in tap water
Corroded in salt water
60 80 100 120 140 160 180 200 220 240 260
Tensile strength S_u (ksi)

TABLE 15.3 **Reliability Correction Factor k_r, from Figure 6.19 with Assumed Standard Deviation of 8 Percent**

Reliability (%)	50	90	99	99.9	99.99	99.999
Factor k_r	1.000	0.897	0.814	0.753	0.702	0.659



(a) 20° full-depth teeth

where

$$P = 10 \text{ and } b = 1.25 \text{ in. (given)}$$

$J = 0.235$ (pinion) (for $N = 18$, no load sharing because of inadequate precision of manufacture)

$= 0.28$ (gear) (for $N = 36$, which is needed to provide the given speed ratio)

Dynamic factor K_v involves pitch line velocity V, calculated as

$$V = \frac{\pi d_p n_p}{12}$$

$$= \frac{\pi(18 \text{ teeth}/10 \text{ teeth per inch})(1720 \text{ rpm})}{12}$$

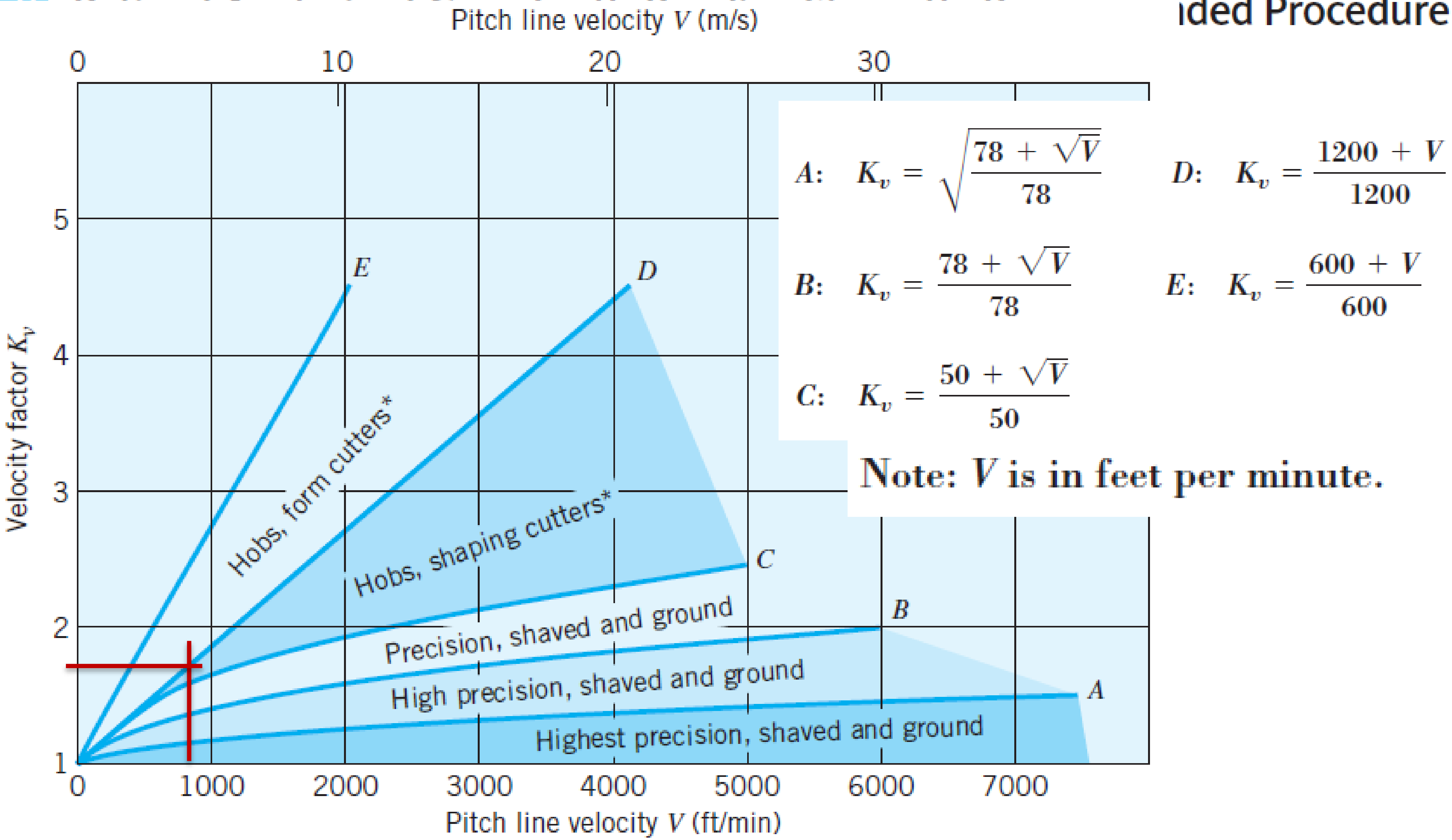
$$= 811 \text{ fpm}$$

We thus have

$K_v = 1.68$ (from Figure 15.24)

$K_o = 1.25$ (from Table 15.1)

$K_m = 1.6$ (from Table 15.2)



* Limited to about 350 Bhn

FIGURE 15.24

Velocity factor K_v. (Note: This figure, in a very rough way, is intended to account for the effects of tooth spacing and profile errors, tooth stiffness, and the velocity, inertia, and stiffness of the rotating parts.)

Table 15.1 Overload Correction Factor K_o

	Driven Machinery		
Source of Power	**Uniform**	**Moderate Shock**	**Heavy Shock**
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Table 15.2 Mounting Correction Factor K_m

	Face Width (in.)			
Characteristics of Support	**0 to 2**	**6**	**9**	**16 up**
Accurate mountings, small bearing clearances, minimum deflection, precision gears	1.3	1.4	1.5	1.8
Less rigid mountings, less accurate gears, contact across the full face	1.6	1.7	1.8	2.2
Accuracy and mounting such that less than full-face contact exists	Over 2.2			

Therefore,

$$\sigma = 114F_t \text{ psi (pinion)}, \qquad \sigma = 96F_t \text{ psi (gear)}$$

3. Equating bending fatigue strength and bending fatigue stress, we have

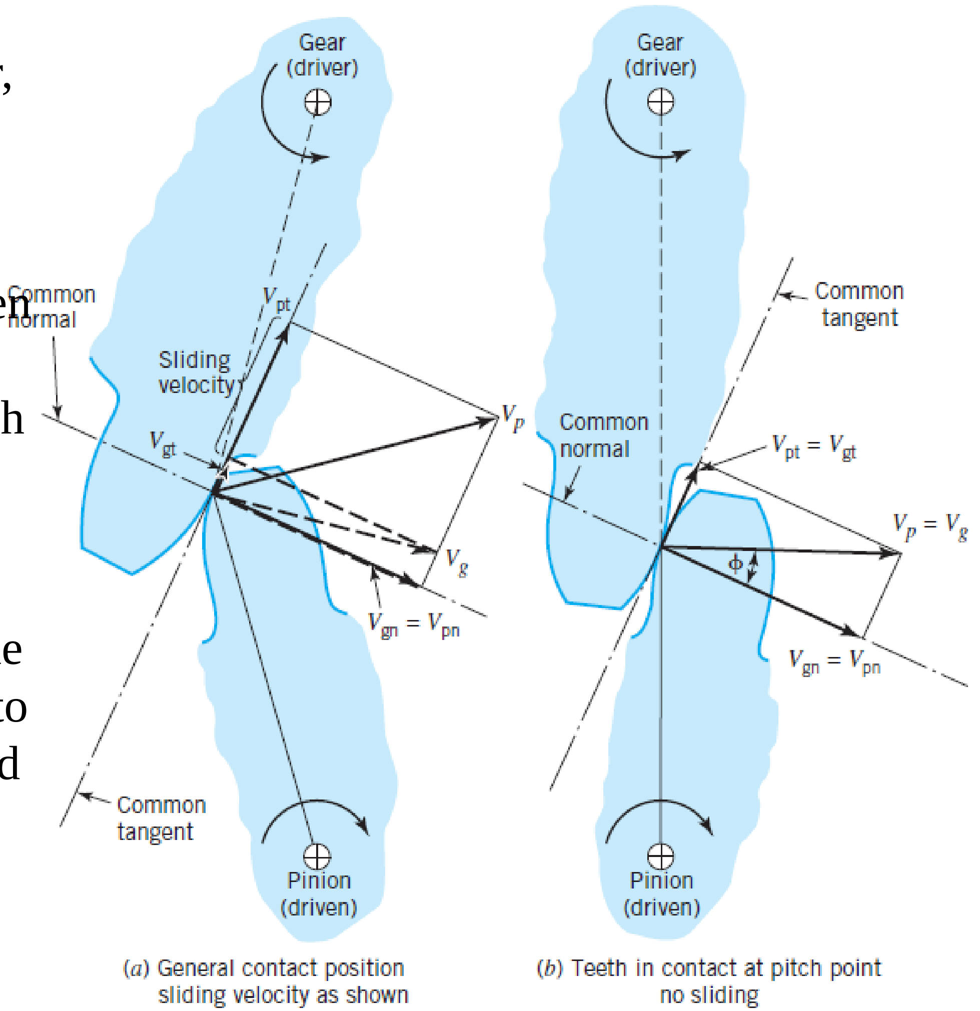
$$63{,}900 \text{ psi} = 114F_t \text{ psi}, \qquad F_t = 561 \text{ (pinion)}$$
$$57{,}800 \text{ psi} = 96F_t \text{ psi}, \qquad F_t = 602 \text{ (gear)}$$

4. In this case the pinion is the weaker member, and the power that can be transmitted is (561 lb)(811 fpm) = 456,000 ft · lb/min. Dividing by 33,000 to convert to horsepower gives *13.8 hp* (without provision for a safety factor).

Comments: Gear teeth generally experience several modes of failure simultaneously. Aside from tooth-bending fatigue, various other modes such as wear, scoring, pitting, and spalling may occur. These failure modes are discussed in the next section.

15.9 Gear-Tooth Surface Durability—Basic Concepts

- Gear teeth undergo various types of surface damage Excessive loading and lubrication breakdown - cause abrasion, pitting, and scoring.
- Nothing has previously been said about the rubbing velocity of the contacting surfaces. Figure shows the same conjugate gear teeth, with vectors showing, V_p & V_g, of the instantaneous contact points on the pinion and gear teeth, V_p & V_g are tangential wrt their centers of rotation.
- If the teeth do not separate or crush together, V_{pn} and V_{gn} normal to the surface are same.
- This results in V_{pt} and V_{gt} tangential components being different.
- The sliding velocity is the difference between V_{pt} and V_{gt} .
- Figure b shows that when contact at the pitch point P , sliding velocity = 0, and the tooth relative motion is one of pure rolling.
- For contact at all other points, the relative motion is one of rolling plus sliding, with the sliding velocity being directly proportional to the distance between the point of contact and the pitch point.
- Three basic types of surface deterioration that occur in gear teeth.



(a) General contact position sliding velocity as shown

(b) Teeth in contact at pitch point no sliding

15.9 Gear-Tooth Surface Durability—Basic Concepts

- Abrasive wear, caused by the presence of foreign particles, such as gears that are not enclosed, enclosed gears that were assembled with abrasive particles present, and gears lubricated by an oil supply with inadequate filtration.
- Scoring (a form of adhesive wear), which occurs at high speeds, when adequate lubrication is not provided. This causes a high coefficient of sliding friction that, together with high tooth loading and high sliding velocities, produces a high rate of heat generation in the localized regions of contact. This results in temperatures and pressures that cause welding and tearing apart.
- Scoring can often be prevented by directing an adequate flow (to provide cooling) of appropriate lubricant to the teeth as they come into mesh. An appropriate lubricant is generally one sufficiently resistant to extreme pressures that it maintains hydrodynamic lubrication.
- Surface finish is also important, with finishes as fine as 20 μinches being desirable where scoring is a factor. Allowing the gears to smooth themselves during an initial "break-in" period of moderate load will increase their resistance to scoring.
- Pitting and spalling, which are, respectively, surface and subsurface fatigue failures brought about by the complex stresses within the contact zone.

15.9 Gear-Tooth Surface Durability—Basic Concepts

- Gears should not fail because of abrasive wear. With proper lubrication and cooling, they will not fail because of scoring. If the best heat transfer available can be provided, but an adequate lubricant cannot be found, then loads and speeds must be reduced, more score-resistant materials used, or the gears made larger.
- Unlike scoring, which occurs early in the operating life, pitting is typical of fatigue failures because it occurs only after accumulating a sufficient number of load cycles.
- Since contact stress S–N curves do not level off after 10^6 or 10^7 cycles even with steel members, this type of potential surface failure must be considered in every gear design.
- Generally good correlation has been observed between spur gear surface fatigue failure and the computed elastic surface stress (Hertz stress).
- Adapting the Hertz equation to spur gear teeth was done by Buckingham who noted that gear-tooth pitting occurs predominantly in the vicinity of the pitch line where, because of zero sliding velocity, the oil film breaks down.
- Hence, he treated a pair of gear teeth as two cylinders of radii equal to the radii of curvature of the mating involutes at the pitch point.

15.9 Gear-Tooth Surface Durability—Basic Concepts

- From basic involute geometry, these radii are

$$R_p = (d_p \sin \phi)/2 \quad \text{and} \quad R_g = (d_g \sin \phi)/2 \qquad (15.20)$$

- Giving surface (Hertz) fatigue stress as

$$\sigma_H = 0.564 \sqrt{\frac{F_t[2/(d_p \sin \phi) + 2/(d_g \sin \phi)]}{b \cos \phi \left(\frac{1 - \nu_p^2}{E_p} + \frac{1 - \nu_g^2}{E_g} \right)}} \qquad (15.21)$$

- Where b is the gear face width
- Because of the increased contact area with load, stress increases only as the square root of load F_t (or square root of load/inch of face width, F_t/b).
- Contact area ↑ses (and stress ↓ses) with ↓sed moduli, E_p and E_g.
- Larger gears have greater radii of curvature, hence lower stress.
- Contact stresses are influenced by manufacturing accuracy, pitch line velocity, shock loading, shaft misalignment and deflection, and moment of inertia and torsional elasticity of the connected rotating members.
- Similarly, surface fatigue strength of the material is affected by the reliability requirement and by possible temperature extremes.

15.10 Gear-Tooth Surface Fatigue Analysis—Recommended Procedure

- Equation 15.21 can be simplified by (1) combining matl properties as elastic coeff, C_p, and (2) combine terms relating to tooth shape into geometry factor, I:

$$C_p = 0.564\sqrt{\frac{1}{\frac{1-\nu_p^2}{E_p}+\frac{1-\nu_g^2}{E_g}}} \qquad (15.22)$$

$$I = \frac{\sin\phi\cos\phi}{2}\frac{R}{R+1} \qquad (15.23)$$

$$R = \frac{d_g}{d_p}$$

- Where R is the ratio of gear to pinion diameters
- R is +ve for external gears and –ve for a pinion and internal gear.
- Substituting C_p and I into Eq. 15.21, and introducing factors K_v, K_o, and K_m, which were used with the bending fatigue analysis, gives

$$\sigma_H = C_p\sqrt{\frac{F_t}{bd_pI}K_vK_oK_m} \qquad (15.24)$$

- I can be calculated from 15.23 and C_P can be determined from the tables

TABLE 15.4a Values of Elastic Coefficient C_p for Spur Gears, in $\sqrt{\text{psi}}$ (Values Rounded Off)

Pinion Material (ν = 0.30 in All Cases)	Gear Material: Steel	Cast Iron	Aluminum Bronze	Tin Bronze
Steel, E = 30,000 ksi	2300	2000	1950	1900
Cast iron, E = 19,000 ksi	2000	1800	1800	1750
Aluminum bronze, E = 17,500 ksi	1950	1800	1750	1700
Tin bronze, E = 16,000 ksi	1900	1750	1700	1650

TABLE 15.4b Values of Elastic Coefficient C_p for Spur Gears, in $\sqrt{\text{MPa}}$ (Values Converted from Table 15.4a)

Pinion Material (ν = 0.30 in All Cases)	Gear Material: Steel	Cast Iron	Aluminum Bronze	Tin Bronze
Steel, E = 207 GPa	191	166	162	158
Cast iron, E = 131 GPa	166	149	149	145
Aluminum bronze, E = 121 GPa	162	149	145	141
Tin bronze, E = 110 GPa	158	145	141	137

15.10 Gear-Tooth Surface Fatigue Analysis—Recommended Procedure

- The actual stress state at the point of contact is influenced by several factors not considered in the simple Hertz equation
- These include thermal stresses, changes in pressure distribution because a lubricant is present, stresses from sliding friction etc. So the stresses calculated from Eq. 15.24 must be compared with surface fatigue strength S–N curves that have been obtained experimentally from tests in which these additional factors were at least roughly comparable with those for the situation under study.
- Table gives representative values of information about surface fatigue strength.

TABLE 15.5 Surface Fatigue Strength S_{fe}, for Use with Metallic Spur Gears (10^7-Cycle Life, 99 Percent Reliability, Temperature <250°F)

Material	S_{fe} (ksi)	S_{fe} (MPa)
Steel	0.4 (Bhn)–10 ksi	28 (Bhn)–69 MPA
Nodular iron	0.95[0.4 (Bhn)–10 ksi]	0.95[28 (Bhn)–69 MPa]
Cast iron, grade 20	55	379
grade 30	70	482
grade 40	80	551
Tin bronze AGMA 2C (11 percent tin)	30	207
Aluminum bronze (ASTM B 148—52) (Alloy 9C—H.T.)	65	448

15.9 Gear-Tooth Surface Durability—Basic Concepts

- It is desirable for one of the contacting members to be harder than the other. In case of steel set, the pinion is made harder because pinion is subjected to a higher of fatigue cycles and it is economical to manufacture the smaller pinion to higher hardness.
- Typically, the hardness differential ranges from about 30 Bhn for gears in the 200-Bhn range to about 100 Bhn for the 500-Bhn range and 2 Rockwell C for the 60RC range.
- For hardness differentials not exceeding these values, the average hardness can be used for checking both pinion and gear.
- For surface-hardened steel gears, the hardness used with Table 15.5 is the surface hardness, but the depth of the hardened case should extend down to the peak shear stresses.
- This would normally be at least 1 mm, or 0.040 in.
- For fatigue lives other than 10^7 cycles, multiply the values of S_{fe} (Table 15.5) by a life factor, C_{Li}, from Figure 15.27 representing average shape of S-N curve of steel

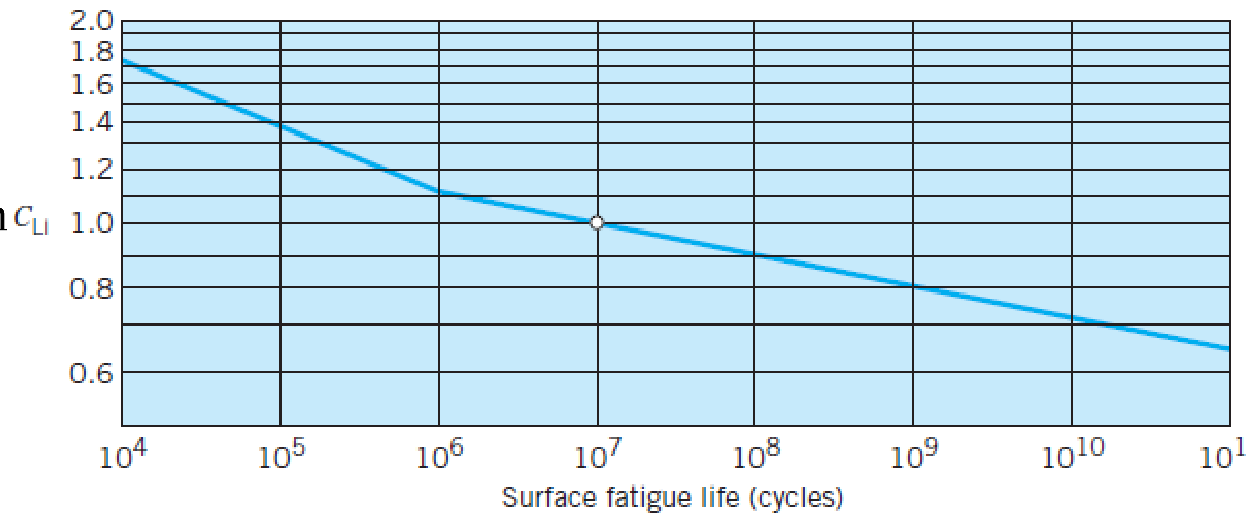


FIGURE 15.27
Values of C_{Li} for steel gears (general shape of surface fatigue S–N curve).

15.9 Gear-Tooth Surface Durability—Basic Concepts

- Rough appropriate reliability factor C_R, in Table, is used (in Eq. 15.25).
- When gear-tooth surface temperatures are high (above about 120ºC, or 250ºF), we must determine the appropriate surface fatigue strength for the material and temperature of the gear teeth. (A temperature correction factor for surface fatigue strength has not been included in Eq. 15.25.)
- Applying the information about surface fatigue strength just given, the resulting equation for gear-tooth surface fatigue strength, which should be compared to the gear-tooth surface fatigue stress from Eq. 15.24, is

$$S_H = S_{fe} C_{Li} C_R \tag{15.25}$$

- The Safety Factor, defined as the multiplier of F_t needed to make σ_H equal to S_H, can be small (1.1 to 1.5). Many of the factors often included in "SF" are already accounted
- The consequences of failure are low as pitting failures develop slowly and give warning by increasing gear noise.
- The extent of surface fatigue damage constituting "failure" is arbitrary, and the gears will continue to operate for some period of time after their surface endurance "life" has expired.

TABLE 15.6 Reliability Factor C_R

Reliability (%)	C_R
50	1.25
99	1.00
99.9	0.80

SAMPLE PROBLEM 15.4 Gear Horsepower Capacity for Tooth Surface Fatigue Failure

For the gears in Sample Problem 15.3, estimate the maximum horsepower that the gears can transmit with only a 1 percent chance of a surface fatigue failure during 5 years of 40 hours/week, 50 weeks/year operation.

SOLUTION

Known: The steel pinion of Sample Problem 15.3 with 330 Bhn hardness and given diametral pitch, number of teeth, rotational speed, and 20° full-depth teeth drives a steel gear of 290 Bhn at 860 rpm with only a 1 percent chance of surface fatigue failure during a specified time period.

Find: Estimate the maximum horsepower that the gears can transmit.

Schematic and Given Data: See Figure 15.25.

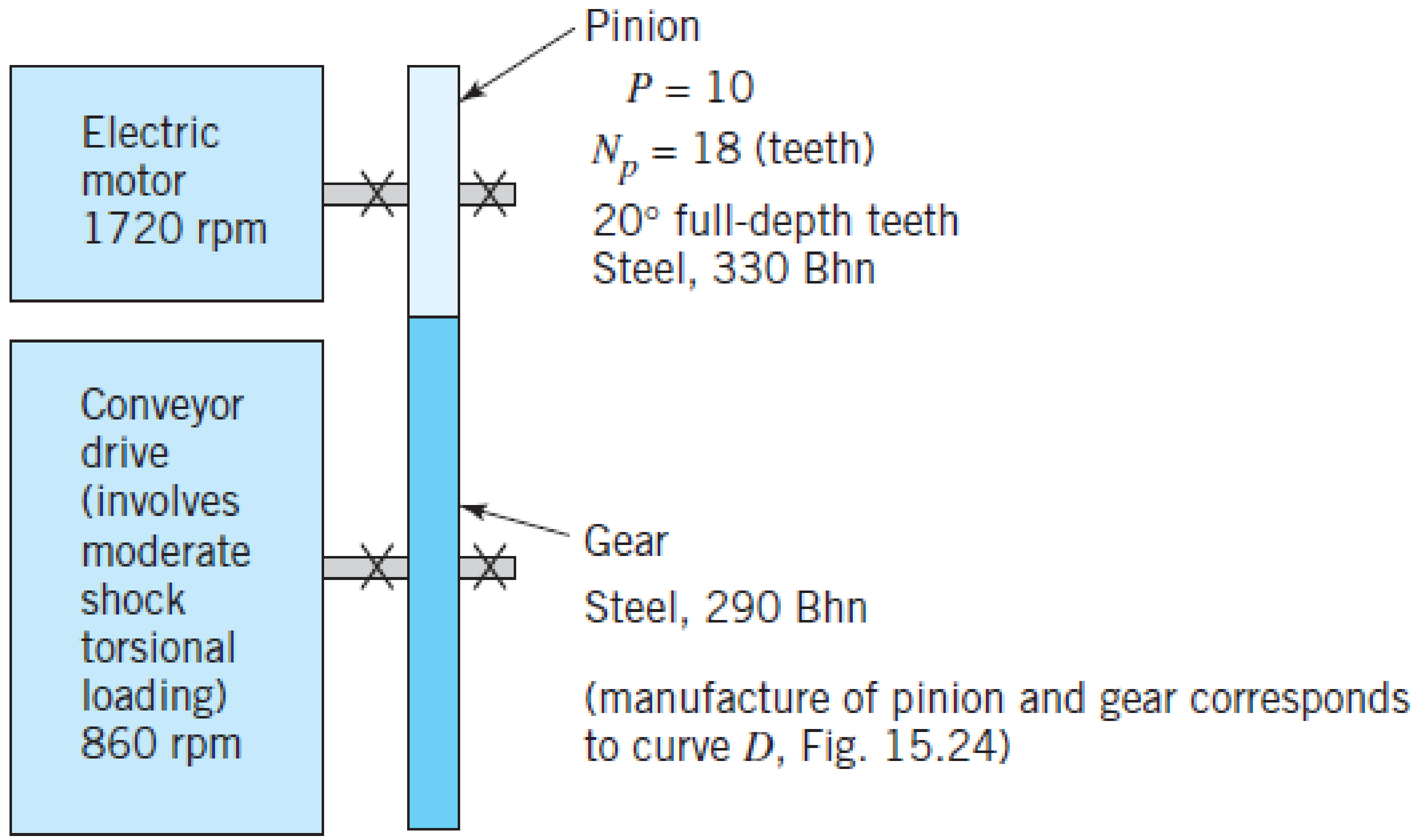


FIGURE 15.25

Data for Sample Problem 15.3.

15.9 Gear-Tooth Surface Durability—Basic Concepts

Assumptions:

1. The gear-tooth surface temperatures are below 120°C (250°F).
2. The surface fatigue endurance limit can be calculated from the surface hardness—see Table 15.5.
3. The surface fatigue stress is a maximum at the pitch point (line).
4. The manufacturing quality of the pinion and gear corresponds to curve *D*, Figure 15.24.
5. The output gear experiences moderate torsional shock.
6. The characteristics of support include less rigid mounting, less accurate gears, and contact across the full face.
7. No factor of safety will be necessary.
8. The tooth profiles of the gears are standard involutes. The contact surfaces at the pitch point can be approximated by cylinders.
9. The gears are mounted to mesh at the pitch circles.
10. The effects of surface failure from abrasive wear and scoring are eliminated by enclosure and lubrication—only pitting needs consideration.
11. The stresses caused by sliding friction can be neglected.
12. The contact pressure distribution is unaffected by the lubricant.
13. Thermal stresses and residual stresses can be neglected.

14. The gear materials are homogeneous, isotropic, and linearly elastic.
15. The surface endurance limit and life factor data available are sufficiently accurate. The velocity factor K_v, the overload factor K_o, and the mounting factor K_m obtained from available data are reasonably accurate.

Analysis:

1. The surface endurance strength is estimated from Eq. 15.25 as

$$S_H = S_{\text{fe}} C_{\text{Li}} C_R$$

where

$S_{\text{fe}} = 114$ ksi [from Table 15.5 for steel, $S_{\text{fe}} = 0.4\,(\text{Bhn}) - 10\text{ ksi} = 0.4\,(330) - 10 = 122$ ksi]

$C_{\text{Li}} = 0.8$ [from Figure 15.27, life $= (1720)(60)(40)(50)(5) = 1.03 \times 10^9$ cycles]

$C_R = 1$ (from Table 15.6 for 99 percent reliability)

$S_H = (122)(0.8)(1) = 97.6$ ksi

2. The surface (Hertz) fatigue stress is estimated from Eq. 15.24 as

$$\sigma_H = C_p \sqrt{\frac{F_t}{b d_p I} K_v K_o K_m}$$

TABLE 15.5 Surface Fatigue Strength S_{fe}, for Use with Metallic Spur Gears (10^7-Cycle Life, 99 Percent Reliability, Temperature <250°F)

Material	S_{fe} (ksi)	S_{fe} (MPa)
Steel	0.4 (Bhn)–10 ksi	28 (Bhn)–69 MPA
Nodular iron	0.95[0.4 (Bhn)–10 ksi]	0.95[28 (Bhn)–69 MPa]
Cast iron, grade 20	55	379
grade 30	70	482
grade 40	80	551
Tin bronze AGMA 2C (11 percent tin)	30	207
Aluminum bronze (ASTM B 148—52) (Alloy 9C—H.T.)	65	448

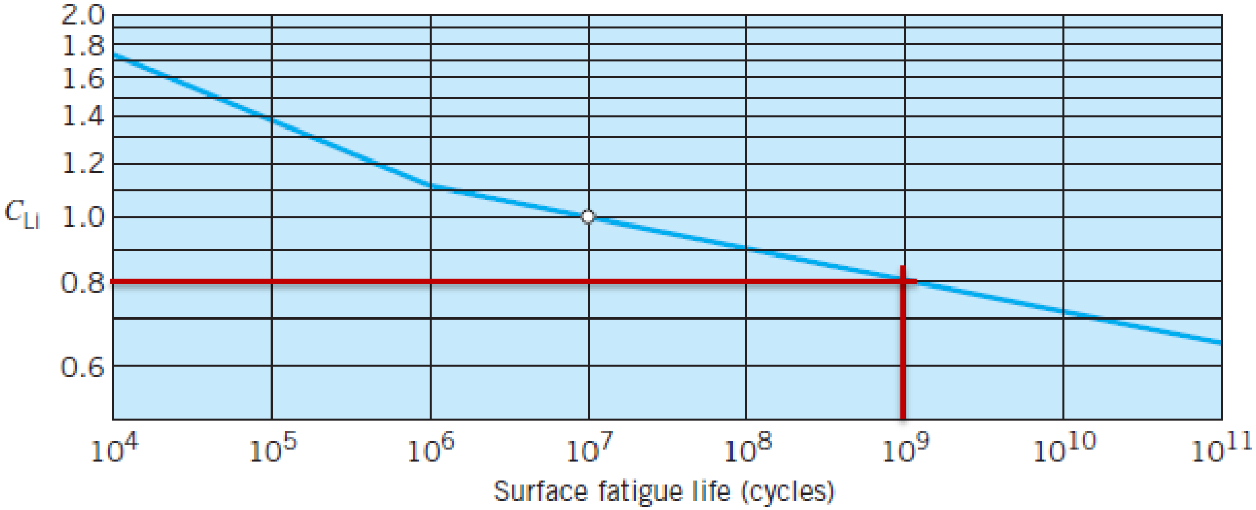


FIGURE 15.27
Values of C_{Li} for steel gears (general shape of surface fatigue S–N curve).

TABLE 15.6 Reliability Factor C_R

Reliability (%)	C_R
50	1.25
99	1.00
99.9	0.80

TABLE 15.4a Values of Elastic Coefficient C_p for Spur
(Values Rounded Off)

Pinion Material (ν = 0.30 in All Cases)	Steel
Steel, E = 30,000 ksi	2300

where

$$C_p = 2300\sqrt{\text{psi}}\ \text{(from Table 15.4)}$$

$b = 1.25$ in., $d_p = 1.8$ in., $K_v = 1.68$, $K_o = 1.25$, and $K_m = 1.6$ (all are the same as in Sample Problem 15.3)

$$I = \frac{\sin\phi\cos\phi}{2}\frac{R}{R+1} = 0.107\ \text{(from Eq. 15.23)}$$

$$\sigma_H = 2300\sqrt{\frac{F_t}{(1.25)(1.8)(0.107)}(1.68)(1.25)(1.6)} = 8592\sqrt{F_t}$$

3. Equating surface fatigue strength and surface fatigue stress gives

$$8592\sqrt{F_t} = 97{,}600\ \text{psi} \quad \text{or} \quad F_t = 129\ \text{lb}$$

(This value applies to both of the mating gear-tooth surfaces.)

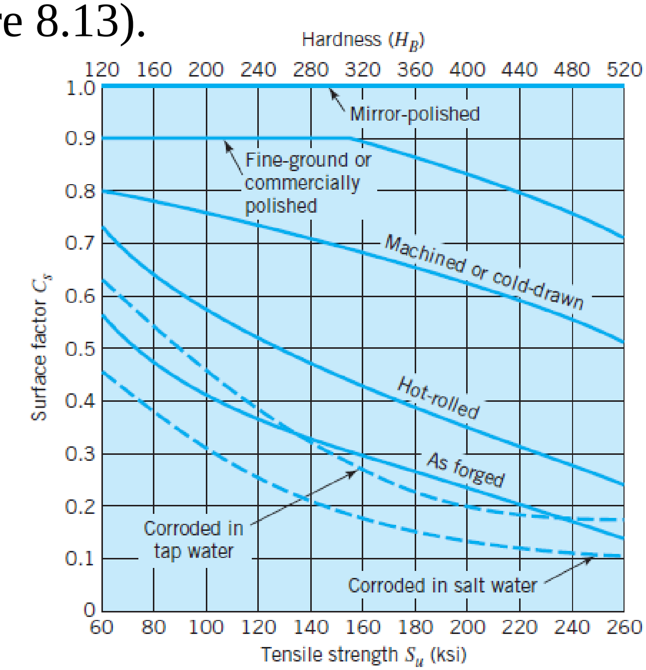
4. The corresponding power is $\dot{W} = F_t V = (129\ \text{lb})(811\ \text{fpm}) = 104{,}620$ ft · lb/min, or 3.2 hp.

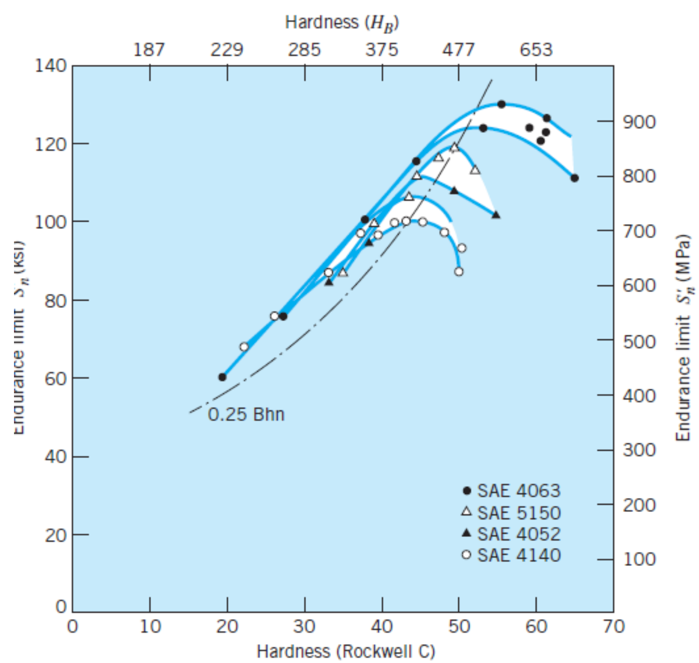
Comment: This power compares with a bending fatigue-limited power of nearly 14 hp and illustrates the usual situation of steel gears being stronger in bending fatigue. Although much of the 14-hp bending fatigue capacity is obviously wasted, a moderate excess of bending capacity is desirable because bending fatigue failures are sudden and total, whereas surface failures are gradual and cause increasing noise levels to warn of gear deterioration.

15.11 Spur Gear Design Procedures

- Sample Problems so far shows the analysis of estimated capacity of a given pair of gears. As is generally the case with machine components, it is a more challenging task to design a suitable pair of gears for a given application. Few general observations.

1. Increasing the surface hardness of steel gears pays off handsomely in terms of surface endurance. Table 15.5 indicates that doubling the hardness more than doubles surface fatigue strength (allowable Hertz stress); Eq. 15.24 shows that doubling the allowable Hertz stress quadruples the load capacity F_t.
2. Increases in steel hardness also increase bending fatigue strength, but the increase is far less. For example, doubling the hardness will likely not double the basic endurance limit, (flattening of curves in Figure 8.6). Furthermore, doubling hardness substantially reduces C_S (see Figure 8.13).





15.11 Spur Gear Design Procedures

3. Increasing tooth size (using a coarser pitch) increases bending strength more than surface strength. This fact, together with points 1 and 2, correlates with two observations. (a) A balance between bending and surface strengths occurs typically in the region of P = 8 for high-hardness steel gears (above about 500 Bhn, or 50RC), with coarser teeth failing in surface fatigue and finer teeth failing in bending fatigue. (b) With progressively softer steel teeth, surface fatigue becomes critical at increasingly fine pitches. Other materials have properties giving different gear-tooth strength characteristics.

4. In general, the harder the gears, the more costly they are to manufacture. On the other hand, harder gears can be smaller and still do the same job. And if the gears are smaller, the housing and other associated parts may also be smaller and lighter. Furthermore, if the gears are smaller, pitch line velocities are lower, and this reduces the dynamic loading and rubbing velocities. Thus, overall cost can often be reduced by using harder gears.

5. If minimum-size gears are desired (for any given gear materials and application), it is best in general to start by choosing the minimum acceptable number of teeth for the pinion (usually 18 teeth for 20º pinions, and 12 teeth for 25º pinions), and then solving for the pitch (or module) required.

15.11 Spur Gear Design Procedures

Sample Problem 15.5D Design of a Single Reduction Spur Gear Train

Using a standard gear system, design a pair of spur gears to connect a 100-hp, 3600-rpm motor to a 900-rpm load shaft. Shock loading from the motor and driven machine is negligible. The center distance is to be as small as reasonably possible. A life of 5 years of 2000 hours/year operation is desired, but full power will be transmitted only about 10 percent of the time, with half power the other 90 percent. Likelihood of failure during the 5 years should not exceed 10 percent.

Solution

Known: A spur gear pair is to transmit power from a motor of known horsepower and speed to a driven machine shaft rotating at 900 rpm. Full power is transmitted 10 percent of the time, half power the other 90 percent. The likelihood of failure should not exceed 10 percent when the gears are operated at 2000 hours/year for 5 years. Center distance is to be as small as reasonably possible. (See Figure 15.28.)

Find: Determine the geometry of the gearset.

Schematic and Given Data:

15.11 Spur Gear Design Procedures

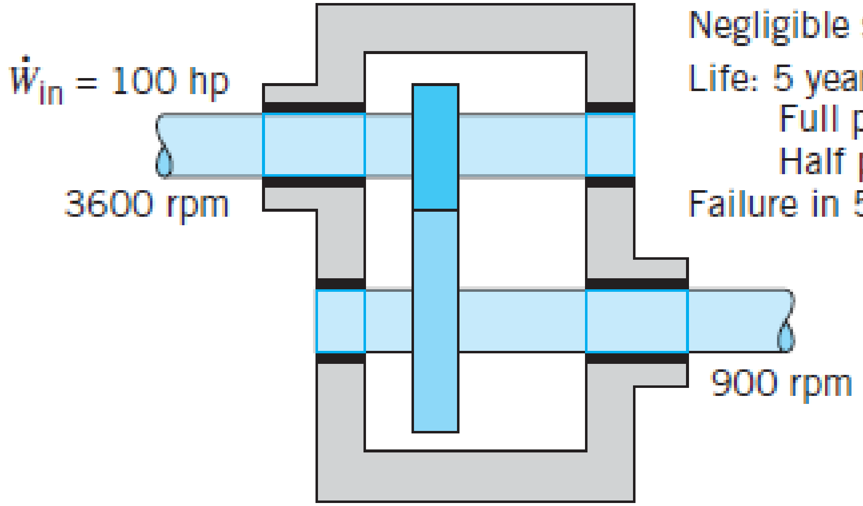


FIGURE 15.28
Single-reduction spur gear train.

Decisions:

1. Choose hardened-steel gears corresponding to the spur gear curve in Figure 9.21, which shows a 10 percent probability of failure. Steel gear material will be selected to provide relatively high strength at relatively low cost. The pinion and gear will be machined and then ground. In accordance with good practice, specify a case-hardening procedure that will leave compressive residual stresses in the gear-tooth surfaces.
2. Specify high surface hardness of 660 Bhn and 600 Bhn, respectively, for pinion and gear to obtain the minimum center distance and the pinion-tooth hardness that will exceed the gear-tooth hardness by 10 percent.
3. For these hardnesses (which are too hard for normal machining), specify a ground finish and precision manufacture corresponding to the average of curves *A* and *B* in Figure 15.24.

15.11 Spur Gear Design Procedures

4. Choose the more common 20° full-depth involute tooth form.
5. Choose 18 teeth, the minimum number of pinion teeth possible to avoid interference.
6. For minimum center distance (i.e., minimum gear diameters), tentatively choose width b at the maximum of the normal range, $14/P$.
7. Choose a safety factor of 1.25 for failure by surface fatigue.
8. A nominal value for face width will be used.
9. A standard diametral pitch will be selected.

Assumptions:

1. The Palmgren–Miner cumulative-damage rule applies.
2. The ground-surface finish will correspond to the average of curves A and B in Figure 15.24, and $K_v = 1.4$.
3. The characteristics of support are accurate mountings, small bearing clearances, minimum deflection, and precision gears.
4. The spur gear curve in Figure 9.21 represents about the highest contact strength that is obtainable for steel gears, and this curve is a plot of $S_H = S_{fe} C_{Li} C_R$ for a 10 percent probability of failure versus the number of cycles constituting the life of the spur gear.

5. There is no load sharing between gear teeth.
6. In the limiting case, the fatigue strength of the core material must be equal to the bending fatigue stresses at the surface. Under the surface C_s is 1.
7. For the steel core material, $S'_n = 250$ (Bhn).

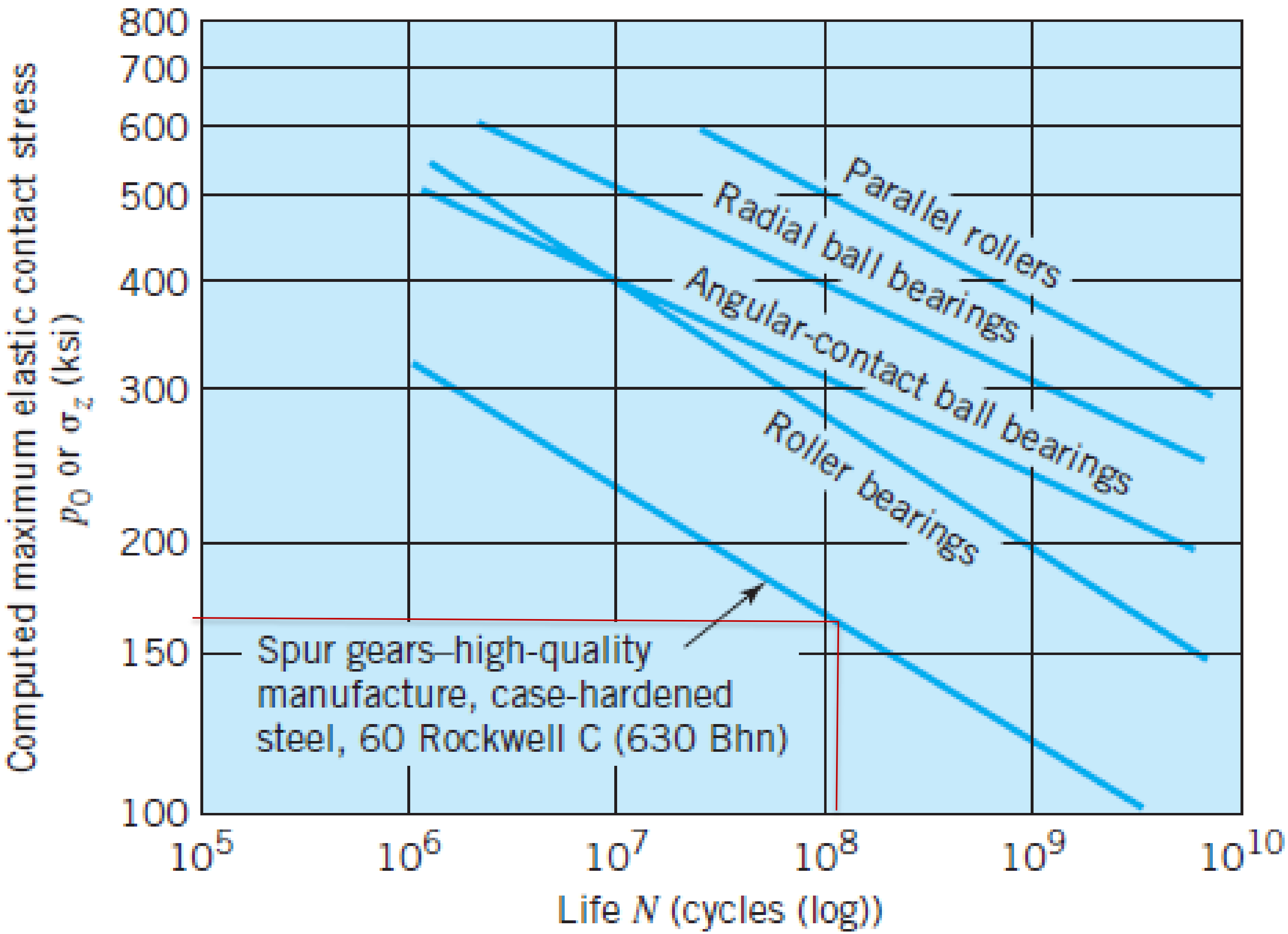
Design Analysis:

1. Total life required = 3600 rev/min × 60 min/h × 2000 h/yr × 5 yr = 2.16×10^9 revolutions of the pinion. Only 2.16×10^8 cycles are at full power. Looking at the spur gear curve in Figure 9.21, we note that if the stresses for 2×10^8 cycles of full power are on the curve, stresses for 50 percent power would correspond to over 10^{10}-cycle life. Considering the Palmgren–Miner cumulative-damage rule (Section 8.12), and recognizing the approximate nature of our solution, we appear justified in designing for the full-load cycles only and in ignoring the half-load cycles.

2. Anticipating that surface fatigue will likely be more critical than bending fatigue, we solve for the value of P that will balance σ_H and S_H with a small safety factor, SF, of say 1.25:

$$\sigma_H \text{ (from Eq. 15.24)} = S_H \text{ (from Eq. 15.25)}$$

$$C_p\sqrt{\frac{F_t(SF)}{bd_pI}K_vK_oK_m} = S_{\text{fe}}C_{\text{Li}}C_R$$



800
700
600
500
400
300
200
150
100
Computed maximum elastic contact stress
p_O or σ_z (ksi)
Parallel rollers
Radial ball bearings
Angular-contact ball bearings
Roller bearings
Spur gears–high-quality
manufacture, case-hardened
steel, 60 Rockwell C (630 Bhn)
10^5
10^6
10^7
10^8
10^9
10^{10}
Life N (cycles (log))

15.11 Spur Gear Design Procedures

A few auxiliary calculations are required:

$$V = \pi d_p(3600 \text{ rpm})/12 = 942 d_p = 942(18/P) = 16{,}960/P$$

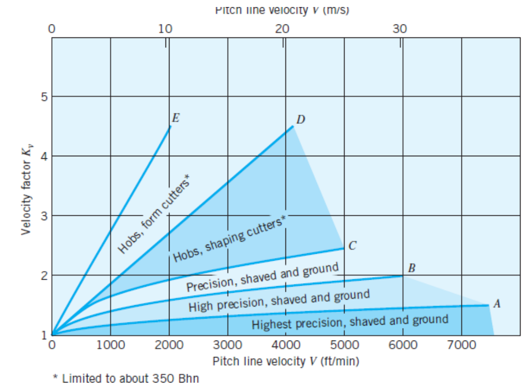
$$K_v \approx 1.4 \text{ (This value is a rough estimate from Figure 15.24, and must be confirmed or modified after } P \text{ is determined.)}$$

$$K_m = 1.3 \text{ (This value must be increased if } b > 2 \text{ in.)}$$

$$F_t = 100 \text{ hp } (33{,}000)/V = 195P$$

$$I = [(\sin 20° \cos 20°)/2](4/5) = 0.128$$

$$S_{fe} C_{Li} C_R = 165{,}000 \text{ psi} \quad \text{(directly from Figure 9.21)}$$



Substituting gives

$$2300\sqrt{\frac{(195P)(1.25)}{(14/P)(18/P)(0.128)}}(1.4)(1)(1.3) = 165{,}000$$

from which

$$P = 7.21 \text{ teeth/in.}$$

3. Tentatively choose a standard pitch of 7, compute the corresponding value of V, refine the estimate of K_v, and compute the value of b required to balance σ_H and S_H. (Note that if $P = 8$ were chosen, b would have to exceed $14/P$ to balance σ_H and S_H.)

$$V = \frac{\pi d_p n_p}{12} = \frac{\pi(18/7)(3600)}{12} = 2424 \text{ fpm}$$

From Figure 15.24, $K_v = 1.5$, and

$$2300\sqrt{\frac{(195 \times 7)(1.25)}{b(18/7)(0.128)}(1.5)(1)(1.3)} = 165{,}000$$

from which $b = 1.96$ in. Round off to $b = 2$ in. For this value of b, $K_m = 1.3$ is satisfactory. Also note that b remained at $14/P$ because decreasing P from 7.21 to 7 offsets increasing K_v from 1.4 to 1.5.

4. Check the contact ratio, using Eq. 15.9.

The pitch radii are $r_p = 9/7$ and $r_g = 36/7$.

The addendum, $a = 1/P$; and hence, $r_{\text{ap}} = 10/7$, $r_{\text{ag}} = 37/7$.

Center distance, $c = r_p + r_g = 45/7$.

From Eq. 15.11, $r_{\text{bp}} = (9/7) \cos 20°$, $r_{\text{bg}} = (36/7) \cos 20°$.

From Eq. 15.10, $p_b = \pi(18/7)\,(\cos 20°)/18 = 0.422$ in.

Substituting in Eq. 15.9 gives CR $= 1.67$.

This is satisfactory, but it means that a single pair of teeth carries the load in the vicinity of the pitch line, where pitting is most likely to occur. Thus there can be no sharing of the surface fatigue load, regardless of manufacturing precision. (Note that no sharing was assumed in the preceding calculations.)

5. We need to design the gears to provide adequate bending fatigue strength. Detailed consideration of gear-tooth-bending fatigue for case-hardened gears must include an analysis of stress and strength gradients, as represented in Figure 8.29. Since we anticipate no problem in satisfying this requirement, let us, as previously stated, make the conservative assumption that the fatigue strength of the *core* material (Eq. 15.18) must be equal to the bending fatigue stresses at the surface (Eq. 15.17):

$$S'_n C_L C_G C_S k_r k_t k_{\mathrm{ms}} = \frac{F_t P}{bJ} K_v K_o K_m$$

The manufacturing accuracy is in a "gray area" with respect to load sharing. There will likely be at least a partial sharing, meriting a value of J at least intermediate between the "sharing" and "not sharing" curves (i.e., between $J = 0.235$ and 0.32). But since we conservatively assumed no sharing, there is no need to consider the matter further. In calculating a value for C_s, remember that we are considering fatigue strength *under* the surface, where surface roughness would not be involved:

$$S'_n(1)(1)(1)(0.897)(1)(1.4) = \frac{1365(7)}{2(0.235)}(1.5)(1)(1.3)$$

From this equation $S'_n = 31{,}600$ psi, which requires a (core) hardness of 126 Bhn, a value that will be satisfied or exceeded by any steel selected to meet the case-hardened surface requirement.

6. In summary, our tentatively proposed design has 20° full-depth teeth, precision-manufactured with ground finish (between curves *A* and *B* of Figure 15.24) from case-hardening steel, surface-hardened to 660 Bhn and 600 Bhn, respectively, for pinion and gear, and with core hardness of at least 126 Bhn. The design also has $P = 7$, $N_p = 18$, $N_g = 72$, $b = 2$ in. ($D_p = 2.57$ in., $D_g = 10.29$ in., $c = 6.43$ in.). As decided, we will specify a case-hardening procedure leaving compressive residual stresses in the surfaces.

Comment: This sample problem represents but one of a great many situations and approaches encountered in the practical design of spur gears. The important thing for the student is to gain a clear understanding of the basic concepts and to understand how these may be brought to bear in handling specific situations. We have seen that a great amount of empirical data is needed in addition to the fundamentals. It is always important to seek out the best and most directly relevant empirical data for use in any given situation. Textbooks such as this can include only sample empirical information. Better values for actual use are often found in company files, contemporary specialized technical literature, and current publications of the AGMA.

15.12 Gear Materials

- The least expensive gear material is CI, ASTM grade 20. Grades 30, 40, 50, and 60 are progressively stronger and more expensive.
- CI gears typically have greater surface fatigue strength than bending fatigue strength. Their internal damping tends to make them quieter than steel gears.
- Nodular CI gears have substantially greater bending strength, together with good surface durability. A good combination is often a steel pinion mated to a CI gear.
- Steel gears that are not heat-treated are inexpensive, but have low surface endurance capacity. Heat-treated steel gears must be designed to resist warpage; hence, alloy steels and oil quenching are usually preferred.
- For hardnesses > 250 to 350 Bhn, machining must usually be done before hardening.
- Greater profile accuracy is obtained if the surfaces are finished after heat treating, as
- by grinding. (But if grinding is done, avoid residual tensile stresses at surface.)
- Through-hardened gears generally have 0.35 to 0.6 % carbon. Surface or case-hardened gears are usually processed by flame hardening, induction hardening, carburizing, or nitriding.
- Of the nonferrous metals, bronzes are most often used for making gears.

15.12 Gear Materials

- Nonmetallic gears made of acetal, nylon, and other plastics are generally quiet, durable, reasonably priced, and can often operate under light loads without lubrication.
- Their teeth deflect more easily than those of corresponding metal gears. This promotes effective load sharing among teeth in simultaneous contact, but results in substantial hysteresis heating if the gears are rotating at high speed.
- Since non-metallic materials have low thermal conductivity, special cooling provisions may be required.
- Also, these materials have relatively high coefficients of thermal expansion, and thus may require installation with greater backlash than metal gears.
- Often the base plastics used for gears are formulated with fillers, such as glass fibers, for strength, and with lubricants such as Teflon for reduced friction and wear.
- Nonmetallic gears are usually mated with CI or steel pinions.
- For best wear resistance, hardness of mating metal pinion should be at least 300 Bhn.
- Design procedures for gears made of plastics are similar to those for gears made of metals, but are not yet as reliable.
- Hence, prototype testing is even more important than for metal gears.