# CHAPTER 3

## Frequency Response of Basic BJT and MOSFET Amplifiers

(*Review materials in Appendices III and V*)

In this chapter you will learn about the general form of the frequency domain transfer function of an amplifier. You will learn to analyze the amplifier equivalent circuit and determine the critical frequencies that limit the response at low and high frequencies. You will learn some special techniques to determine these frequencies. BJT and MOSFET amplifiers will be considered. You will also learn the concepts that are pursued to design a wide band width amplifier. Following topics will be considered.

- Review of Bode plot technique.
- Ways to write the transfer (i.e., gain) functions to show frequency dependence.
- Band-width limiting at low frequencies (i.e., DC to $f_L$). Determination of lower band cut-off frequency for a single-stage amplifier – short circuit time constant technique.
- Band-width limiting at high frequencies for a single-stage amplifier. Determination of upper band cut-off frequency- several alternative techniques.
- Frequency response of a single device (BJT, MOSFET).
- Concepts related to wide-band amplifier design – BJT and MOSFET examples.

### 3.1 A short review on Bode plot technique

*Example:* Produce the Bode plots for the magnitude and phase of the transfer function $T(s) = \dfrac{10s}{(1+s/10^2)(1+s/10^5)}$, for frequencies between 1 rad/sec to $10^6$ rad/sec.

We first observe that the function has zeros and poles in the numerical sequence 0 (zero), $10^2$ (pole), and $10^5$ (pole). Further at ω=1 rad/sec i.e., lot less than the first pole (at ω=$10^2$ rad/sec), $T(s) \cong 10s$. Hence the first portion of the plot will follow the asymptotic line rising at 6 dB/octave, or 20 dB/decade, in the neighborhood of ω=1 rad/sec. The magnitude of *T(s)* in decibels will be approximately 20 dB at ω= 1 rad/sec.

The second asymptotic line will commence at the pole of $\omega=10^2$ rad/sec, running at -6 dB/octave slope relative to the previous asymptote. Thus the overall asymptote will be a line of slope zero, i.e., a line parallel to the ω- axis.

The third asymptote will commence at the pole $\omega=10^5$ rad/sec, running at -6 dB/Octave slope relative to the previous asymptote. The overall asymptote will be a line dropping off at -6 dB/octave beginning from $\omega=10^5$ rad/sec.

Since we have covered all the poles and zeros, we need not work on sketching any further asymptotes. The three asymptotic lines are now sketched as shown in figure 3.1.

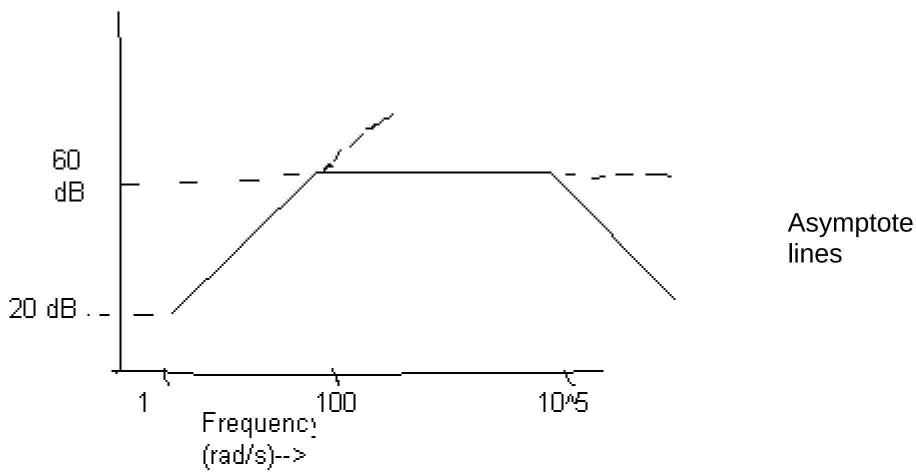



Figure 3.1: The asymptotic line plots for the *T(s)*.

The actual plot will follow the asymptotic lines being 3 dB below the first corner point (i.e.,at ω=100)i.e., 57 dB ,and 3 dB below the second corner point (i.e., ω=10^5), i.e. 57 dB. In between the two corner point the plot will approach the asymptotic line of constant value 60 dB. The magnitude plot is shown in figure 3.2.

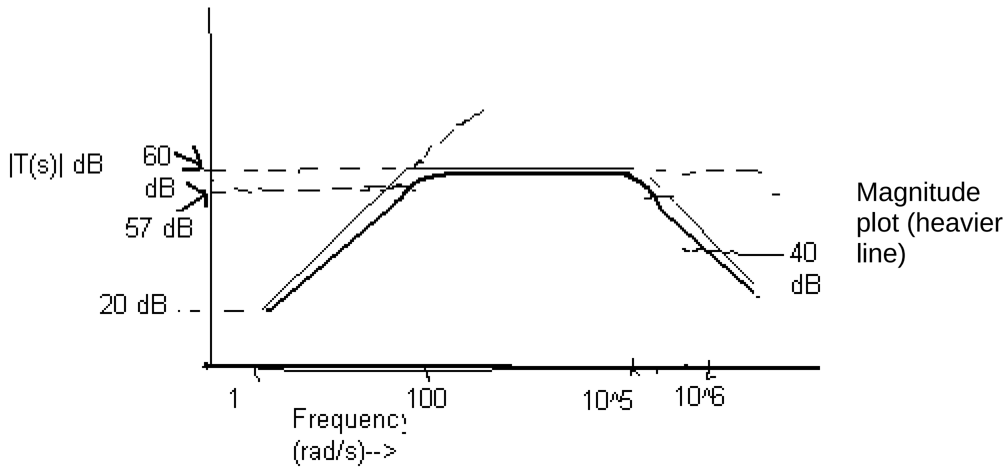



Figure 3.2:Bode magnitude plot for *T(s)*

For phase plot, we note that the 's' in the numerator will give a constant phase shift of +90$^o$ degrees (since $s \to j\omega \to 0 + j\omega,$ angle: $\tan^{-1}(\omega/0) \to \tan^{-1}(\infty) \to 90^o$), while the terms in the denominator will produce angles of $-\tan^{-1}(\omega/10^2)$, and $-\tan^{-1}(\omega/10^5)$ respectively. The total phase angle will then be:

$$\phi(\omega) = 90^o - \tan^{-1}(\omega/10^2) - \tan^{-1}(\omega/10^5) \qquad (3.1)$$

Thus at low frequency (<< 100 rad/sec), the phase angle will be close to 90$^o$. Near the pole frequency ω=100, a -45$^o$ will be added due to the ploe at making the phase angle to be close to +45$^o$. The phase angle will progressively decrease, because of the first two terms in φ(ω). Near the second pole ω=10$^5$, the phase angle will approach

$$\phi(\omega) = 90^o - \tan^{-1}(10^5/10^2) - \tan^{-1}(10^5/10^5) \cong 90^o - 90^o - 45^o$$ i.e., -45$^o$ degrees.

(The student in encouraged to draw the curve)

3.2 Simplified form of the gain function of an amplifier revealing the frequency response limitation

3.2.1 Gain function at low frequencies

Electronic amplifiers are limited in frequency response in that the response magnitude falls off from a constant mid-band value to lower values both at frequencies below and above an intermediate range (the mid-band) of frequencies. A typical frequency response curve of an amplifier system appears as in figure3.3.

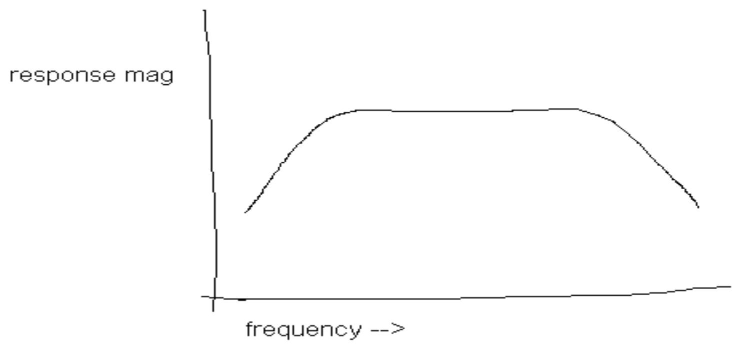


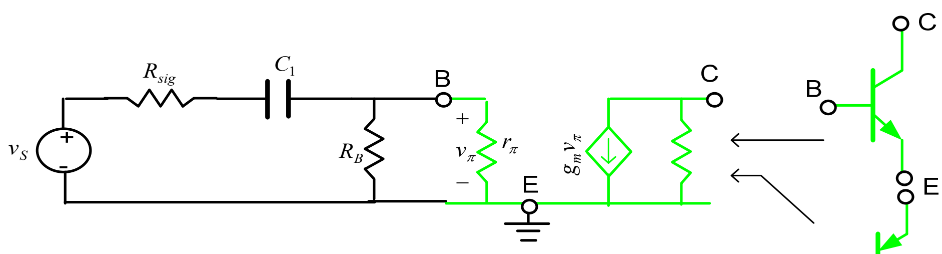

Figure 3.3: Typical frequency response function magnitude plot for an electronic amplifier

Using the concepts of Bode magnitude plot technique, we can approximate the low-frequency portion of the sketch above by an expression of the form $T_L(s) = \frac{Ks}{s+a}$, or $T_L(s) = \frac{K}{1+a/s}$. In this $K$ and $a$ are constants and $s=j\omega$, where $\omega$ is the (physical, i.e., measurable) angular frequency (in rad/sec). In either case, when the signal frequency is very much smaller than the pole frequency 'a', the response $T_L(s)$ takes the form $Ks/a$. This function increases progressively with the frequency $s = j\omega$, following the asymptotic line with a slope of +6 dB per octave. At the pole frequency 'a', the response will be 3 dB below the previous asymptotic line, and henceforth follow an asymptotic line of slope (-6+6=0) of zero dB/ octave. Thus $T_L(s)$ will remain constant with frequency, assuming the mid-band value. Note that $T_L(s)$ is a first order function in 's' (a single time-constant function).

The *frequency* at which the magnitude plot reaches *3 dB below the mid-band* (i.e., the flat portion of the magnitude response curve) gain value is known as the *-3 dB frequency* of the gain function. For the low-frequency segment (i.e., $T_L$(s)) of the magnitude plot this will be designated by $f_L$ (or $\omega_L = 2\pi f_L$).

In a practical case the function *$T_L(s)$* may have several poles and zeros at low frequencies. The pole which is *closest* to the flat mid-band value is known as the *low frequency dominant pole* of the system. Thus it is the pole of *highest* magnitude among all the poles and zeros at low frequencies. Numerically the *dominant* pole differs from the -3 dB frequency. But for simplicity, one can approximate dominant pole to be of same value as the -3dB frequency. The -3dB frequency at low frequencies is also sometimes referred to as the *lower* cut-off frequency of the amplifier system.

The frequency response limitation at low frequency occurs because of coupling and by-pass capacitors used in the amplifier circuit. For single-stage amplifiers, i.e., CE, CB..CS,CG amplifiers these capacitors come in series with the signal path (i.e., they form a loop in the signal path), and hence impedes the flow of signal coupled to the internal nodes ( i.e., BE nodes of the BJT, GS nodes of the MOSFET) of the active device. The students can convince themselves by considering the simple illustration presented in figure 3.4.




$$v_\pi = v_S \frac{(r_\pi \parallel R_B) s C_1}{1 + s C_1 (R_{sig} + r_\pi \parallel R_B)} \qquad T_1(s) = \frac{v_\pi}{v_S} \rightarrow \frac{Ks}{s + a}$$

Figure 3.4: Illustrating the formation of s *zero* in the voltage transfer function because of a capacitor in the signal loop. The controlling voltage $v_\pi$ for the VCCS has a *zero* because of the presence of $C_1$.

### 3.2.2 Gain function at high frequencies

A similar scenario exists for the response at high frequencies. By considering the graph in Fig.3.3 at frequencies beyond (i.e., higher than) the mid-band segment, we can propose the form of the response function as: $T_H(s) = \frac{K}{s+b}$. $K$ and $b$ are constants. Other alternative forms are: $T_H(s) = \frac{K_o b}{s+b}$, or $T_H(s) = \frac{K_o}{1+s/b}$. Note that in all cases, for frequencies << the pole frequency 'b', the response function assumes a constant value (i.e., the mid-band response). For $T_H(s)$, which is a *first-order* function, the frequency $b$ becomes the *-3db frequency* for high frequency response, or the *upper* cut-off frequency. When there are several poles and zeros in the high frequency range, the pole with the *smallest* magnitude and hence closest to the mid-band response zone is referred to as the *high frequency dominant* pole. Again, numerically the high frequency dominant pole will be different from the upper cut-off frequency. But in most practical cases, the difference is small. In case the high frequency response has several poles and zeros, one can formulate the function as

$$T_H(s) = \frac{(1+s/\omega_{z1})(1+s/\omega_{z2})..}{(1+s/\omega_{p1})(1+s/\omega_{p2})..} \qquad (3.2)$$

In an integrated circuit scenario coupling or by-pass capacitors are absent. The frequency dependent gain function (i.e., *transfer function*) is produced because of the intrinsic capacitances (*parasitic* capacitances) of the devices. As a consequence the zeros occur at very high frequencies and only one of the poles fall in the signal frequency range of interest, with the other poles at substantially higher frequencies. Thus if $\omega_{p1}$ is the pole of smallest magnitude, the amplifier will have $\omega_{p1}$ as the dominant pole. In such case $T_H(s) \cong \frac{\omega_{p1}}{s+\omega_{p1}}$, and $\omega_{p1}$ will also be the -3 dB or upper cut-off frequency of the system. Otherwise, the -3 dB frequency $\omega_H$ can be calculated using the formula[1]

$$\omega_H \cong 1 \Big/ \left[\left(\frac{1}{\omega_{p1}^2} + \frac{1}{\omega_{p2}^2} + ...\right) - 2\left(\frac{1}{\omega_{z1}^2} + \frac{1}{\omega_{z2}^2} + ..\right)\right]^{1/2} \qquad (3.3)$$

3.2.3 Simplified (first order) form of the amplifier gain function

[1] Sedra and Smith, "Microelectronic Circuits", 6th edn., ch.9, p.722, Oxford University Press, ©2010.

Considering the discussions in sections 3.2.1-2 we can formulate the simplified form of the amplifier gain function can then be considered as :

$$A(s)=A_M\, F_L(s)\, F_H(s) \qquad (3.4)$$

In (3.4), $A_M$ is independent of frequency, $F_L$ has a frequency dependence of the form $s/(s+w_L)$, while $F_H$ has a frequency dependence of the form $w_H/(s+w_H)$. Thus for frequencies higher than $w_L$ and for frequencies lower than $w_H$ the gain is close to $A_M$. This is a constant gain and the frequency band $w_H$ - $w_L$ is called the mid-band frequencies. So in the mid-band frequencies the gain is constant i.e., $A_M$. At frequencies << $w_L$, $F_L(s)$ increases with frequency (re: Bode plot) by virtue of the '*s*' in the numerator, at 6dB/octave. As the frequency increases, the rate of increase slows down and the Bode plot merges with the constant value $A_M$ shortly after $w=w_L$. At $w=w_L$ the response falls 3 dB below the initial asymptotic line of slope 6dB/octave. Similarly, as frequency increases past $w_H$, the response A(s) tends to fall off, passing through 3dB below $A_M$ (in dB) at $w=w_H$ and then following the asymptotic line with slope *minus* 6dB/octave drawn at $w=w_H$. It is of interest to be able to find out these two critical frequencies for basic single stage amplifiers implemented using BJT or MOSFET.

## 3.3 Simplified high-frequency *ac* equivalent circuits for BJT and MOSFET devices

It can be noted that for amplifiers implemented in integrated circuit technology only the upper cut-off frequency $w_H$ is of interest. To investigate this we must be familiar with the ac equivalent circuit of the transistor at high frequencies. The elements that affect the high frequency behavior are the parasitic capacitors that exist in a transistor. These arise because a transistor is made by laying down several semiconductor layers of different conductivity (i.e., p-type and n-type materials). At the junction of each pair of dissimilar layers, a capacitance is generated. We will consider the simplified high-frequency equivalent circuits for the BJT and MOSFET as shown in Figs.3.5-3.6. In these models each transistor is assigned with only two parasitic capacitance associated with its internal nodes. These arise out of the semiconductor junctions that are involved in building the transistor. For the BJT, the base material produces a small resistance $r_x$, which assumes importance for high (signal) frequency applications (signal processing). The models for N-type (i.e., NPN, NMOSFET) and P-type (i.e., PNP, PMOSFET) transistors are considered same. In more advanced models (used in industries) more number of parasitic capacitances and resistances are employed.

### 3.3.1 High frequency response characteristics of a BJT

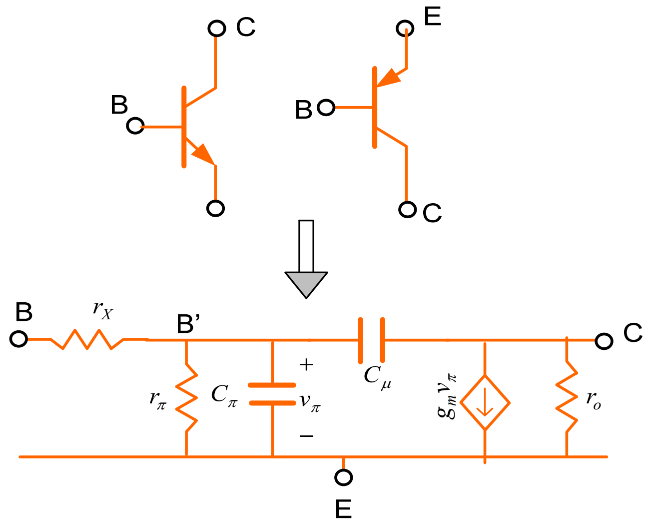



Figure 3.5: Simplified ac equivalent circuit for a BJT device for high signal frequency situation.

An important performance parameter of a BJT device is the small signal short circuit current gain of the device under CE mode of operation. Thus in Fig.3.5, if we insert an ac current source at terminal **B** and seek the ac short-circuit output current at node C, we can construct the CE ac equivalent circuit as in Fig.3.6. The short-circuit current gain $i_o/i_i$ of the device can be derived from the KCL equations (returning terminal C to *ac* ground) at the nodes B and B'. Writing $g_i = 1/r_i$ in general, we get

$$i_i = g_x(v_B - v_\pi), \quad 0 = -g_x v_B + (g_x + g_\pi + sC_\pi + sC_\mu)v_\pi \qquad (3.5)$$

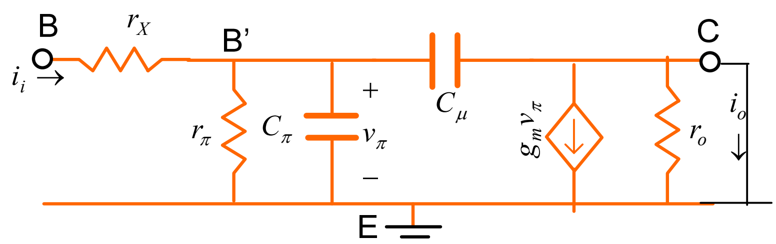


Figure 3.6: Configuring the BJT device for CS short-circuit current gain calculation.

Solving for $v_\pi$ and noting that at C node (which is short circuited for *ac*) $i_o = -g_m v_\pi + sC_\mu$, we can finally derive the *short-circuit* current gain of the BJT under CE mode of operation as:

$$h_{fe}(s) = {}^{i_o}\!/_{i_i} = -\frac{-(g_m - sC_\mu)g_x}{g_x(g_\pi + s(C_\pi + C_\mu))} = -\frac{g_m - sC_\mu}{C_\pi + C_\mu}\frac{1}{s + \dfrac{1}{r_\pi(C_\pi + C_\mu)}} \qquad (3.6)$$

Eq.(3.6) represents a transfer function with a low-frequency (i.e., $\omega \approx 0$) value of $h_{fe}|_{\text{low-frequency}} = h_{fe}(s)|_{s=j\omega=0} = -g_m r_\pi = -\beta$, the familiar symbol for the current gain of a BJT in CE operation. Because $C_\mu$ is very small, the *zero* of $h_{fe}(j\omega)$ i.e., $g_m/C_\mu$ lies at very high frequencies. Using the symbol $h_{fe}$ (0) for low-frequency ($\omega \approx 0$) value of $h_{fe}$, and for frequencies << $\omega_z = {}^{g_m}\!/_{C_\mu}$, the Bode magnitude plot of $h_{fe}$ appears as in Fig. 3.7.

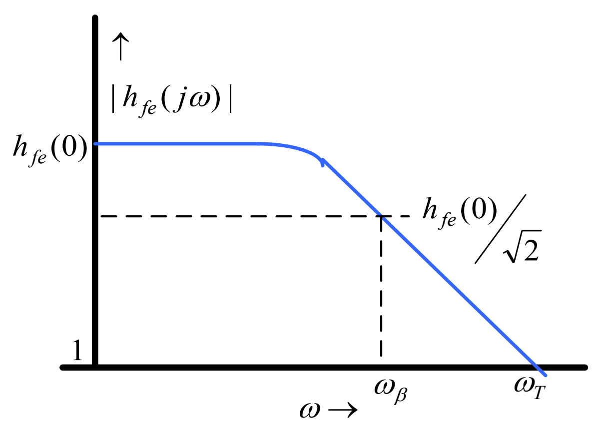



Figure 3.7: The Bode magnitude plot of | $h_{fe}(j\omega)$ |.

It is observed that at the frequency $\omega_\beta = {}^{1}\!/_{r_\pi(C_\pi + C_\mu)}$, $|h_{fe}|$ drops to ${}^{h_{fe}(0)}\!/_{\sqrt{2}}$, i.e., -3db below $h_{fe}(0)$. This frequency is known as the $\beta$ cut-off frequency for the BJT under CE mode of operation.

At frequencies much higher than $\omega_\beta$, $h_{fe}(j\omega)$ changes as (see eq.(3.6)) $\approx -\dfrac{g_m}{j\omega(C_\pi + C_\mu)}$. This reaches a magnitude of *unity* (i.e. =1), at a frequency

$$\omega_T = {g_m}/{(C_\pi + C_\mu)} \qquad (3.7)$$

This is known as the *transition frequency* of the BJT for operation as CE amplifier. The *transition frequency* $\omega_T = 2\pi f_T$ is a very *important* parameter of the BJT for high-frequency applications. For a given BJT, the high-frequency operational limit of the device can be increased by increasing $\omega_T$ via an increase in $g_m$ , the *ac* transconductance of the BJT. This, however, implies an increase in the DC bias current (since $g_m = I/V_T$) and hence an increase in the DC power consumption of the system. Recalling the relation $g_m r_\pi = \beta+1$, we can deduce that

$$\omega_T = (1+\beta)\omega_\beta = (1+h_{fe}(0))\omega_\beta \qquad (3.8)$$

In real BJT devices $C_\pi >> C_\mu$, and $C_\pi + C_\mu >> C_\mu$. Hence, the *zero* frequency $\omega_z = {g_m}/{C_\mu}$ will be >> the *transition* frequency $\omega_T$. Since $|h_{fe}(j\omega)|$ becomes <1 beyond $\omega_T$, the *zero* frequency bears no practical interest.

### 3.3.2 High frequency response characteristics of a MOSFET

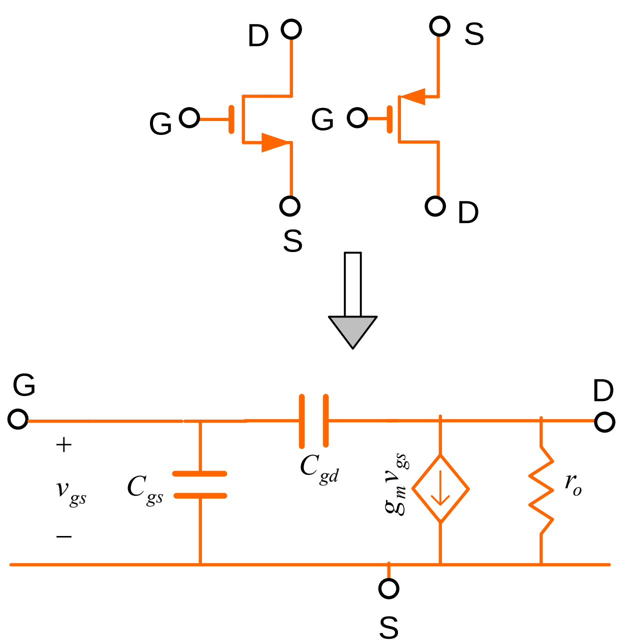



Figure 3.8: Simplified ac equivalent circuit for a MOSFET device for high signal frequency situation.

A simplified ac equivalent circuit for the MOSFET is shown in figure 3.8. The body terminal (B) for the MOSFET , and the associated parasitic capacitances as well as the body transconductance ($g_{mb}$) have not been shown. By following a procedure similar to that of a BJT, it can be shown that the *short circuit current gain* of the MOSFET configured as a CS amplifier is given by

$i_o/i_i = -\dfrac{g_m - sC_{gd}}{s(C_{gs} + C_{gd})}$ which can be approximated as $\dfrac{i_o}{i_i} = -\dfrac{g_m}{s(C_{gs} + C_{gd})}$ for frequencies well below the *zero* frequency $g_m/sC_{gd}$.

Under the above assumption the frequency at which the magnitude of the current gain becomes *unity* i.e., the *transition frequency*, becomes:

$$\omega_T = \frac{g_m}{(C_{gs} + C_{gd})} \tag{3.9}$$

The *transition frequency* of a MOSFET is a very important parameter for high frequency operation. This can be increased via an increase in $g_m$ with the attendant increase in the DC bias current and hence increase in DC power dissipation.

3.4 Calculation of $\omega_L$ – the lower cut-off frequency (*Short Circuit Time Constant method*)

Figure 3.9(a) depicts a typical CE-BJT amplifier with coupling capacitors $C_1$, $C_3$, and the by-pass capacitor $C_E$. Each of these capacitors fall in the *signal path* for the operation of the amplifier and hence influences the voltage gain function in terms of introducing several poles and zeros in the gain transfer function.

A simplified method to determine *the poles* is to consider *only one of the capacitors* effective at a time and assume that the other capacitors behave approximately as *short circuits*. Because only one capacitor is present in the system, it is easy to determine the *time constant* parameter of the *associated ac equivalent* circuit. Hence the method is known as *short circuit time constant* method (SCTC). Figures 3.9(b)-(d) show the three *ac* equivalent circuits under the assumption of only one of $C_1$, $C_2$, or $C_E$ present in the circuit. The location(s) to be used for the calculation of the equivalent *Thevenin* resistance for each of the capacitors ($C_1$, $C_E$, $C_3$) are shown in blue lines on the diagrams.

The internal capacitances of the BJT offer very high impedance at low frequencies and hence they are considered as open circuits (so these are not shown).

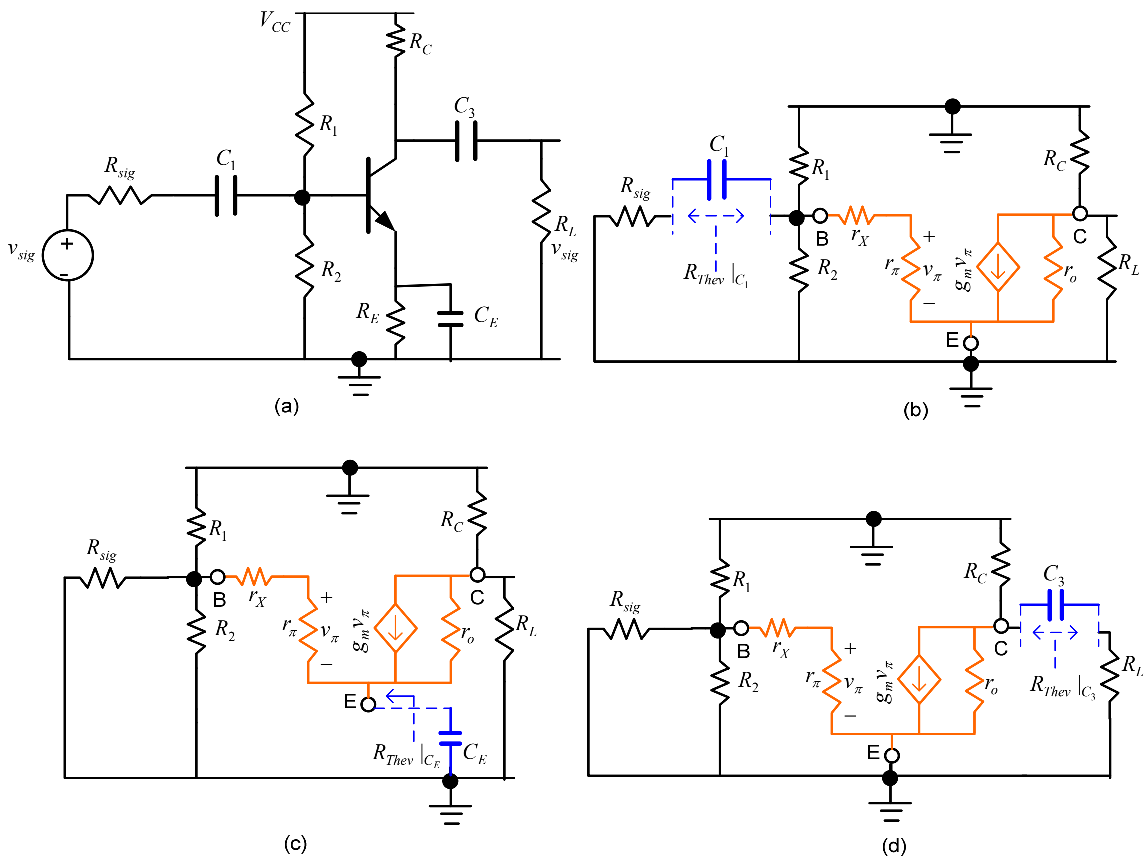



Figure 3.9: (a) Schematic of a CE amplifier with four resistor biasing; (b) the *ac* equivalent circuit with $C_E$ ,$C_3$ as *short circuits*; (c) the *ac* equivalent circuit with $C_1$ ,$C_3$ as *short circuits*, and (d) the *ac* equivalent circuit with $C_E$ ,$C_1$ as *short circuits*.

Analysis of the equivalent circuit in Fig.3.9(b) is straightforward. By inspection, the *Thevenin* resistance associated with $C_1$ is $R_{Th1} = R_{sig} + R_1 \| R_2 \| (r_x + r_\pi)$, where the notation $\|$ implies *in parallel with*. The associated time-constant is $C_1R_{Th1}$, and the corresponding *pole-frequency* is $\omega_{L1} = 1/(C_1R_{Th1})$. Similarly, the *Thevenin* resistance for $C_3$ is $R_{Th3} = R_C \| r_o + R_L$ (see Fig.3.9(d)). The corresponding *pole-frequency* is $\omega_{L3} = 1/(C_3R_{Th3})$. The calculation of the *Thevenin* resistance associated with $C_E$ can be simplified considerably by assuming $r_o$ as *inifinity*. Then by inspection (see Fig.3.9(c)), $R_{ThE} = R_E \| (\frac{r_\pi + r_x + R_1 \| R_2 \| R_{sig}}{1 + h_{fe}})$. The corresponding *pole-frequency* is $\omega_{LE} = 1/(C_ER_{ThE})$.

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A more adventurous student may discard the assumption of $r_o \rightarrow$ *infinity* and proceed to set up a 3 by 3 nodal admittance matrix (NAM) (see *Appendix III*) by using the substitutions

$r_{\pi p} = r_\pi + r_x$, $R_{sp} = R_{sig} \| R_1 \| R_2$, $R_{cp} = R_c \| R_L$, and by inserting a *dummy* current source $i_x$ at the node labeled as E in Fig. 3.9(c). The NAM will appear as

$$\begin{bmatrix} g_E + g_{\pi p} + g_o & -g_{\pi p} & -g_o \\ -g_{\pi p} & g_{\pi p} + g_{sp} & 0 \\ -g_o & 0 & g_o + g_{cp} \end{bmatrix} \begin{bmatrix} V_E \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} i_x + g_m v_\pi \\ 0 \\ -g_m v_\pi \end{bmatrix} \qquad (3.10)$$

In the above $g_E = 1/R_E$,, $g_{\pi p} = 1/r_{\pi p}$, $g_o = 1/r_o$, and so on, have been used. With the further assumption (it is very good if $r_x$ is << $r_\pi$) of $v_\pi = V_B - V_E$, the matrix equation (3.10), becomes, after rearrangement (i.e., changing sides):

$$\begin{bmatrix} g_E + g_{\pi p} + g_o + g_m & -g_{\pi p} - g_m & -g_o \\ -g_{\pi p} & g_{\pi p} + g_{sp} & 0 \\ -g_o - g_m & 0 + g_m & g_o + g_{cp} \end{bmatrix} \begin{bmatrix} V_E \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} i_x \\ 0 \\ 0 \end{bmatrix} \qquad (3.11)$$

Then $R_{ThE}$ is given by $V_E/i_x$. The result is (using *Maple* program code):

$$R_{ThE} = \frac{(g_{\pi p} + g_{sp})(g_o + g_{cp})}{g_E g_{\pi p} g_o + g_E g_{\pi p} g_{cp} + g_E g_{sp} g_o + g_E g_{sp} g_{cp} + g_{\pi p} g_{sp} g_o + g_{\pi p} g_{sp} g_{cp} + g_o g_{\pi p} g_{cp} + g_o g_{sp} g_{cp} + g_m g_{sp} g_{cp}}$$

Now introducing the assumption $g_o \rightarrow 0$ (i.e., $r_o \rightarrow$ *infinity*), one will get

$$R_{ThE} = \frac{(g_{\pi p} + g_{sp}) g_{cp}}{g_E g_{\pi p} g_{cp} + g_E g_{sp} g_{cp} + g_{\pi p} g_{sp} g_{cp} + g_m g_{sp} g_{cp}} \qquad (3.12)$$

Substituting back in terms of the resistance notations, i.e., $g_E = 1/R_E$,, $g_{\pi p} = 1/r_{\pi p}$, $g_o = 1/r_o$, and so on, one can get

$$R_{ThE} = \frac{(R_{sp} + r_{\pi p}) R_E}{R_{sp} + r_{\pi p} + R_E + g_m r_{\pi p} R_E} \qquad (3.13)$$

Using $g_m\, r_{\pi p} = h_{fe}$, and simplifying, one arrives at $R_{ThE} = \dfrac{(R_{sp} + r_{\pi p}) R_E / (1 + h_{fe})}{(R_{sp} + r_{\pi p})/(1 + h_{fe}) + R_E}$, i.e.,

$$R_{ThE} = R_E \| (\frac{R_{sp} + r_{\pi p}}{1 + h_{fe}}) = R_E \| (\frac{r_\pi + r_x + R_1 \| R_2 \| R_{sig}}{1 + h_{fe}}) .$$

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The overall lower -3 dB frequency is calculated approximately by the formula $\omega_L = \omega_{L1} + \omega_{L3} + \omega_{LE}$. If out of the several poles of the low-frequency transfer function $F_L(s)$, one is very large compared to all other poles and zeros, the overall lower -3 dB frequency $\omega_L$ becomes $\cong$ dominant pole (i.e., largest of $\omega_{L1}$ or $\omega_{LE}$ or $\omega_{L3}$).

If the numerical values of the various pole frequencies are known (by exact circuit analysis followed by numerical computation), the lower 3-dB frequency can be calculated approximately by a formula of the form $\omega_L = \sqrt{\omega_1^2 + \omega_2^2 + \omega_3^2 + ...}$ where, $\omega_1$, $\omega_2$ , .. are the individual pole frequencies and the zero-frequencies are very small compared with the pole frequencies.

*Example 3.4.1*: Consider the following values in a BJT amplifier.
$R_{sig}$ =50Ω, $R_B$=$R_1$||$R_2$ = 10 kΩ, $r_\pi$ =2500, $r_x$ =25Ω, $h_{fe}$ =100 and $R_E$ =1kΩ, $R_C$ =1.5kΩ, $R_L$ =3.3 kΩ, $V_A$ =20 volts, $I_C \approx$ 1 mA. Further, $C_1$ =1uF and $C_E$= 10uF and $C_3$ = 1uF. What is $\omega_L$ ?

According to above formulas, $R_{Th1}$ = 2.05kΩ , $R_{ThE}$ =25.25Ω and $R_{Th3}$ =1.39kΩ+3.3kΩ = 4.69kΩ. Then $\omega_{L1}$ = 487.8 rad/s, $\omega_{LE}$ = 3.96E3 rad/sec and $\omega_{L3}$ = 213.2 rad/sec. Then , $\omega_L = \sqrt{487.8^2 + 3.96E3^2 + 213.2^2} = 3.9956E3$, which is pretty close to $\omega_{LE}$.

*Example 3.4.2*: What if , $\omega_L$= 1800 rad/sec is to be designed? We can assume, for example, $\omega_{L1} = 0.8\omega_L$, , $\omega_{LE}$= $\omega_{L2}$= $0.1\omega_L$ and $\omega_{L3}$=$0.1\omega_L$. Then, design the values of the capacitors $C_1$, $C_E$ and $C_3$. The student can try other relative allocations too.

3.5: Calculation of $\omega_H$ – the higher cut-off frequency

Several alternative methods exist in the literature. The following are presented.

3.5.1 <u>Open circuit time-constant (OCTC) method</u>

This is similar to the case as with low frequency response. For high frequency operation, we are interested in the capacitor which will have lower reactance value since this capacitance will start to degrade the high frequency response sooner than the other. Thus, we can consider one capacitor at a time and assume that the other capacitors are too small and have reasonably high reactance values (for a C, the reactance is $\propto$ 1/C) so that they could be considered as open circuits. We then calculate the associated time constant. Thus the method is named as *open circuit time constant* (OCTC) method. We shall illustrate the method using the case of a CE BJT amplifier.

Consider figure 3.10(a) which shows the *ac* equivalent circuit of the CE-BJT amplifier of Fig.3.9(a). The *high frequency* equivalent circuit for the BJT has been included. The coupling and by-pass capacitors are assumed as *short circuits* for *high frequency* situation.

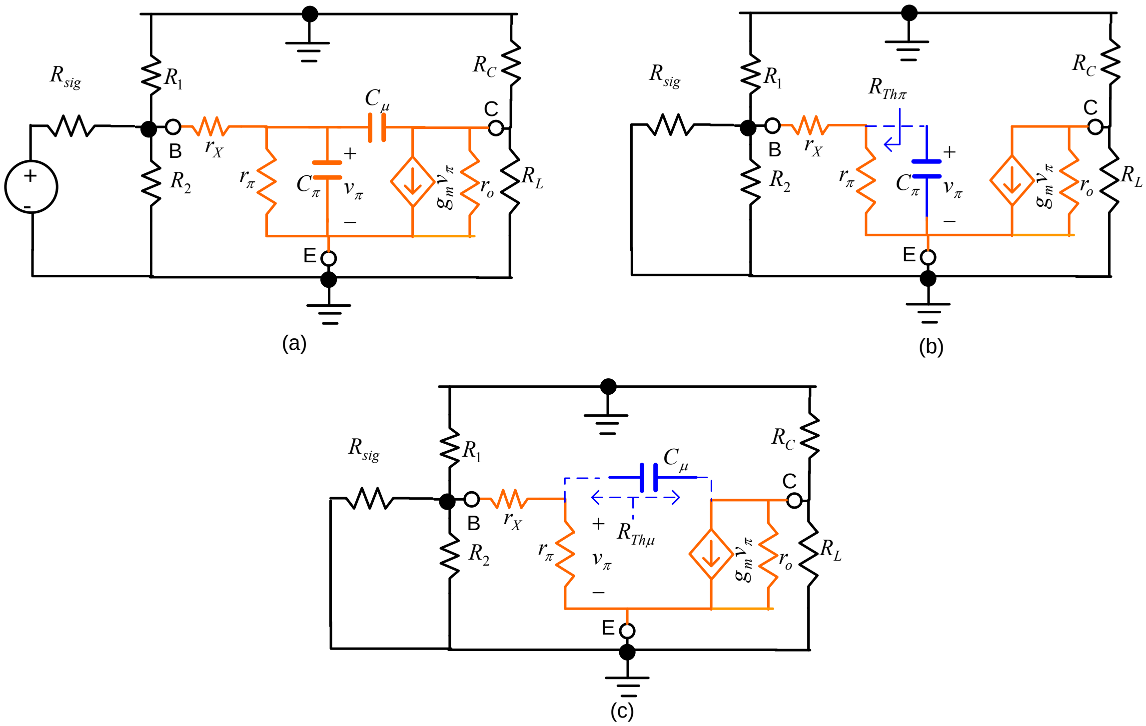



Figure 3.10: (a) high frequency equivalent circuit of the amplifier in Fig.3.9(a); (b) the equivalent circuit with $C_\mu$ open; (c) the equivalent circuit with $C_\pi$ open.

*Case 1: Cμ open*

The ac equivalent circuit to determine the *Thevenin* equivalent resistance $R_{Th\pi}$ across $C_\pi$ is shown in Fig.3.10(b). By simple inspection $R_{Th\pi} = r_\pi \,||\, (r_x + R_{sig} \,||\, R_1 \,||\, R_2)$

The high frequency pole due to this situation is $\omega_{H1} = 1/(C_\pi R_{Th\pi})$.

*Case 2: $C_\pi$ open*

We now need to determine the Thevenin equivalent resistance $R_{Th\mu}$ across $C_\mu$. The associated equivalent circuit is shown in Fig.3.10(c). We can use a dummy signal current source $i_x$ and carry out few steps of basic circuit analysis (see Fig.3.11).

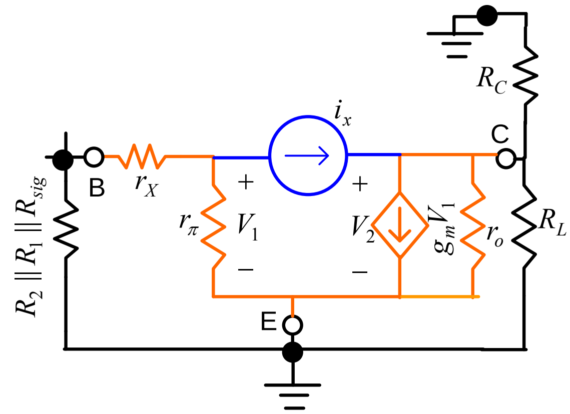



Figure 3.11: Equivalent circuit for calculating $R_{Th\mu}$

KCL at $V_1$ node gives: $V_1 G'_S + i_x = 0, \quad G'_S = \dfrac{1}{r_\pi \| (r_x + R_{sig} \| R_1 \| R_2)} = 1/R'_S$ (3.13a)

KCL at $V_2$ node gives: $-i_x + g_m V_1 + V_2 G'_L = 0, \quad G'_L = \dfrac{1}{r_o \| R_C \| R_L} = 1/R'_L$ (3.13b)

Solving (3.13(a),(b)) for $V_1$ and $V_2$ we can find $R_{Th\mu} = (V_2 - V_1)/i_x = R_s' + (1 + g_m R_s')R_L'$

The high frequency pole for $C_\mu$ is $\omega_{H2} = 1/\, C_\mu R_{Th\mu}$.

When the two pole frequencies are comparable in values, the upper -3 dB frequency is given approximately by $\omega_H = 1/(C_\pi R_{Th\pi} + C_\mu R_{Th\mu}) = 1/(\tau_1 + \tau_2)$ (3.14)

If, however, the two values are widely apart (say, by a factor of 5 or more), the upper -3 dB frequency will be called as the *dominant high frequency pole* and will be equal to the lesser of $\omega_{H1}$ and $\omega_{H2}$.

3.5.2 Application of Miller's theorem

This theorem helps simplifying the ac equivalent circuit of the BJT CE amplifier by removing the $C_\mu$ capacitor, which runs between two floating nodes (i.e., between the base side to the collector side). In principle, if an admittance $Y_3$ runs between nodes 1 and 2 with $Y_1$ at node 1 (to ground) and $Y_2$ at node 2 (and ground) and if K is the voltage gain ($V_2/V_1$) between nodes 1 and 2, then $Y_3$ can be split into two parts – one being in parallel with $Y_1$ with a value $Y_3$ (1-K) and another becoming in parallel with $Y_2$ with a value $(1-1/K)Y_3$. The theorem can be applied to all cases of floating elements connected between two nodes in a system.

As a result of this principle, the high frequency equivalent circuit of the CE BJT amplifier (see Fig.3.5) simplifies to figure 3.12.

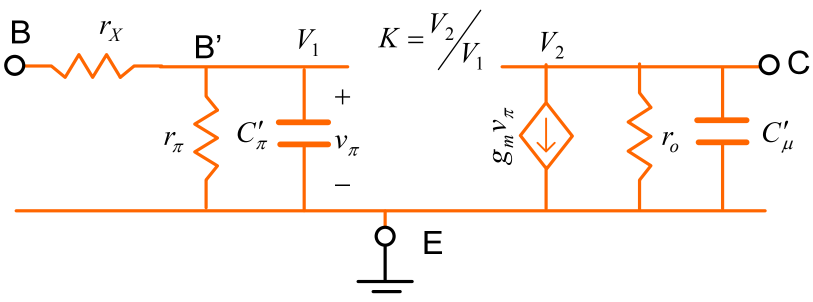



Figure 3.12: High frequency equivalent circuit of a CE-BJT device after application of Miller's theorem

In the above $C'_\pi = C_\pi + C_\mu(1-K)$, $C'_\mu = C_\mu(1-1/K)$. Figure 3.12 is a simple two node circuit and can be conveniently analyzed. Miller's theorem is very effective when the admittances $Y_1$ and $Y_2$ have one of their ends grounded for ac signals. After the equivalent circuit is simplified as above, one can apply the OCTC method to determine the high frequency poles.

*Example 3.5.2.1*: Consider Fig.3.14(a) which shows the high frequency equivalent circuit for the amplifier in Fig.3.10(a). Figure 3.13(b) is a reduced form of Fig.3.14(a), suitable for analysis by nodal matrix formulation. In Fig.3.14(b), the following expressions hold:

$$R_S = r_X + R_{sig} \| R_1 \| R_2, \quad i_S = v'_S / R_S, \quad v'_S = v_S \frac{R_B}{R_{sig} + R_B}, R_B = R_1 \| R_2$$

(a) Calculate the *low frequency* voltage gain between nodes labeled 1 and 2 in Fig.3.14(b). This amounts to ignoring the presence of $C_\pi$ and $C_\mu$ for this calculation. Let this gain be K.

(b) Use Miller's theorem to find the new circuit configuration in the form of Fig.3.12.

(c) Apply OCTC method to derive the pole frequencies for high frequency response of the amplifier.

(d) Given that $R_{sig}$ =50Ω, $R_1$=82 kΩ, $R_2$=47 kΩ, $r_X$ =10Ω, $g_m$ =40 m mhos, $r_o$=50 kΩ, $R_C$ =2.7 kΩ, $R_L$=4.7 kΩ, $R_E$=270 Ω, $h_{fe}$ (0)=$h_{FE}$= 49, $C_\pi$ =1.2 pF, $C_\mu$ =0.1 pF, find the high frequency poles by using

(i) The OCTC method discussed in section 3.5.1.

(ii) The OCTC method after applying Miller's theorem (introduced in section 3.5.2).

*Solution* :

By inspection of Fig.3.14(b), the voltage gain K=$v_2/v_1$=$v_2/v_\pi = -g_m r_o \| R_C \| R_L$ =-66.14

Further, recalling $g_m r_\pi = h_{fe}(0)$ =49, we get $r_\pi$ =1225 Ω.

**Part (c)** : $R_{Th\pi} = r_\pi \| (r_x + R_{sig} \| R_1 \| R_2)$ =57.1 Ω. $\omega_{H1}$ =1/($C_\pi R_{Th\pi}$) =$14.59\times10^9$ rad/s

$R_{Th\mu}$ = $R_s' + (1+g_m R_s')R_L'$ (see derivations in 3.5.1)=5503.5 Ω, $\omega_{H2}$ = 1/ $C_\mu R_{Th\mu}$ =$1.817\times10^9$ rad/sec. The above is the result by OCTC method *without taking recourse to Miller's theorem.*

**Part (b):** Using Miller's theorem the equivalent circuit of Fig.3.14(b) transform to figure 3.13. Using K=-66.14, the new capacitance values become:

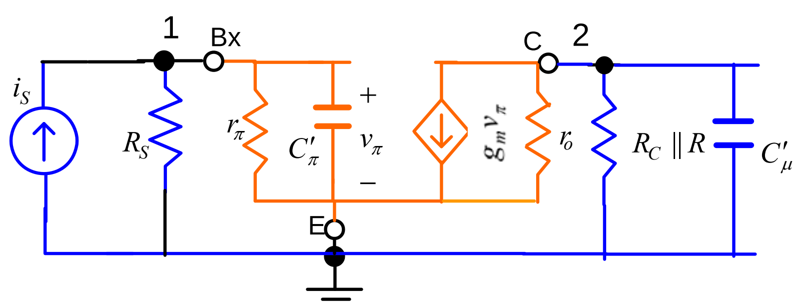



Figure 3.13: Transformed ac equivalent circuit after application of Miller's theorem

$$C'_\pi = C_\pi + C_\mu(1-K) = 7.93\times10^{-12}, \quad C'_\mu = C_\mu(1-1/K) = 1.01\times10^{-13}$$

Now, we need to recalculate the Thevenin resistances $R'_{Th\pi} = r_\pi \| R_S = 59.9 \| 1225 = 57\,\Omega$,

$R'_{Th\mu} = R_C \| R_L \| r_o = 1658\,\Omega$.

Then, $\omega'_{H1} = {}^{1}\!\!\diagup_{C'_{\pi} R'_{Th\pi}} = 2.21 \times 10^{9}$ rad/sec, and $\omega'_{H2} = {}^{1}\!\!\diagup_{C'_{\mu} R'_{Th\mu}} = 5.94 \times 10^{9}$ rad/sec.

**Part d(i):** Since $\omega_{H2}$ is << $\omega_{H1}$, $\omega_{H2}$ is the *dominant* high-frequency pole for the amplifier. Hence the -3dB frequency is approximately $1.817 \times 10^{9}$ rad/sec, i.e., the *dominant* high frequency pole of the system.

**Part d(ii):** Since the $\omega'_{H1}$, $\omega'_{H2}$ values are not widely apart (i.e., differ by a factor of 5 or more), we will estimate the higher -3dB frequency by adopting the formula

$$\omega'_{H} = \frac{1}{C'_{\pi} R'_{Th\pi} + C'_{\mu} R'_{Th\mu}} = 1.6118 \times 10^{9} \text{ rad/sec.}$$

**Part d(i) revised**: If we had used the same formula with the values found in **part (c),** we would get $\omega_{H} = \dfrac{1}{C_{\pi} R_{Th\pi} + C_{\mu} R_{Th\mu}} = 1.6157 \times 10^{9}$ rad/sec.

*Conclusion:* In practice the more conservative value should be chosen, i.e., the upper -3 dB frequency will be $1.6118 \times 10^{9}$ rad/sec.

### 3.5.3 <u>Transfer function analysis method</u>

The time-constant methods discussed above lead to determination of the poles (and zeros) of the transfer function. We need yet to determine the mid-band gain function and combine this with the poles (and zeros) to form the overall transfer function for high (or low) frequency response of the amplifier. This involves a two-step process. Alternatively, we can apply nodal matrix analysis (see *Appendix III*) technique to determine the overall transfer function as the first operation. Since the high frequency equivalent circuit of the transistor has two capacitors, the transfer function will be of order two in 's' (i.e., second degree in 's'). For such a transfer function there exists a simple rule to determine the *dominant* pole of the transfer function. Further, by ignoring the frequency dependent terms (i.e., coefficients of $s$ ) in the transfer function, we can derive the mid-band gain of the system. Hence the transfer function analysis opens up avenues for deriving several important network functions for the amplifier on hand. Following examples illustrate several cases.

*Example 3.5.3.1: Derivation of the voltage gain transfer function of a CE-BJT amplifier*

Consider the ac equivalent circuit of the BJT CE amplifier of Fig.3.10(a) which is re-drawn in Fig.3.14(a) for convenience. Applying Thevenin-Norton equivalence principle to the left of the arrow-head (in blue), we can convert the circuit in Fig.3.14(a) to

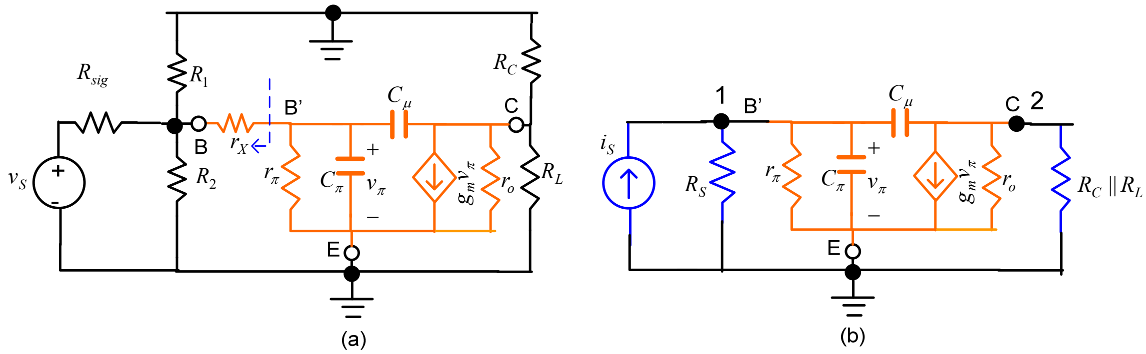



Figure 3.14: (a) high frequency equivalent circuit of the complete amplifier, (b) the equivalent circuit adjusted for nodal admittance matrix (NAM) analysis.

Fig.3.14(b), which is convenient for nodal analysis (i.e., it has fewer number of nodes and is driven by a current source. In Fig. 3.13(b) note that

$$R_S = r_X + R_{sig} \| R_1 \| R_2, \quad i_S = v_S' / R_S, \quad v_S' = v_S \frac{R_B}{R_{sig} + R_B}, R_B = R_1 \| R_2 \qquad (3.15)$$

Using the notations $g_S$= $1/R_S$, $g_\pi$ =$1/r_\pi$, $R_C'$=$R_C\|R_L$, $g_C'$ =$1/R_C'$,and so on, we can formulate the admittance matrix for the two nodes (labeled as 1,2) system in Fig.3.13(b). Thus,

$$\begin{bmatrix} g_S + g_\pi + s(C_\pi + C_\mu) & -sC_\mu \\ -sC_\mu & g_o + g_C' + sC_\mu \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} i_S \\ -g_m v_\pi \end{bmatrix} \qquad (3.16)$$

Noting that $v_\pi = V_1$, and bringing $-g_m v_\pi$ ($=g_m V_1$) on the left side inside the matrix,

$$\begin{bmatrix} g_S + g_\pi + s(C_\pi + C_\mu) & -sC_\mu \\ g_m - sC_\mu & g_o + g_C' + sC_\mu \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} i_S \\ 0 \end{bmatrix} \qquad (3.17)$$

The output signal voltage of the system is $v_o = V_2$, given by

$$v_o = V_2 = \frac{\begin{vmatrix} g_S + g_\pi + s(C_\pi + C_\mu) & i_S \\ g_m - sC_\mu & 0 \end{vmatrix}}{\begin{vmatrix} g_S + g_\pi + s(C_\pi + C_\mu) & -sC_\mu \\ g_m - sC_\mu & g_o + g_C' + sC_\mu \end{vmatrix}} \qquad (3.18)$$

On carrying out the tasks of evaluation of the determinants, one can find

$$v_o = \frac{-i_S(g_m - sC_\mu)}{s^2C_pC_\mu + s(g_SC_\mu + g_\pi C_\mu + g_oC_\mu + g'_CC_\mu + g_mC_\mu + g_oC_\pi + g'_CC_\pi) + g_Sg_o + g_Sg'_C + g_\pi g_o + g_\pi g'_C}$$

Writing $D(s)$ =

$s^2C_pC_\mu + s(g_SC_\mu + g_\pi C_\mu + g_oC_\mu + g'_CC_\mu + g_mC_\mu + g_oC_\pi + g'_CC_\pi) + g_Sg_o + g_Sg'_C + g_\pi g_o + g_\pi g'_C$ , and

substituting for $i_S$ we find $v_o = -\dfrac{g_m - sC_\mu}{D(s)}\dfrac{1}{r_X + R_{sig} \parallel R_B}\dfrac{R_B}{R_B + R_{sig}}v_S$ (3.19)

The voltage gain transfer function is: $T(s) = \dfrac{v_o}{v_S} = -\dfrac{g_m - sC_\mu}{D(s)}\dfrac{1}{r_X + R_{sig} \parallel R_B}\dfrac{R_B}{R_B + R_{sig}}$ (3.20)

We can make two important derivations from the result in (3.20)

- *Low-frequency voltage gain $A_M$* : This is obtained from (3.20) on approximating *$\omega \to 0$,* i.e., *$s \to 0$.*

  Thus $A_M = -\dfrac{g_m}{g_Sg_o + g_Sg'_C + g_\pi g_o + g_\pi g'_C}\dfrac{R_B}{(r_X + R_{sig} \parallel R_B)(R_B + R_{sig})}$ (3.21)

Assuming the component and device parameter values as in *Example 3.5.2.1*, we get $A_M$ =-63.12

- High frequency dominant pole $\omega_{HD}$

When the denominator *D(s)* of the transfer function is of second order, one can estimate the dominant pole from the least valued root of the denominator polynomial. The technique is explained as follows.

We can write *D(s)* in the form : $s^2 + bs + c$. Then the high frequency dominant pole (i.e., the pole with the least magnitude of all the high frequency poles) is given by $\omega_{pD} = c/b$ i.e., the *ratio of the constant term* in *D(s)* to the *coefficient of the s term* in *D(s)*. This follows easily by writing $s^2 + bs + c = (s+\alpha)(s+\beta) = s^2 + (\alpha+\beta)s + \alpha\beta.$ Then, $\alpha + \beta = b \cong \beta$, if $\alpha \ll \beta$. Further, from $\alpha\beta = c$, we deduce $\alpha = c/\beta \cong c/b$, as the dominant pole of the voltage gain transfer function. Hence,

$\omega_{HD} = (g_Sg_o + g_Sg'_C + g_\pi g_o + g_\pi g'_C)/(g_SC_\mu + g_\pi C_\mu + g_oC_\mu + g'_CC_\mu + g_mC_\mu + g_oC_\pi + g'_CC_\pi)$ (3.22)

Consider the voltage gain function of the CE BJT amplifier in *Example 3.5.2.1* as an illustration. We can find $\omega_{HD}$ approximate the by substituting the values for the pertinent parameters $g_S$, $g_o$, and so on. Thus, the upper cut-off frequency (i.e., upper -3dB frequency) is calculated as 1.6158 $\times 10^9$ rad/sec. The student may compare this value with the values obtained previously, i.e.,

$1.6157\times10^{9}$ rad/sec (using OCTC method), and $1.6118\times10^{9}$ rad/sec (using Miller's theorem followed by OCTC method), respectively.

*Example 3.5.3.2: Derivation of the voltage gain transfer function* (VTF) *of a* CB-BJT *amplifier*

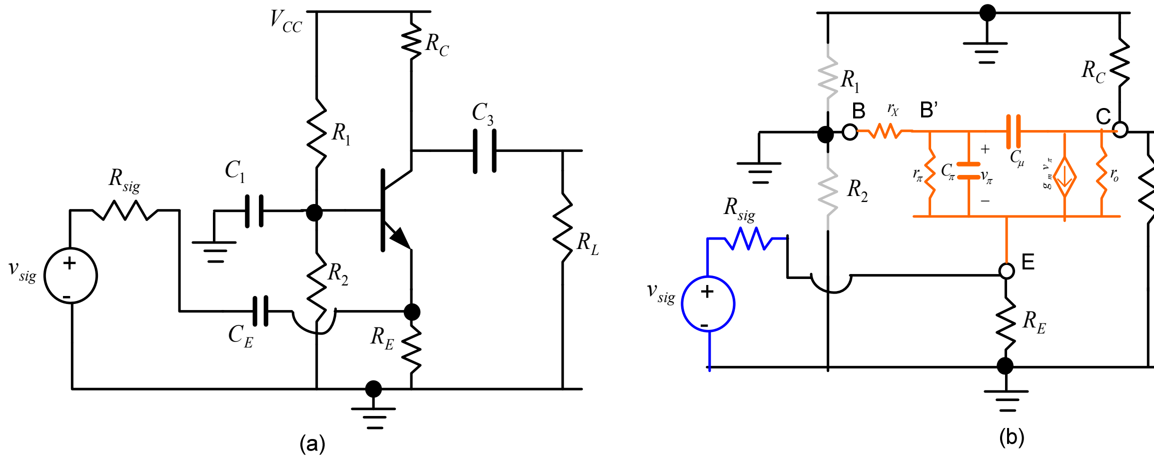



Figure 3.15: (a) Schematic of the CB-BJT amplifier stage, (b) the high-frequency equivalent circuit.

Figures 3.15(a)-(b) depict respectively the schematic and high frequency equivalent circuits of the CB amplifier stage. All coupling and by-pass capacitors behave as short circuits and hence they do not appear in Fig.3.15(b).

For nodal admittance matrix (NAM) formulation we need to transform the voltage source with its internal resistance to its Norton equivalent, i.e., a signal current source $i_S = {}^{v_{sig}}\!/\!_{R_{sig}}$ in parallel with $R_{sig}$ (the student is encouraged to complete this part to modify Fig.3.15(b)). The NAM by inspection will be:

$$\begin{bmatrix} g_{sig}+g_E+g_\pi+g_o+sC_\pi & -g_\pi-sC_\pi & -g_o \\ -g_\pi-sC_\pi & g_x+g_\pi+sC_\pi+sC_\mu & -sC_\mu \\ -g_o & -sC_\mu & g_o+g'_C \end{bmatrix}\begin{bmatrix} v_E \\ v_{B'} \\ v_C \end{bmatrix}=\begin{bmatrix} i_S \\ 0 \\ -g_m v_\pi \end{bmatrix} \quad (3.23)$$

But $v_\pi = v_{B'} - v_E$. Hence (3.23) reduces to

$$\begin{bmatrix} g_{sig}+g_E+g_\pi+g_o+sC_\pi & -g_\pi-sC_\pi & -g_o \\ -g_\pi-sC_\pi & g_x+g_\pi+sC_\pi+sC_\mu & -sC_\mu \\ -g_m-g_o & g_m-sC_\mu & g_o+g'_C+sC_\mu \end{bmatrix}\begin{bmatrix} v_E \\ v_{B'} \\ v_C \end{bmatrix}=\begin{bmatrix} i_S \\ 0 \\ 0 \end{bmatrix} \quad (3.24)$$

In order to compare the performance of the CB and CE BJT amplifiers, it will be convenient to assume $r_X$ as negligible. Then node *B'* will be at *zero* ac potential and (3.24) will approximate to (by discarding the *row* and *column* associated with the ***B'*** node)

$$\begin{bmatrix} g_{sig}+g_E+g_\pi+g_o+sC_\pi & -g_o \\ -g_m-g_o & g_o+g'_C+sC_\mu \end{bmatrix}\begin{bmatrix} v_E \\ v_C \end{bmatrix}=\begin{bmatrix} i_S \\ 0 \end{bmatrix} \qquad (3.25)$$

The VTF is given by

$v_o/v_{sig} = v_C/v_{sig} =$

$$\frac{g_{sig}(g_m+g_o)}{s^2C_\pi C_\mu + s[C_\pi(g_o+g'_C)+C_\mu(g_\pi+g_E+g_{sig}+g_o)]+g_o(g_{sig}+g_\pi+g_E+g'_C-g_m)+g'_C(g_{sig}+g_E+g_o+g_\pi)}$$

Using the VTF we can deduce

- The mid-band gain (i.e., $s \rightarrow j\omega$, $\omega \rightarrow 0$) for CB mode of operation is:

$$A_M\,|_{CB} = \frac{g_{sig}(g_m+g_o)}{g_o(g_{sig}+g_\pi+g_E+g'_C-g_m)+g'_C(g_{sig}+g_E+g_o+g_\pi)} \qquad (3.26)$$

- The dominant high frequency pole for CB mode of operation is

$$\omega_{HD}|_{\mathrm{CB}} = \frac{g_o(g_{sig}+g_\pi+g_E+g'_C-g_m)+g'_C(g_{sig}+g_E+g_o+g_\pi)}{C_\pi(g_o+g'_C)+C_\mu(g_\pi+g_E+g_{sig}+g_o)} \qquad (3.27)$$

*Example 3.5.3.3: Determine the mid-band gains and dominant high frequency poles of the BJT amplifier operated as CE (see Fig.3.9a) and CB (see Fig.3.15a) modes of operation. The devices, the DC biasing conditions, and the circuit components are assumed identical for both the configurations.*

[Given $R_{sig}$ =50Ω, $R_1$=82 kΩ, $R_2$=47 kΩ, $R_E$=270 Ω, $R_C$ =2.7 kΩ, $R_L$=4.7 kΩ

$g_m$ =40 m mhos, $r_o$=50 kΩ, $r_X$ =10Ω ,$h_{fe}(0)$=$h_{FE}$ = 49

$C_\pi$ =1.2 pF, $C_\mu$ =0.1 pF]

On substitution into the expressions (3.26) and (3.27), we get: $A_M$ =57.17 v/v, $\omega_{HD}|_{\mathrm{CB}}$=4.5796 $\times 10^9$ rad/sec.

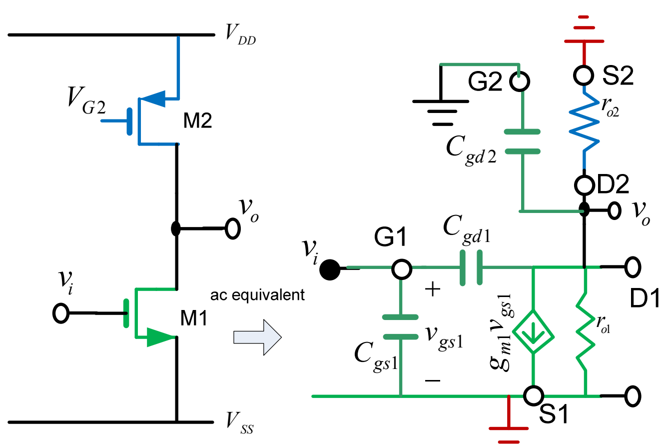
***Note I**:* We observed that the CB-BJT amplifier has somewhat *lower voltage gain* (i.e., 57.17) compared with that of CE-BJT (i.e., 63.12 in magnitude) amplifier, but has a *higher high-frequency bandwidth* (~ dominant high-frequency pole=$4.5796\times 10^9$ rad/sec) compared with that of CE-BJT (i.e., $1.6158\times 10^9$ rad/sec) amplifier.

***Note II***: It is known that the CE-BJT amplifier as moderate to high (kΩ to tens of kΩ) input resistance, as well as moderately high (kΩ) output resistance. In comparison, a CB-BJT amplifier has low (Ω to tens of Ω) input resistance while a moderately high (kΩ) output resistance.

***Note III:*** A CB-BJT is preferred over a CE-BJT amplifier for most *radio-frequency* (MHz and above) applications because it has a *higher* high-frequency bandwidth (for identical device and biasing conditions), and affords to provide better impedance matching at the input with the *radio-frequency* (RF) source ($R_{sig}$ in the range of 50Ω to 75Ω).

*Example 3.5.3.4: Derivation of the voltage gain transfer function* (VTF) *of a* CS-MOSFET *amplifier with active load*

Consider figures 3.16(a)-(b) which depict respectively the schematic of a CS-MOSFET amplifier and the associated high frequency equivalent circuit.




(a) (b)

Figure 3.16: (a) Schematic of a CS-MOSFET amplifier with active load, (b) associated high-frequency equivalent circuit.

Using a dummy input current source $i_x$ (when not shown explicitly, the student should adopt this principle), the admittance matrix is

$$\begin{bmatrix} s(C_{gs1}+C_{gd1}) & -sC_{gd1} \\ -sC_{gd1} & g_{o1}+g_{o2}+s(C_{gd1}+C_{gd2}) \end{bmatrix} \begin{bmatrix} v_i \\ v_o \end{bmatrix} = \begin{bmatrix} i_x \\ -g_{m1}v_{gs1} \end{bmatrix} \tag{3.28}$$

After re-arranging (the student is suggested to work out the details), we can find the VTF given by

$$\frac{v_o}{v_i} = \frac{\begin{vmatrix} s(C_{gs1}+C_{gd1}) & i_x \\ g_m - sC_{gd1} & 0 \end{vmatrix}}{\begin{vmatrix} i_x & -sC_{gd1} \\ 0 & g_{o1}+g_{o2}+s(C_{gd1}+C_{gd2}) \end{vmatrix}} \tag{3.29}$$

*Exercise* 3.5.3.4*:* Can you sketch the Bode magnitude plot for the above VTF? Assume, $g_m$=250 μA/V, $r_{o1}$, $r_{o2}$ =1 MΩ (each), $C_{gd1}$, $C_{gd2}$= 20 fF (each f→ femto i.e., $10^{-15}$), $C_{gs1}$= 100 fF.

*Example 3.5.3.5: Derivation of the voltage gain transfer function* (VTF) *of a* CG-MOSFET *amplifier.*

Figures 3.17(a)-(b) show the schematic diagrams of a CG-MOSFET amplifier (M1) under *ideal* loading and *practical* loading conditions respectively. Figure 3.17(c) depict the high frequency equivalent circuit. Note that now we need to include the body-

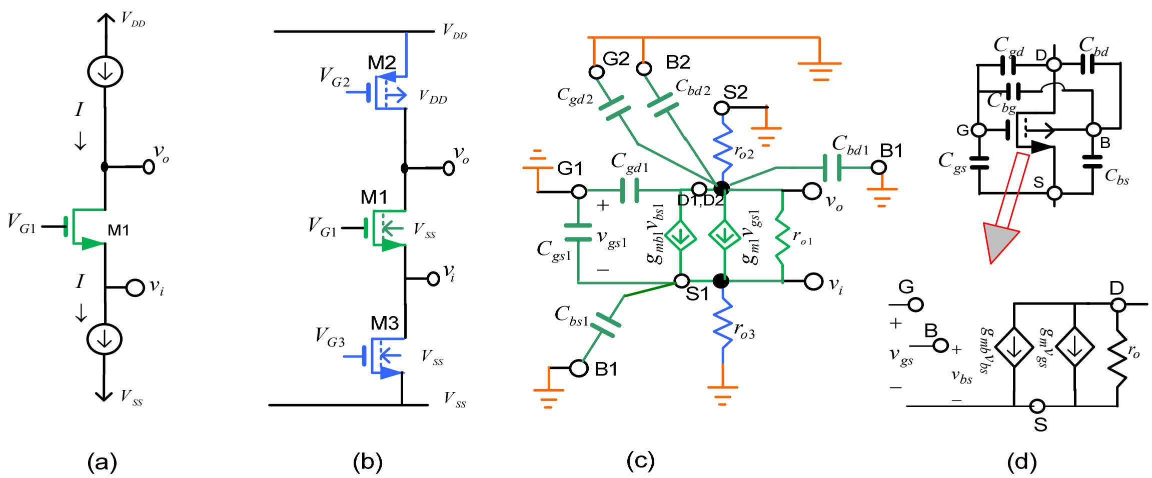



Figure 3.17: (a) Schematic of the CG-MOSFET amplifier with ideal load and bias source, (b) practical biasing and active load arrangement, (c) high-frequency equivalent circuit model for the CG-MOSFET stage, (d) more complete high-frequency model for a MOSFET.

transconductance $g_{mb}$ of the MOSFET, since the topology is such (i.e., the amplifying device is in a *totem pole* connection) that the source and body terminals of the amplifying transistor M1 cannot be connected together!

It may be noted that despite the complicated look of Fig.3.17(c), there are only two signal nodes in the equivalent circuit. Thus the nodal admittance matrix will appear as:

$$\begin{bmatrix} g_{o3}+g_{o1}+s(C_{gs1}+C_{bs1}) & -g_{o1} \\ -g_{o1} & g_{o1}+g_{o2}+s(C_{bd1}+C_{gd1}+C_{bd2}+C_{gd2}) \end{bmatrix} \begin{bmatrix} v_i \\ v_o \end{bmatrix} = \begin{bmatrix} i_x+g_{m1}v_{gs1}+g_{mb1}v_{bs1} \\ -g_{m1}v_{gs1}-g_{mb1}v_{bs1} \end{bmatrix}$$

Writing $C_1$=$C_{gs1}$+$C_{bs1}$, $C_2$=$C_{gd1}$+$C_{bd1}$+$C_{gd2}$+$C_{bd2}$, and observing that the gate and body terminals of the MOSFETs are at *zero* ac potentials so that $v_{gs1}$=-$v_{s1}$=-$v_i$, $v_{bs1}$=-$v_{s1}$=-$v_i$, we can re-write the admittance matrix equation as

$$\begin{bmatrix} g_{o3}+g_{o1}+sC_1 & -g_{o1} \\ -g_{o1} & g_{o1}+g_{o2}+sC_2 \end{bmatrix} \begin{bmatrix} v_i \\ v_o \end{bmatrix} = \begin{bmatrix} i_x-g_{m1}v_i-g_{mb1}v_i \\ g_{m1}v_i+g_{mb1}v_i \end{bmatrix} \quad (3.30)$$

Re-arranging, we get

$$\begin{bmatrix} g_{o3}+g_{o1}+g_{m1}+g_{mb1}+sC_1 & -g_{o1} \\ -g_{m1}-g_{mb1}-g_{o1} & g_{o1}+g_{o2}+sC_2 \end{bmatrix} \begin{bmatrix} v_i \\ v_o \end{bmatrix} = \begin{bmatrix} i_x \\ 0 \end{bmatrix} \quad (3.31)$$

The VTF is given by:

$$\frac{v_o}{v_i} = \frac{\begin{vmatrix} g_{o3}+g_{o1}+g_{m1}+g_{mb1}+sC_1 & i_x \\ -g_{m1}-g_{mb1}-g_{o1} & 0 \end{vmatrix}}{\begin{vmatrix} i_x & -g_{o1} \\ 0 & g_{o1}+g_{o2}+sC_2 \end{vmatrix}} \quad (3.32)$$

*Exercise* 3.5.3.5-I: *Find the explicit expression for the* VTF *using* (3.32).

*Exercise* 3.5.3.5-II: *Using* (3.31), *and the knowledge that the driving point impedance* (DPI) *at the input is given by* $Z_{dpi} = {}^{v_i}\!/_{i_x}$, *find the expression for the* $Z_{dpi}$.

*Exercise* 3.5.3.5-III: *Given that* $r_{o1} = r_{o3} = 1M\Omega, r_{o2} = 2M\Omega, g_{m1} = 200\mu\, mho, g_{mb1} = 0.2g_{m1}$, $C_{gs1} = 100fF, C_{bs1} = 20fF, C_{gd1} = 5fF, C_{bd1} = 20fF, C_{gd2} = 10fF, C_{bd2} = 15fF$ *find the expressions for the* VTF *and the* $Z_{dpi}$

*Exercise* 3.5.3.5-IV: *Find the expression for the trans-impedance function* TIF $(= {}^{v_o}\!/_{i_x})$ *for the amplifier stage*.

*Example 3.5.3.6*: *Derivation of the voltage gain transfer functions* (VTF) *for a CC-BJT and CD-MOSFET amplifiers*

Figures 3.18(a)-(d) show respectively the schematic and high frequency equivalent circuits of a CC-BJT and CD-MOSFET amplifiers. The biasing/loading is arranged by

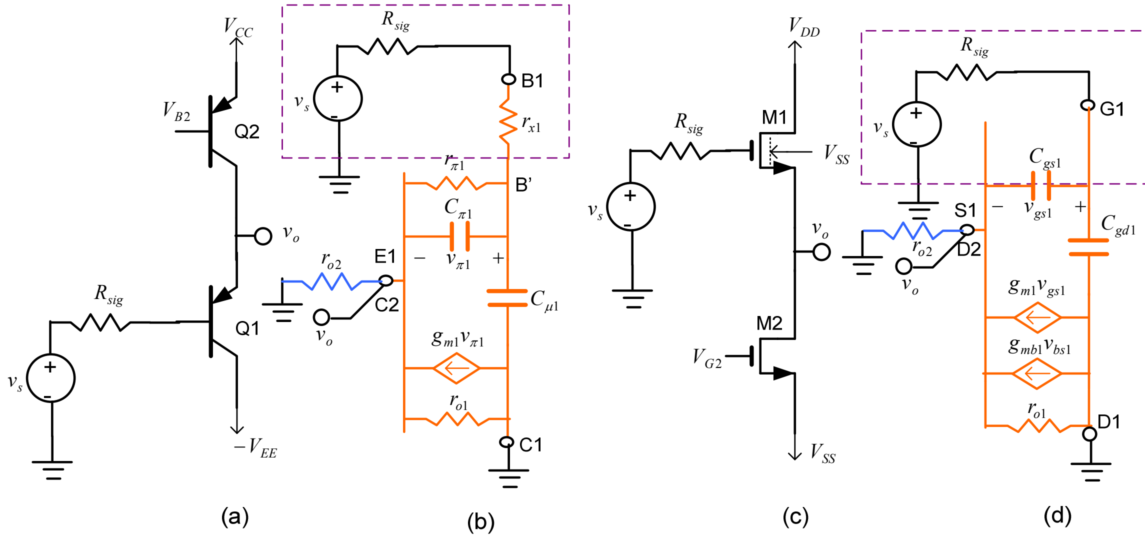



Figure 3.18: (a) Schematic of a CC-BJT amplifier, (b) high frequency equivalent circuit of the CC amplifier, (c) Schematic of a CD-MOSFET amplifier, (d) high frequency equivalent circuit of the CD amplifier

employing active resistors (Q2 in Fig.3.18(a), and M2 in Fig.3.18(c)). It may be noted that since one terminal of the capacitor $C_{\mu 1}$ (and of $C_{gd1}$) is grounded for *ac*, neither of these capacitors are subjected to the *Miller* effect magnification, as are in the cases with CE (BJT and CS (MOSFET) amplifiers.

*The student is encouraged to complete the analysis and derive the expressions for the VTF* $v_o/v_s$ *of the CC and the CD amplifiers.Remember that for the MOSFET the body terminal is connected to a DC voltage.*

3.6: Wide band multi-stage amplifiers

3.6.1: CE-CB Cascode BJT amplifier

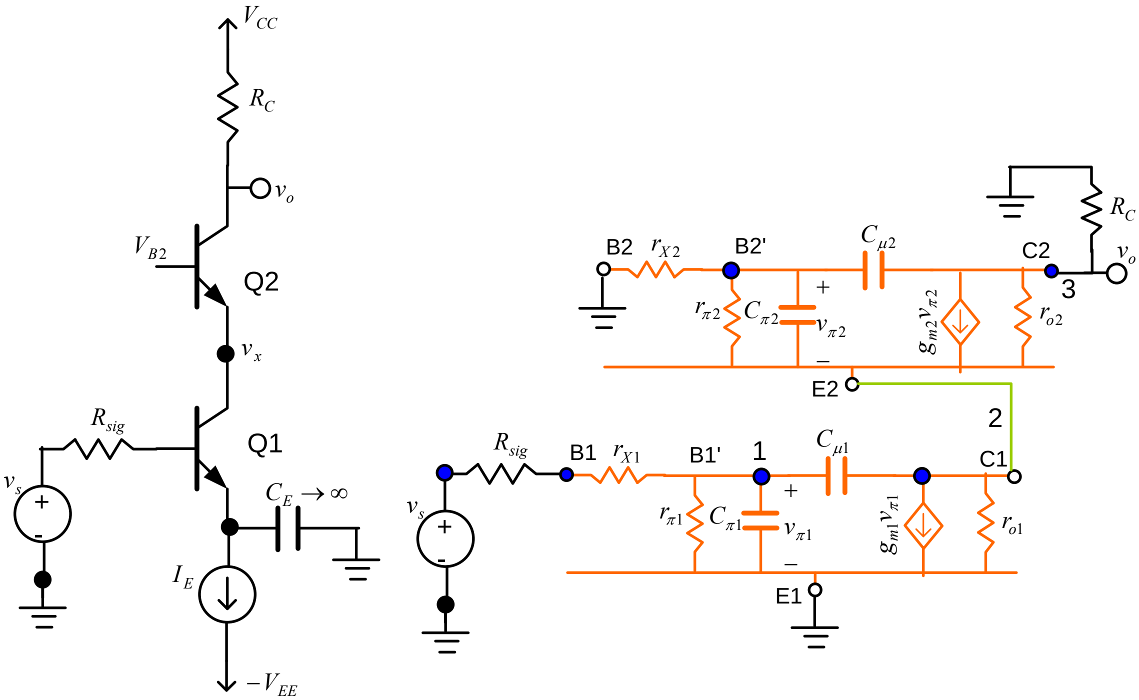
It has been known that a CE stage has high voltage gain and high input resistance while a CB stage has high voltage gain with low input resistance. For high frequency operation CE amplifier will have a smaller band width than the CB amplifier since in the former the capacitance $C_\mu$ is magnified due to Miller effect. This has a corresponding effect on reducing the band width. In the CB stage, since the base is at ac ground, the capacitance $C_\mu$ has already one terminal to ground and is not subjected to Miller effect magnification. So the CB stage affords to higher operating band width.

It is interesting to consider a composite amplifier containing the CE and CB stages so that advantages of both the configurations could be shared. That is what happens in a cascode amplifier where the CE stage receives the input signal, while the CB stage acts as a low resistance load for the CE stage. Because of the low resistance load, the Miller effect magnification of the $C_\mu$ capacitor in the CE stage is drastically reduced. Because of the low resistance load, however, the voltage gain in the CE stage drops. But it is adequately compensated by the large voltage gain of the CB stage which works from a low input resistance to a high output resistance.

*3.6.1.1: Analysis for the bandwidth and mid-band gain*

The schematic connection of a cascode amplifier and the associated ac equivalent circuit are in Figures 3.19(a)-(b). The equivalent circuit has *six* nodes. Converting the signal source in series with $R_{sig}$ and $r_{X1}$ to its Norton equivalent will bring the number of nodes to *four*. For the *common base* transistor Q2 we can ignore $r_{X2}$ as very small (~ zero). Thus the number of nodes reduce to *three*. These are labeled as nodes 1,2,3 in Fig.3.19(b). For quick hand analysis we can adopt the following simplification procedure.

We can now introduce the assumption that $r_{o2}$ is very large (→infinity), and remove it. This separates node#2 and #3 with the controlled current source $g_{m2}v_{\pi 2}$ split into two parts- one running from node#3 to ground, and the other from ground to node#2.




(a)

Figure 3.19: (a) Schematic of a CE-CB cascode amplifier, (b) high frequency equivalent circuit.

The simplified equivalent circuit appears in figure 3.20.

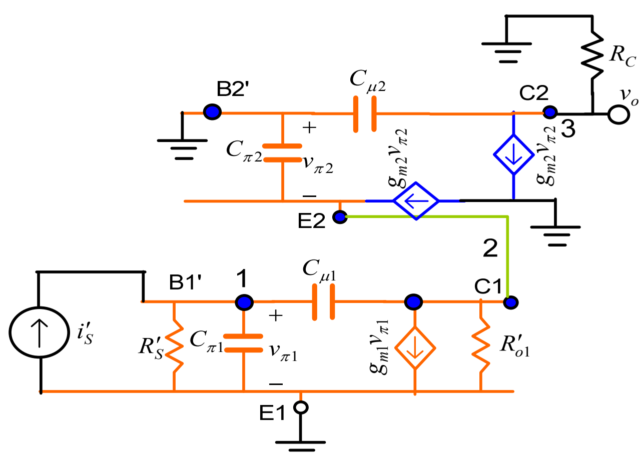



Figure 3.20: Simplified high frequency equivalent circuit for the CE-CB cascode amplifier.

In Fig.3.20 we have used $R'_S = (R_{sig} + r_{X1}) \| r_{\pi1}$, $R'_{o1} = r_{o1} \| r_{\pi2}$.

A careful scrutiny of the controlled current source $g_{m2}v_{\pi2}$ between ground to node #2 and the fact that the controlling voltage $v_{\pi2}$ is also effective across the same pair of nodes with the positive terminal at ground (node labeled $B'_2$) reveals that the controlled current source $g_{m2}v_{\pi2}$ can be equivalently replaced by a conductance of $g_{m2}$ (i.e., resistance $1/g_{m2}$) connected across node#2 and ground. This observation leads to the equivalent circuit shown in figure 3.21.

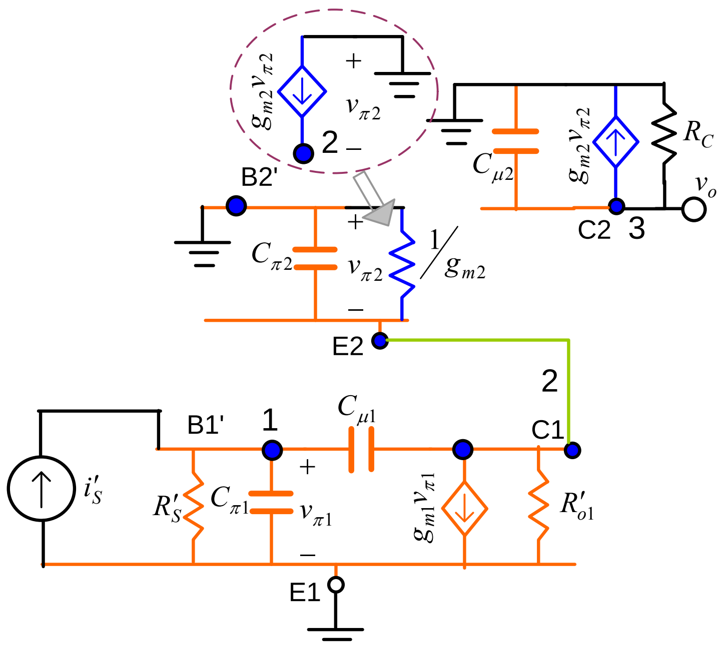



Figure 3.21: Conversion of Fig.3.20 after relocation of the controlled current source segments $g_{m2}v_{\pi2}$ each.

Now we can apply the Miller effect magnification consideration to the floating capacitor $C_{\mu1}$ running between nodes #1 and #2. The voltage gain between these nodes is (ignoring the feed-forward current through $C_{\mu1}$ which is a very small capacitance) approximately given by the product of $-g_{m1}$ and $R'_{o1} \| 1/g_{m2}$. Considering the fact that $R'_{o1} = r_{o1} \| r_{\pi2} \approx r_{\pi2}$, and that $r_{\pi2} >> 1/g_{m2}$, we can approximate $R'_{o1} \| 1/g_{m2}$ as $\approx 1/g_{m2}$. Hence the Miller magnification turns out to be $K \approx -g_{m1}/g_{m2}$, which is very close to $-1$ when the transistors Q1 and Q2 are matched to each other. By the principle of Miller effect the floating capacitor $C_{\mu 1}$ can now be replaced by two grounded capacitor of value $C_{\mu 1}(1\text{-}K)$, i.e., 2 $C_{\mu 1}$ at node #1, and $C_{\mu 1}(1\text{-}1/K)$, i.e., 2 $C_{\mu 1}$ at node#2. This leads to the final simplified form of the equivalent circuit as in figure 3.22.

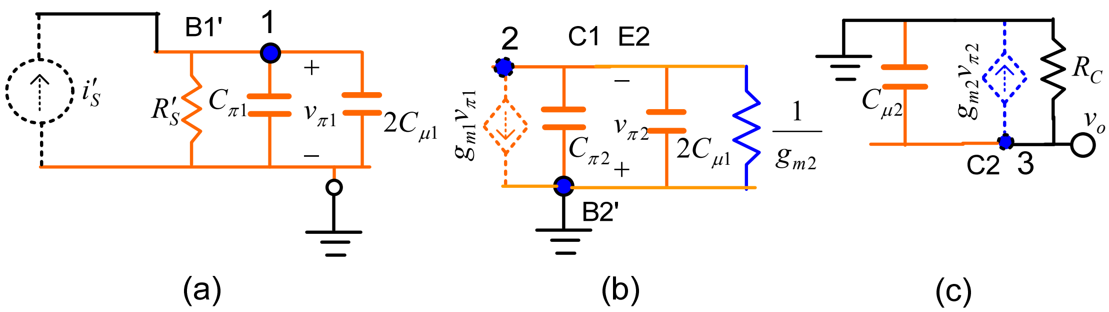



Figure 3.22: Final simplified high frequency equivalent circuit for the cascode amplifier.

Figure 3.22 present *three* disjoint sub-circuits with *three* distinct time constants. For Fig.3.22(a), the time constant is $\tau_a = (C_{\pi 1} + 2C_{\mu 1})R'_S$. For Fig.3.22(b) and (c) the time constants are respectively, $\tau_b = (C_{\pi 2} + 2C_{\mu 1})\frac{1}{g_{m2}}$, and $\tau_c = C_{\mu 2}R_C$. The higher -3dB frequency is then $\omega_H = \frac{1}{\tau_a + \tau_b + \tau_c}$, which can be considered as the bandwidth of the CE-CB cascode amplifier.

The mid-band gain of the cascode amplifier can be obtained by ignoring all the capacitors (as open circuits) and considering the circuit in figure 3.23.

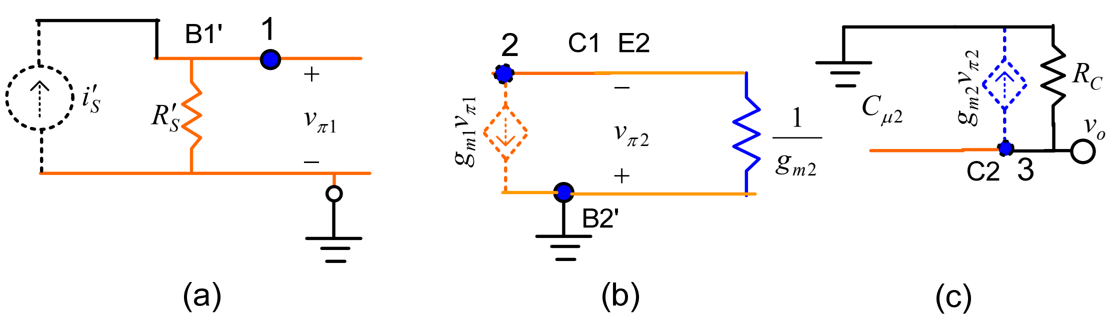



Figure 3.23: Equivalent circuit for the CE-CB cascode amplifier for mid-band (i.e., low frequency) gain calculation.

By inspection, $v_o = -g_{m2} R_C (g_{m1} \dfrac{1}{g_{m2}}) R'_S \dfrac{v_s}{R_{sig} + r_{X1}} = -g_{m1} R_C \dfrac{r_{\pi 1}}{R_{sig} + r_{X1} + r_{\pi 1}} v_s$ (3.33)

The mid-band gain is then $$\frac{v_o}{v_s} = A_M = -g_{m1} \frac{r_{\pi 1}}{R_{sig} + r_{X1} + r_{\pi 1}} R_C \qquad (3.34)$$

*Exercise 3.6.1.1.1:Consider an NPN BJT device with $h_{FE}$=100, $f_T$=6000 MHz, $C_\pi$= 25 pF biased to operate at $I_E$= 1 mA. Given that $r_X$=10 Ω, $V_A$ =50 V, and the signal source resistance $R_{sig}$ =100 Ω. The load resistance $R_C$ is 2.7 kΩ.*

*(a)The BJT is used as a CE amplifier with above given parameters. Find the mid-band gain $A_M$ , the upper cut-off frequency $f_{-3dB}$, and hence the Gain-Bandwidth (GBW) of the amplifier (*note: *GBW=$A_M$ times $f_{-3dB}$)*

*(b)Two matched BJT devices with above specifications are used to construct a CE-CB* **cascode** *amplifier. Find the mid-band gain $A_M$ , the upper cut-off frequency $f_{-3dB}$, and hence the Gain-Bandwidth (GBW) of the amplifier (*note: *GBW=$A_M$ times $f_{-3dB}$).*

*(c)How does the $GBW|_{CE}$ compare with the $GBW|_{Cascode}$ ?*

(Hint: the student need to first determine *$g_m$, $r_\pi$, $r_o$, and $C_\mu$ for the BJT from the given information*)

*Example 3.6.1.1.2: On using a typical BJT devices from SPICE simulation library, we get the following results:*

| Amplifier mode | CE | CB | CE-CB cascode |
|---|---|---|---|
| Gain | 61.1 | 15.1 | 63.8 |
| Band width | 180.1MHz | 793.5MHz | 616.2MHz |

3.6.2 CS-CG MOSFET cascode amplifier

It is easy to construct a cascode of CS-CG amplifier stages using MOSFET devices. The schematic and the high frequency equivalent circuit are shown in figures 3.24(a)-(b) respectively. I should be noted that the source terminals of transistors M1,M2 cannot be connected to the respective body (substrate) terminals and hence the associated parasitic

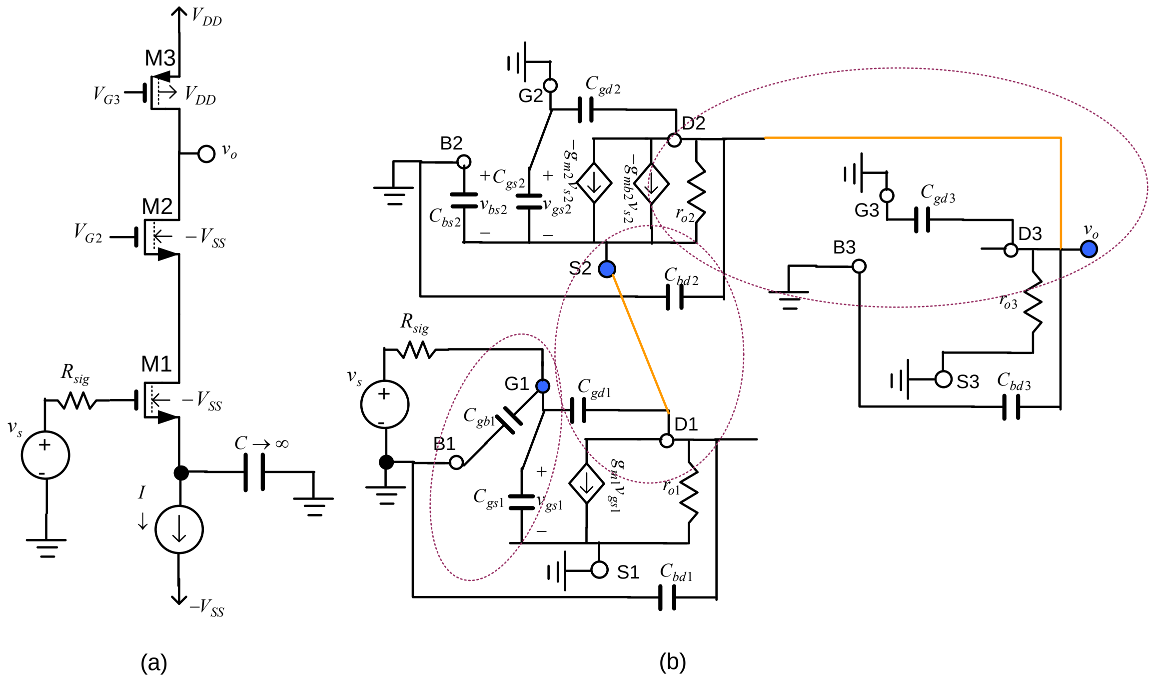



Figure 3.24: (a) Schematic of a CS-CG MOSFET cascode amplifier with active load and current source/mirror biasing, (b) high frequency equivalent circuit.

capacitances need be considered. For M1, however, since the S and B terminals are both at *ac* ground, the capacitance $C_{bs}$ is shorted out.

The equivalent circuit in Fig.3.24(b) has *three* ungrounded nodes (shown in blue shaded circles). The voltage gain $v_o/v_s$ can be calculated using standard nodal matrix formulation. Alternatively, the circuit can be simplified by adopting approximation procedures as used in the CS-CB cascode amplifier. This is left *as an exercise to the interested students*. Note that *Miller* magnification effect will be applicable to $C_{gd1}$.

3.6.3: <u>Wide band differential amplifier with BJT devices</u>

The cascode amplifier is single ended, i.e., has one terminal for signal input. It is of interest to investigate the possibility for a wide band differential amplifier. Towards this we can argue that a common differential amplifier with two CE stages accepting the differential input signals will not be desirable, because each CE stage will suffer degradation in high frequency performance because of Miller effect on $C_\mu$. In the following we discuss several possible configurations for wide-band differential amplifier.

*3.6.3.1: CE-CB cascode doublet as wide band differential amplifier*

The CE-CB cascode amplifier that we discussed already can be considered as a half circuit for constructing a differential amplifier. Figure 3.25 presents the schematic

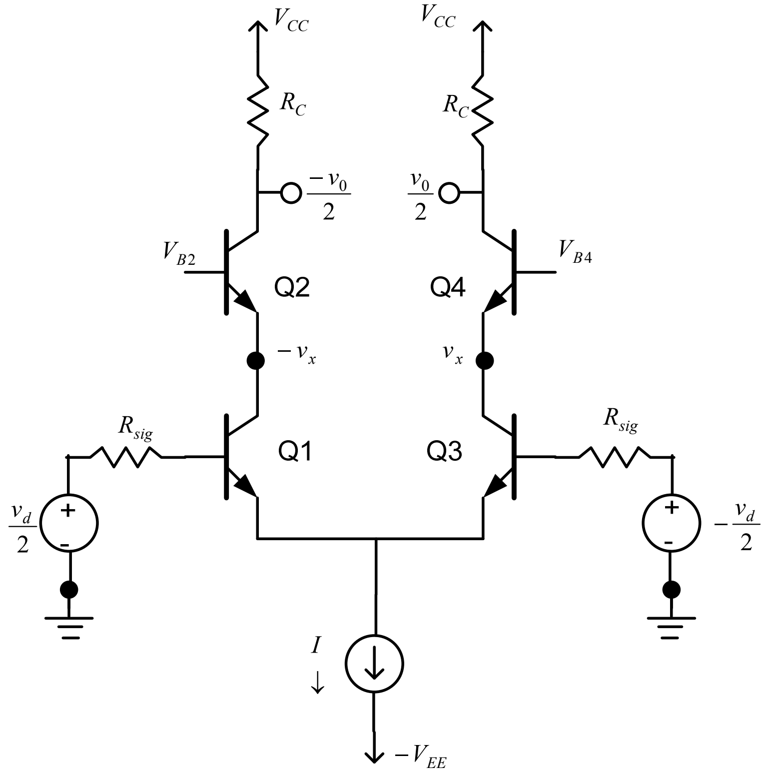



Figure 3.25: CE-CB cascode doublet as wide band differential amplifier

configuration. The analysis of the gain bandwidth (GBW) can be carried out as in the CE-CB cascode amplifier by considering only one half circuit of the configuration. The differential amplifier will preserve all the merits of the parent CE-CB cascode in addition to providing the special features of a differential amplifier (i.e., common mode rejection, removing even harmonic components)

*3.6.3.2: CC-CB cascade doublet as wide band differential amplifier*

The Miller magnification in a CE stage could be entirely removed if $R_C$ were zero. But that implies a CC stage, which has a low voltage gain (less but close to 1). The loss in voltage gain can be compensated partially, by cascading the CC stage with a CB stage.

But then we are getting a voltage gain only from one stage. By using the CC-CB cascade as a half circuit, it is possible to develop a wide band CC-CB doublet differential amplifier. The schematic is shown in figure 3.26.

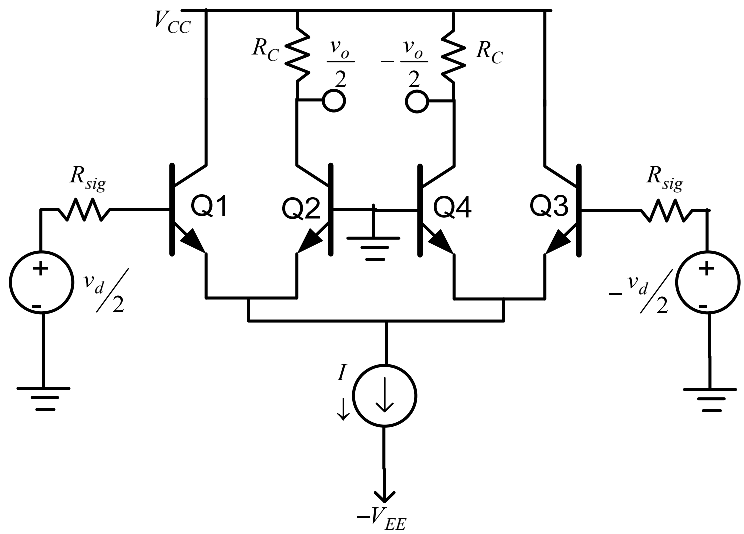



Figure 3.26: A CC-CB cascade doublet wide band differential amplifier.

The amplifier in Fig.3.26 will provide wide band operation with good voltage gain. But there are four transistors which need be supplied with DC bias current. The DC power consumption will be high. A *totem-pole* configuration where two transistors are stacked in one column will result in a lower DC power consumption. This is possible by using complementary (i.e., NPN and PNP) transistors to form the CC-CB cascade. Figure 3.27 presents the configuration.

*3.6.3.3: CC-CB cascade doublet differential amplifier with complementary transistors*

In the configuration of Fig.3.27 DC bias current is to be supplier only to two columns of transistors. So compared with Fig.3.26, the complementary CC-CB cascade doublet differential amplifier (Fig.3.27) will consume less DC power.

- *Analysis for gain bandwidth*

A simplified analysis for the band width of the differential amplifier can be carried out on the assumption that the NPN and the PNP transistors are matched pairs (it is *seldom true* in practice) and then working on one half circuit of the system. Thus considering the pair

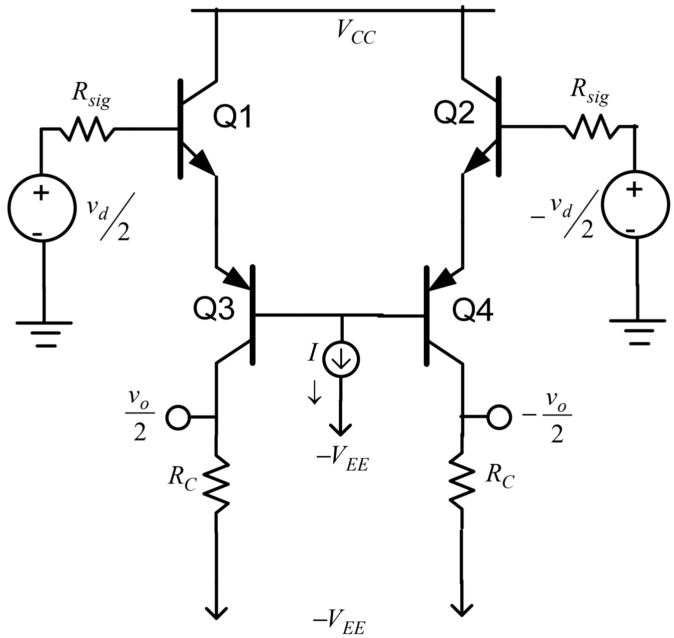



Figure 3.27: Schematic of complementary CC-CB cascade doublet differential amplifier

Q1,Q3 we can construct the high frequency equivalent circuit as shown in figure 3.28. We have ignored[1] $r_x$ and $r_o$ to further simplify the analysis.

We can now rearrange the direction of one of the controlled current sources by reversing the direction of the controlling voltage $v_\pi$. This leads to figure 3.29(a). Figure 3.29(b) reveals further simplification by combining the two identical parallel $C_\pi$ , $r_\pi$ circuits which occur in series connection. The current $g_m v_\pi$ coming *toward* and *leaving* from the node marked E1,E2 can be lifted off from this junction and has been shown as a single current $g_m v_\pi$ meeting the node C2.

[1] A *rule –of- thumb* regarding such simplifications is: a resistance in *series* connection can be neglected as *short circuit* if it is *small* compared with other resistances. Similarly, a resistance in *shunt* connection can be neglected *as an open circuit* if it is *high* compared with other resistances.

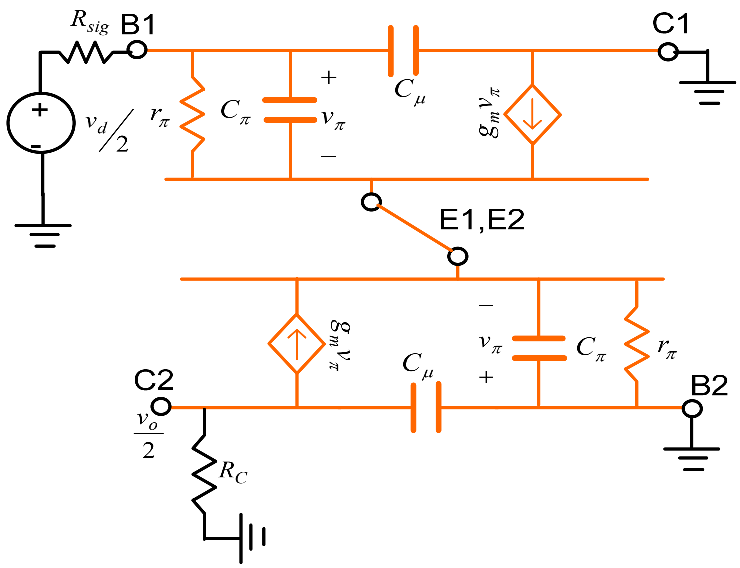



Figure 3.28: Approximate high frequency equivalent circuit for the schematic in Fig.3.27.

In Fig.3.29(b) we can see two disjoint circuits (shown by blue and *olive* green lines) with associated distinct time constants. The time constants are:

$\tau_1 = (\frac{C_\pi}{2} + C_\mu) \times (2r_\pi \,\|\, R_{sig})$, and $\tau_2 = C_\mu R_C$. The dominant high frequency pole is:

$\omega_H = \dfrac{1}{\tau_1 + \tau_2}$, which can be regarded as the band width of the amplifier.

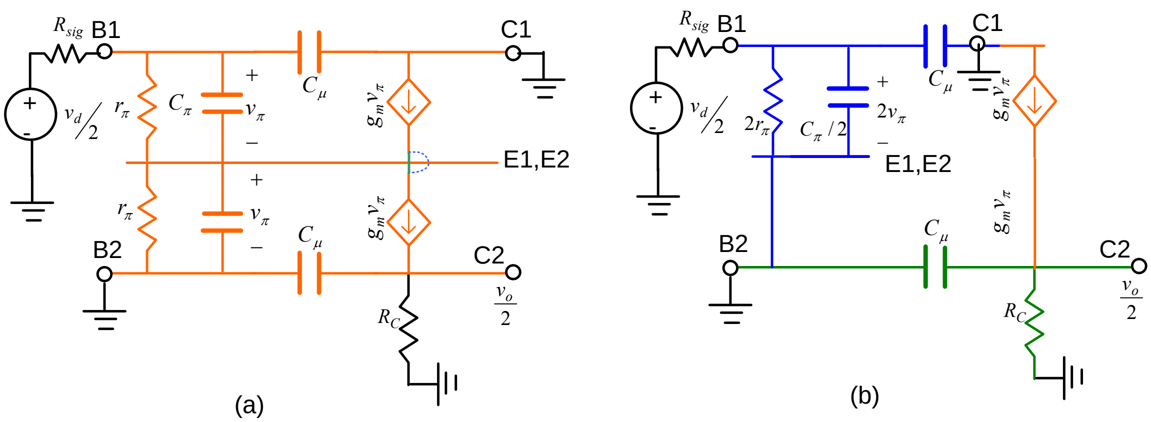



Figure 3.29: Further simplification and compaction of the circuit in Fig.3.28.

The *student is suggested to construct* the low-frequency form of the equivalent circuit in Fig.3.29(b) and find the expression for the mid-band gain $A_M$. The gain bandwidth is then $A_M \, \omega_H$.

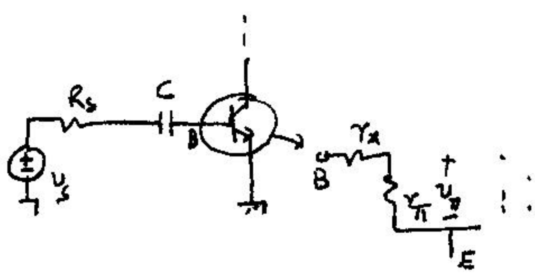
3.6.4: Wide band amplifiers with MOSFET transistors

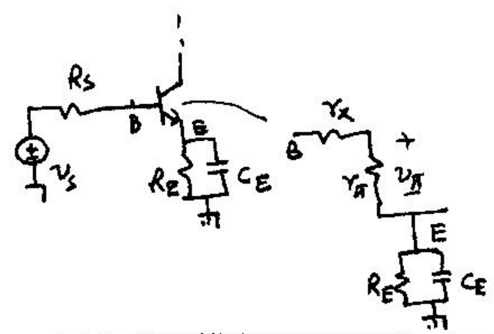
Enhancement mode MOSFET can be connected in the same way as BJT stages to derive cascode and wide band differential amplifiers for high frequency applications. The analysis follows similarly.

*The student is encouraged to draw the schematics with MOSFET devices by following the corresponding BJT configurations in sections 3.6.3.1-3.*

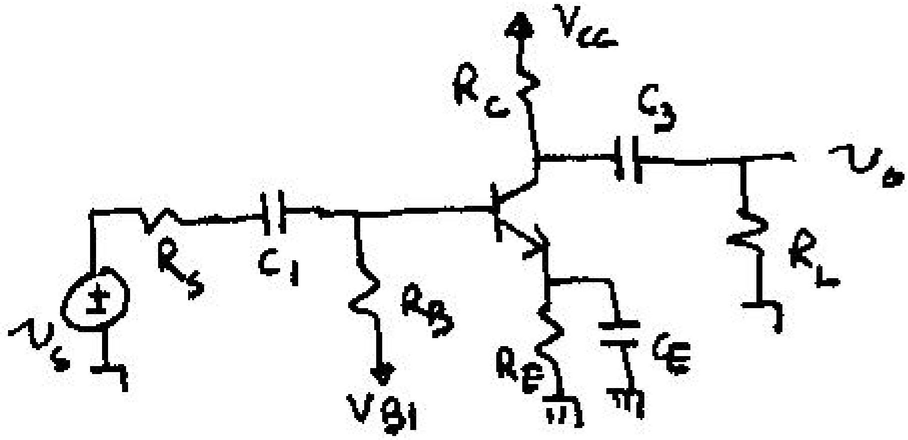
3.7: Practice Exercises

3.7.1: Show by appropriate analysis (KCL/KVL/Nodal matrix) that the voltage signal coupled across the internal base-emitter junction of a BJT (i.e., $v_\pi$) has a (a) zero at $\omega$=0, when a voltage source signal is fed to the base of the BJT via a series capacitor, and (b) has a zero at a finite frequency when parallel R,C network is in series with the voltage source. Use the low-frequency equivalent circuit for the transistor. Consider the representative cases as shown below.




3.7.2: For the BJT CE amplifier below, given $R_S$=600 ohms, $R_B$= 22 k ohms, $h_{fe}$=99, $I_C$=2 mA, $R_E$= 1.5 k ohms, $R_C$=2.2 k ohms, $R_L$=1 k ohms, $C_1$ =1 μF, $C_2$=25 μF, $C_3$= 10 μF. What will be the lower -3dB frequency for the amplifier?



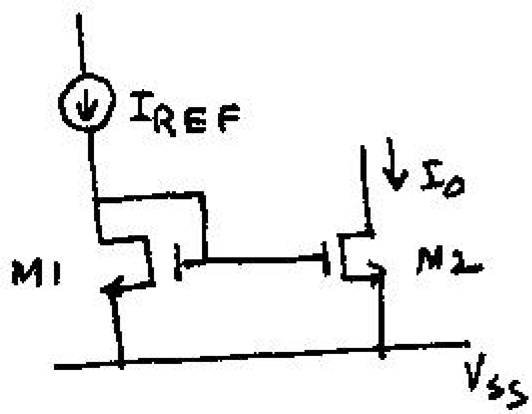

3.7.3: In a BJT, CE amplifier the network parameters are: $R_S$= 100 ohms, $R_B$=1 k ohms, $r_x$= 50 ohms, $I_C$= 1 mA, $h_{fe}$= 99, $C_\pi$=1.2 pF, $C_\mu$=0.1 pF, $V_A$=50 V, $R_C$=1.5 k ohms. Find,

(a) The time constants associated with the capacitors using open-circuit time constant method.
(b) What is the approximate upper cut-off frequency?
(c) In the equivalent circuit of the amplifier use Miller's theorem assuming a gain of $-g_mR_C$ between the internal collector and base terminals of the BJT, and re-draw the equivalent circuit.
(d) Determine the pole frequencies in the equivalent circuit derived in step (c) above.
(e) What will be the approximate upper cut-off frequency using the results in (d)?
(f) Use the full transfer function determination method to the equivalent circuit of the CE amplifier and determine the pole frequencies using exact solution of *D(s)=0*, where the voltage gain function is : *N(s)/D(s)*.
(g) Determine the 'dominant' pole from the *D(s)* derived in step (f) above.
(h) What are your estimates about the upper cut-off frequencies if you use the results in (f) and (g)?
(i) Tabulate the upper cut-off frequency values obtained in steps (b), (e), (f), and (g).

3.7.4: In a MOSFET amplifier, you are given the following: $R_S$=100 ohms, $C_{gs}$=0.1 pF, $C_{gd}$=20 fF, $g_m$ =50 μ mho, $I_{DC}$=50 μA, $V_A$=20 V, and $R_L$= 5 k ohms. The MOSFET amplifier is configured to operate as CS amplifier. Find the dominant high-frequency pole of the amplifier using:

(a) Miller's theorem
(b) Full nodal analysis
(c) Open-circuit time constant method

3.7.5: Consider a basic MOSFET current mirror circuit. Find an expression for the high frequency current transfer function $i_o(s)/i_{in}(s)$. Use the high frequency ac equivalent circuit model for the transistors.




3.7.6: For the BJT amplifier shown below, determine the high frequency voltage gain transfer function in the form: $A(s) = A_M \frac{\omega_H}{s+\omega_H}$. Given $C_\mu$ =0.5 pF, $f_T$=600 MHz, $h_{fe}$=49.

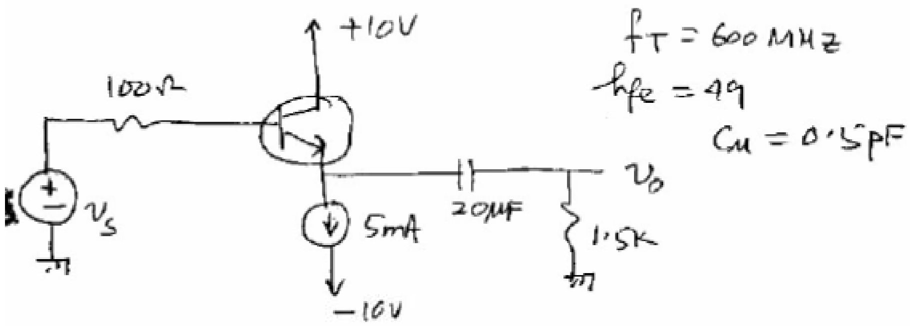



Fig. for Problem ~~3.8.6~~ 3.7.6

3.7.7: For the CD-CS cascade MOSFET composite amplifier, it is given that, $V_A$=20 V, $C_{gs}$=0.5 pF, $C_{gd}$= 0.1 pF, $I_{DC}$= 50$(V_{GS}$- $V_{TH})^2$ μA, $V_{TH}$ =1 V. Estimate $f_H$ for this amplifier system. The output resistance of each current source is 1 Mega ohms.

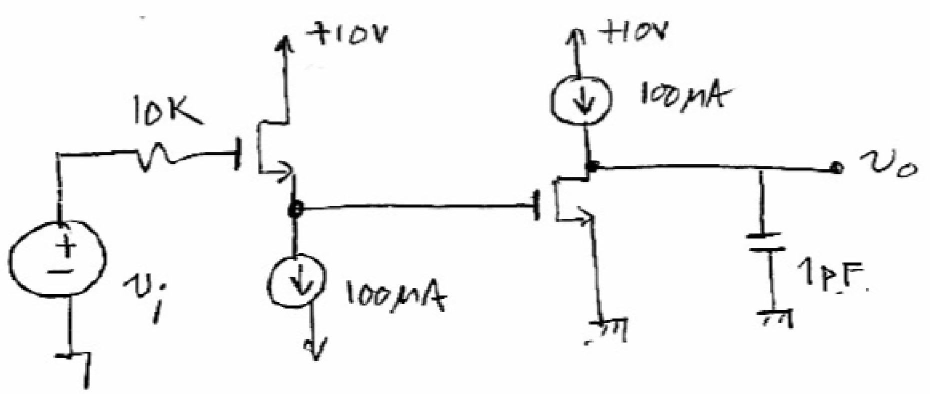
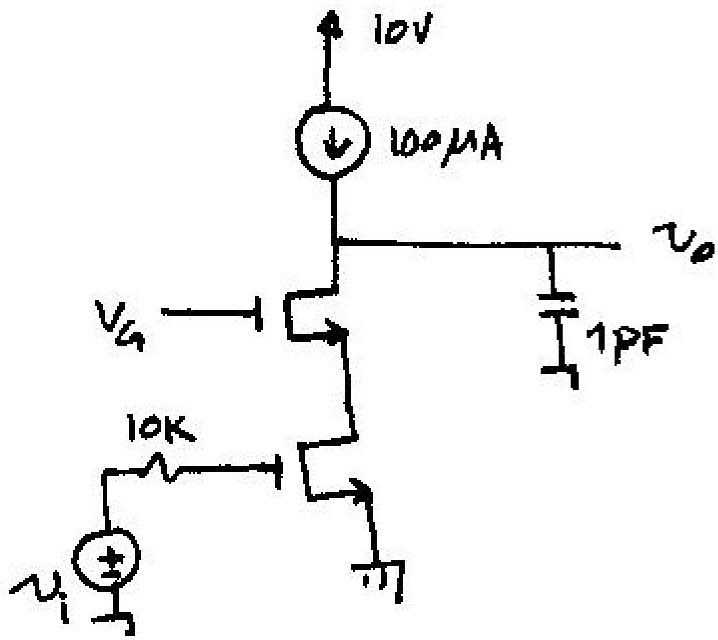



Fig. for Problem ~~3.8.7~~ 3.7.7

3.7.8: Find $f_H$ for the CS-CG cascade MOSFET amplifier system shown below. Use necessary data from problem # 3.7.7 above.



10V
100μA
$v_o$
$V_G$
1PF
10K
$v_i$