Chapter 4

FEEDBACK in AMPLIFIERS

(*Review Appendix 3.5 for background on two-port networks*)

Feedback implies feeding back (i.e., returning back) a part of the processed signal to the input side so as to enhance or diminish the input signal. When the input signal (current or voltage) is diminished, it is considered as negative feedback. When the input signal is enhanced, it is known as positive feedback. Negative feedback is employed in amplifying systems to achieve certain special characteristics that are not obtainable from the basic amplifier. Positive feedback is employed to produce signal generator, such as oscillators. In this chapter we shall consider the case of negative feedback. The following topics will be covered:

- Basic concepts and benefits of negative feedback.
- Interconnections and associated circuit models of the amplifier and the feedback network.
- Analysis techniques with examples for the four basic amplifier configurations (VCVS,CCCS,VCCS,CCVS).
- Negative feedback and stability- phase and gain margins.

4. 1: Basic negative feedback system

Consider the Figure below. The source signal could be a current or voltage. The basic amplifier has a gain A from left to right direction. A part of the output signal x_o is fed back by the factor β, from right to left, and subtracted (added with a phase inversion) from the input signal x_s.

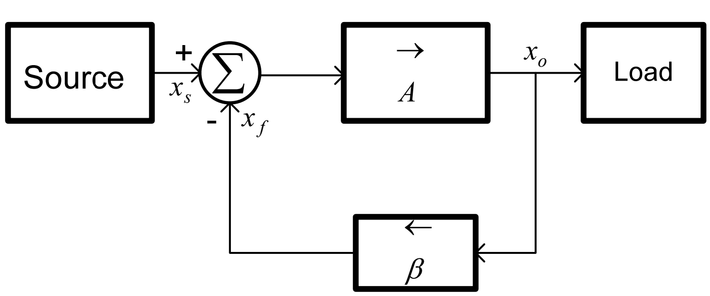


Figure 4.1: Basic negative feedback system

While the basic amplifier has a gain A (i.e., x_o / x_I), the overall gain of the feedback system x_o / x_s is A_f which is $A/(1 + A\beta)$. This gain is called the gain with feedback. The quantities A and A_f could be any one of the four different kinds of function, i.e., (a) voltage gain, (b) current gain, (c) trans-resistance gain and (d) trans-conductance gain. Some special features of the negative feedback system can be appreciated very easily.

The quantity $A\beta$ is called the *loop gain* of the system. The term $1 + A\beta$ is referred to as the *feedback factor*.

Exercise 4.1.1: An audio amplifier has a gain of 100 V/V. It is placed under negative feedback with a feedback gain (=β) of 0.1. What will be the net gain now? (*ans*: 100/11)

4.1.1: Benefits of negative feedback

4.1.1.1: *Gain de-sensitivity*

This implies that if there occurs a variation by certain amount in the gain A of the main amplifier, the gain A_f of the feedback system is not altered as much i.e., the gain variation is desensitized by negative feedback.

Assume β is constant. Then if we take differential of A_f, we get $dA_f = \frac{dA}{(1+A\beta)^2}$

Thus it is evident that the variation dA in A has been reduced by the factor $(1+A\beta)^2$ because of negative feedback

Exercise 4.1.1.1.1: Consider the amplifier in *exercise 4.1.1*. Changes in the DC power supply may cause 20% variations in the gain of the amplifier. What will be the variation if it is connected in negative feedback with β=0.05?

4.1.1.2: *Bandwidth extension*

This implies that if the band width of the gain A has certain values (say 1MHz), by applying negative feedback, it can be increased. The increase, however, happens by sacrificing the value of the gain A.

Thus, consider $A = \frac{A_M}{1 + s/\omega_H}$ This has a high frequency band with of ω_H rad/sec. If we apply negative feedback around the amplifier, the gain A_f will become:

$$A_f(s) = \frac{A(s)}{1+\beta A(s)} = \frac{A_M \Big/ (1+\frac{s}{\omega_H})}{1+\beta \; A_M \Big/ (1+\frac{s}{\omega_H})}$$

After simplification : $A_f(s) = \dfrac{A_M/(1+A_M\beta)}{1+s/[\omega_H(1+A_M\beta)]} = \dfrac{A_{Mf}}{1+\dfrac{s}{\omega_H(1+A_M\beta)}} = \dfrac{A_{Mf}}{1+\dfrac{s}{\omega_{Hf}}}$

where, $A_{Mf} = \dfrac{A_M}{1+A_M\beta}$, and $\omega_{Hf} = \omega_H(1+A\beta)$

The above result implies a mid-band gain of A_{Mf} and a high frequency band width of ω_{Hf}. It can be clearly seen that the new mid-band gain is $(1+A\beta)$ times smaller than the mid-band gain without feedback, but the high frequency band width is $(1+A\beta)$ times larger than the band width without feedback. Thus an extension of band width by the factor $(1+A\beta)$ has been achieved.

Exercise 4.1.1.2.1: Consider the amplifier in *exercise 4.1.1*. It has a bandwidth of 5 kHz. It is required to increase the bandwidth to 25 kHz. What value of β should be used? What will be the new gain of the amplifier under feedback?

4.1.1.3: *Reduction of non-linear distortion in amplifiers*

It is known that most practical amplifiers have a non-linear output input transfer characteristics. The non- linearity arises out of (i) non-linear response characteristic of the devices (i.e., transistors), and/or (ii) finite DC power supply values.

A non-linear transfer curve represents a gain (~ slope of the graph) which varies depending upon the location of operation on the curve (i.e., the operating point). Thus one can define a series of gains, say, A_1, A_2, .. along the transfer characteristics. Under negative feedback the corresponding gains become $A_1/(1 + A_1\beta)$, $A_2/(1 + A_2\beta)$, and so on. If the loop gain values (i.e., $A_1\beta$, $A_2\beta$) are very high (i.e., >>1), the feedback gain values approximate to $1/\beta$ in each case. Thus, the gains under negative feedback at different segments of the transfer characteristic appear to remain constant at a value $1/\beta$. A constant gain (~constant slope) implies a linear curve, i.e., a straight line. This is how the non-linearity in the response characteristic of the amplifier is reduced, and hence the attendant distortion gets reduced.

4. 2: Interconnections for the negative feedback systems

We will now consider the four distinct types of negative feedback connections and their respective characteristics as regards the overall gain, input and output impedances. The classification basically depends upon the four distinct types of amplifiers, namely, (a) voltage amplifier (VCVS), (b) current amplifier (CCCS), (c) trans-resistance amplifier (CCVS), and (d) trans-conductance amplifier (VCCS).

For each case, the topology (i.e., the style of interconnection) of the negative feedback network follows a regular pattern relative to the topology of the basic amplifier. Thus, for a voltage amplifier (VCVS), whose equivalent circuit model has (i) a series connection at the input through the input resistance, and (ii) a series connection at the output (i.e., the output controlled voltage source and its associated resistance connected in series), the feedback circuit will have (i) a series connection at the input, and (ii) a shunt (parallel) connection at the output. A *rule of thumb* is: the feedback *connection at input matches* with that of the basic amplifier, but the *connection at the output is opposite* to that of the basic amplifier.

4.2.1: <u>System diagrams for the feedback connections</u>

Consider figures 2(a)-(d) which depict the *four* possible interconnection styles (*topology*) around the *four* basic electronic amplifier systems.

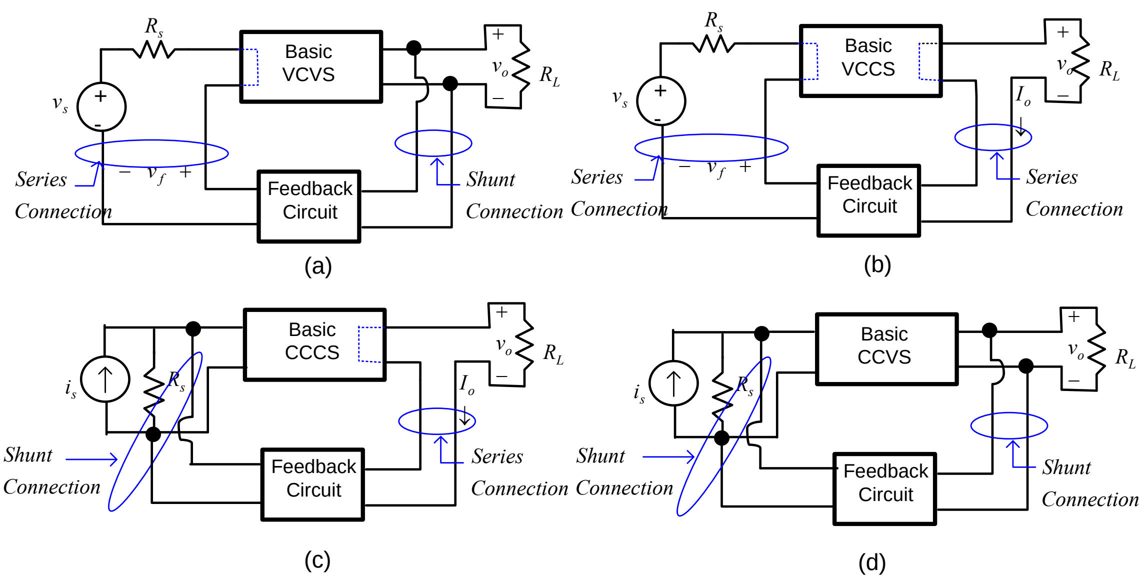


Figure 4.2: *Four* possible feedback connections

Quiz: Given a VCCS amplifier, what will be the topology of the feedback connections at the input and at the output? Apply the *rule of thumb*, then check your answer with Fig.4.2(b). (You need to recollect the topology of the VCCS amplifier though!)

4.2.2: Model of the input source according to the feedback connection style

The operation at the input of a feedback amplifier system involves *mixing* of signals. *Mixing* is possible only for signals of the *same type* i.e., voltage with voltage, current with current, and so on. Similarly, the operation at the output of the amplifier under feedback is called *sampling*. *Sampling* can be done for only *one kind* of signal i.e., a voltage or a current.

Consider Fig.4.2(a). The *mixing* is done with *series* connection at the input of a VCVS. The question is what kind of signal can be *mixed* in *series connection*? Recalling the practice of writing a KVL around a loop (i.e., a succession of *series connected* elements) prompts us to appreciate that such *mixing* in *series connection* is possible *only* with *voltage* signals. Since *voltage* signals are *mixed* in series, we obviously understand that the signal source in this case *must* be a *voltage* source. The *signal* source voltage is *mixed* with the *feedback* voltage in *series* at the input of a VCVS which has a *series* topology at its input.

Same conclusion follows for Fig.4.2(b), where the signal source is to be modeled as a voltage source. The VCCS has a *series* topology at its input. Hence the feedback connection at the input is a *series* connection. Only a *voltage signal* can be mixed in *series* with another *voltage signal* (in this case the feedback signal). Hence the source must be a voltage source.

Hint to remember: Series in implies a voltage signal as the source and a VCxx (*voltage controlled xx*) form of the basic amplifier. VCxx could be VCVS or VCCS.

In the same way, we can understand that for CCCS and CCVS (i.e., CCxx) [Figs. 4.2(c) - (d)], the input topology being a *shunt* (i.e., parallel connection) configuration, *only* current signals can be mixed under this kind of connections. *Shunt* implies a *node* – hence *KCL*- hence *current* signal being *mixed*. Hence the input sources for CCCS and CCVS amplifiers are to be modeled as current sources.

Hint to remember: Shunt in implies a current signal as the source and a CCxx (*current controlled xx*) form of the basic amplifier. CCxx could be CCVS or CCCS.

4.2.3: Model of the basic amplifier according to the feedback connection style

The purpose of this short section paragraph is to emphasize an understanding of the models of the basic amplifiers depicted in Figs. 4.2(a)-(d). From the section 4.2.2, the student can understand that for a *series in* feedback connection the basic amplifier will be of VCxx form. Similarly, for a *shunt in* feedback connection the basic amplifier will be CCxx form. How to figure out the xx part? For this we need to examine the style of the feedback connection at the *output*.

It has been mentioned in section 4.2 that the feedback connection at the *output* is *opposite* to that of the basic amplifier. Thus, a *shunt* connection at the output will imply a *series* connection style at the output of the amplifier. A *series* connection implies a **voltage source** with a series resistance. Now to put both the input and output feedback connections together, a *series in-shunt out* connections will imply a VC (for series in) VS (for shunt out), i.e., a VCVS amplifier model.

Similarly, a *series* connection at the output will imply a *shunt* connection style at the output of the amplifier. A *shunt* connection implies a **current source** with a shunt resistance. Now putting the input and output feedback connections together, a *series in-series out* connections will imply a VC (for series in) CS (for shunt out), i.e., a VCCS amplifier model.

The student may confirm his/her understanding of the models of the basic amplifiers in Figs. 4.2 (c)-(d) in light of the discussions above.

4.2.4: Input, output resistances under negative feedback

If we realize that resistances connected in *series* produces an *increase* in the resistance while resistances connected in *parallel* (shunt connection) produces a *decrease* of resistance, one can immediately infer that at the *series connection location* of the feedback system there will be an increase in the resistance value. Similarly, at the point where the feedback connection is in shunt, the resistance will decrease after feedback connection.

4.2.5: Modeling the feedback circuit by two-port network parameters

The expression for the gain with negative feedback in section 4.1 was derived under the assumption of a smooth interconnection between the amplifier and the feedback network. In practice such smooth interconnection does not happen, a loading is always present. Further, it was implicitly assumed that the signal transmission through the amplifier and the feedback network are unidirectional, being from the *source toward the output* (load) *through the amplifier* (i.e., left to right in our diagrams) and from the *output toward to source* (i.e., right to left in our diagrams) *through the feedback network*. This is an ideal situation. So certain approximations are required and practical design must ensure that these assumptions are reasonably accurate.

The effect of loading of the amplifier can be efficiently taken care of by modeling the feedback circuit as a two port network (*review Appendix 3.5 for necessary background*). The two-port circuit models pertinent to the *four* feedback interconnection styles are presented in **Table 4.1**.

linear circuit elements (i.e., *passive, bilateral* impedance/admittance) and *two* controlled sources. The controlled source on the *right half* (i.e., toward port #2) of the model is *controlled by* a voltage/current present on the *left half* (i.e., toward port #1), and *vice versa*.

For the amplifier since the signal flow is from *left* (i.e., signal source) to *right* (i.e., amplifier output), the controlled source on the *right* half of the two port model is *most* effective and important. For the feedback circuit since the signal flow is from *right* (i.e., amplifier output) to *left* (i.e., signal source), the controlled source on the *left* half of the two port model is *most* effective and important.

We will use the two port model *only* for the feedback circuit. The *passive* linear components of the model will be connected at the input and the output of the basic amplifier in conformity with the feedback connection respectively at the input and at the output. This will produce the *loaded* amplifier. The controlled source on the *left half* of the two port model will be representing the feedback gain β as introduced in section 4.1. The approximated two port models are also shown in **Table 4.1** (in column *three*).

It must be *emphasized* at this point that *the amplifier gain* A and *the feedback factor* β as introduced in section 4.1 in practice correspond to (i) the gain of the *loaded* amplifier, and (ii) the gain represented by the controlled source of the pertinent model in column *three* of Table 4.1. Guidelines to construct the *loaded* amplifier are provided in **Table 4.2**

4.2.6: *Summary of the results for calculating gain and impedances under negative feedback*

An important characteristic of the various feedback systems is that if the *loaded* gain A and the *feedback* gain β can be determined, the expressions for various gain and resistance values can be easily calculated. These follow certain easy to remember standard forms. **Table 4.3** provide various important formulae. The student should pay particular attention to the columns for Z_{if} and Z_{of}.

Table 4.1 (Two port equivalent models for the feedback circuits)

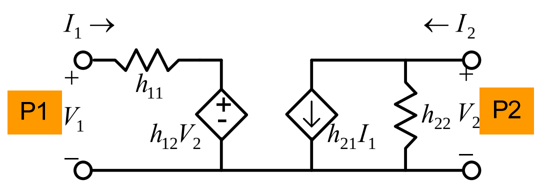
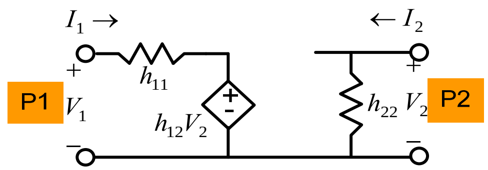
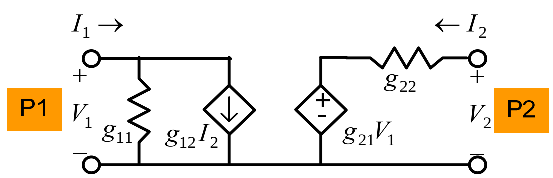
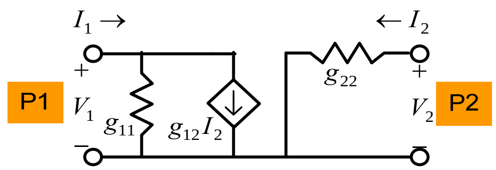
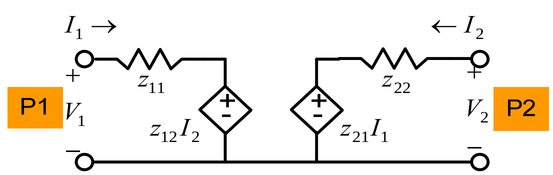
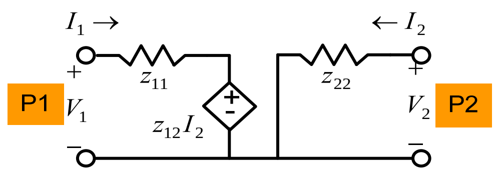
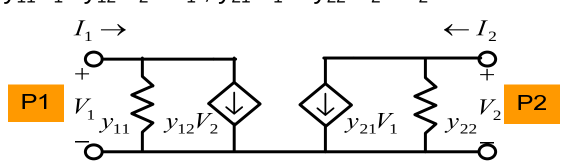
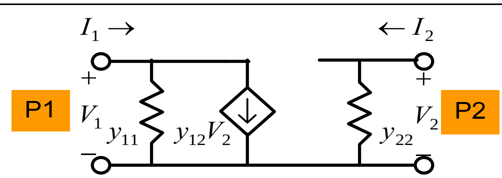
Model name	General formulations (equations & circuit models)	Approximate circuit model	Feedback
[h]	$h_{11}I_1 + h_{12}V_2 = V_1$; $h_{21}I_1 + h_{22}V_2 = I_2$ 		Series-Shunt
[g]	$g_{11}V_1 + g_{12}I_2 = I_1$; $g_{21}V_1 + g_{22}I_2 = V_2$ 		Shunt-Series
[z]	$z_{11}I_1 + z_{12}I_2 = V_1$; $z_{21}I_1 + z_{22}I_2 = V_2$ 		Series-Series
[y]	$y_{11}V_1 + y_{12}V_2 = I_1$; $y_{21}V_1 + y_{22}V_2 = I_2$ 		Shunt-Shunt

Table 4.2 (Guidelines to create the *loaded* amplifier)

Amplifier Model	Pertinent 2-port parameters	Feedback connection	Modifications to basic amplifier to create the *loaded* amplifier
VCVS	[h]	Series in Shunt out	$R_{11} = h_{11}$ in series with input $R_{22}=(h_{22})^{-1}$ in parallel with output
CCCS	[g]	Shunt in Series out	$R_{11} =(g_{11})^{-1}$ in parallel with input $R_{22} =g_{22}$ in series with output
VCCS	[z]	Series in Series out	$R_{11} = z_{11}$ in series with input $R_{22} = z_{22}$ in series with output
CCVS	[y]	Shunt in Shunt out	$R_{11}=(y_{11})^{-1}$ in parallel with input $R_{22}=(y_{22})^{-1}$ in parallel with output

Table 4.3: (Summary of the formulae, $A_f = X_o / X_S = \frac{A}{1+A\beta}$ for all cases)

Feedback connection	A	β	Basic Amplifier	Z_{if}	Z_{of}
Series-shunt	V_o/V_I	V_f/V_o	VCVS	$Z_I(1+A\beta)$	$Z_o/(1+A\beta)$
Shunt-series	I_o/I_I	I_f/I_o	CCCS	$Z_I/(1+A\beta)$	$Z_o(1+A\beta)$
Series-series	I_o/V_I	V_f/I_o	VCCS	$Z_I(1+A\beta)$	$Z_o(1+A\beta)$
Shunt-shunt	V_o/I_I	I_f/V_o	CCVS	$Z_I/(1+A\beta)$	$Z_o/(1+A\beta)$

It may be seen from the above table that whenever the feedback network has series interconnection, the corresponding (i.e., input or output) impedance is *increased* by the factor ($1+A\beta$), while a shunt connection *reduces* the impedance at the pertinent location (i.e., input or output) by the factor ($1+A\beta$). These observations can be of aid to remember the formulae.

4. 3: Analysis examples

4.3.1 Series-in shunt-out feedback (basic amplifier is VCVS)

- *OP-AMP based system example*

An OP-AMP having an open loop gain of 10000, an input differential resistance of 100k and an output resistance of 1k is connected as shown in figure 4.3. The resistances R_1 , R_2 form the feedback network. Find the resulting voltage gain, input resistance and output resistance.

Solution: Since the basic amplifier is a VCVS, the feedback is series-shunt type. In forming the "loaded" amplifier, we have to use *h-parameter* representation of the feedback network. After deriving the loaded amplifier, we shall calculate the gain *A*, R_{in} and R_{out} for the loaded amplifier. We have to calculate the feedback factor β as well. Then we shall apply the formulae in Table 1 to calculate the gain and resistances of the new system (i.e., the OP-AMP with feedback).

Given: $R_S = 10\,k\Omega,\ R_{id} = 100\ k\Omega,\ A_{VO} = 1000,\ R_O = 1k\Omega,\ R_1 = 1k\Omega,\ R_2 = 1M\Omega,\ R_L = 2\,k\Omega$

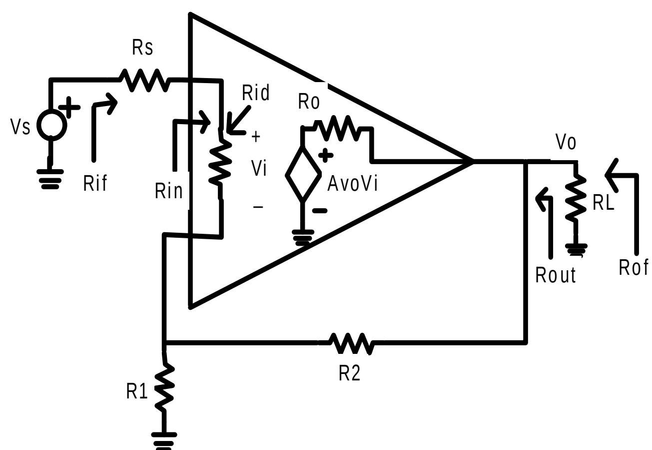


Figure 4.3: A series-in, shunt-out negative feedback system using an OP-AMP

The amplifier ac equivalent circuit using hybrid parameter description for the feedback circuit is shown in Fig.4.4.

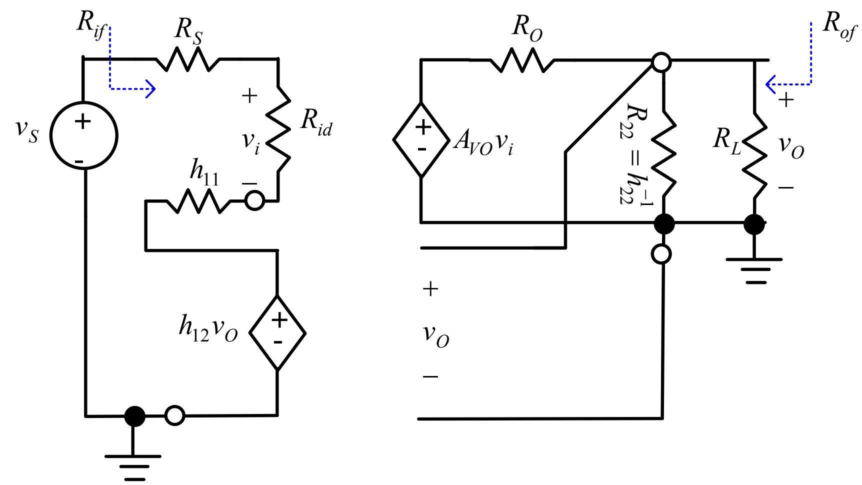
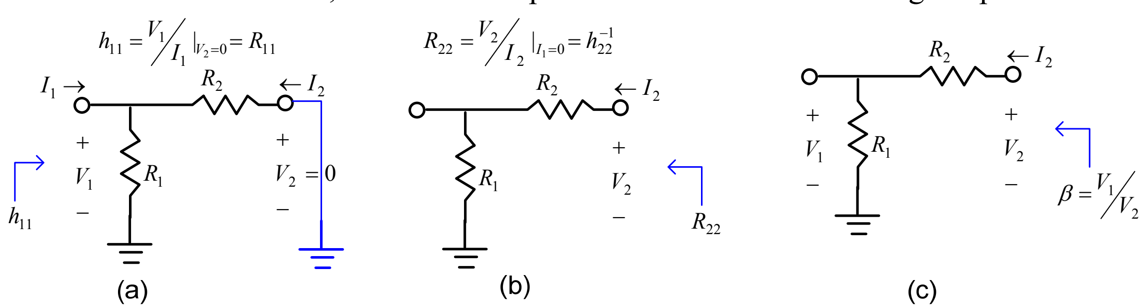


Figure 4.4: *ac* equivalent model of Fig.4.3 including the two-port model of the feedback network (R_1, R_2)

For the feedback network, the relevant *h*-parameters are evaluated using the partial *ac*



4.5 Partial equivalent circuit models to determine the *h* –parameter components.

equivalent circuit models depicted in figure 4.5(a)-(c).

We need now to attach the feedback circuit model to the amplifier circuit model. Figure 4.6(a) shows the amplifier model in black lines and the feedback part in blue lines. This is an intermediate step to form the *loaded* amplifier circuit (*A*-circuit). Figure 4.6(b) shows the *loaded* amplifier in pink lines. The feedback contribution as the controlled source is shown in orange lines. This is called as the β-circuit. At this stage we can apply the standard formulae in **Table 4.3** for the gain (A_f), input and output resistances (R_{if}, R_{of}) under feedback.
Note that for the loaded amplifier $R_{in} = R_s + R_{id} + h_{11}$ = 10+100+1= 111k. Then since β = 10E-3, R_{if}(by formula)=111k*(1+$A\beta$). Now the gain A of the loaded amplifier needs to be found out.

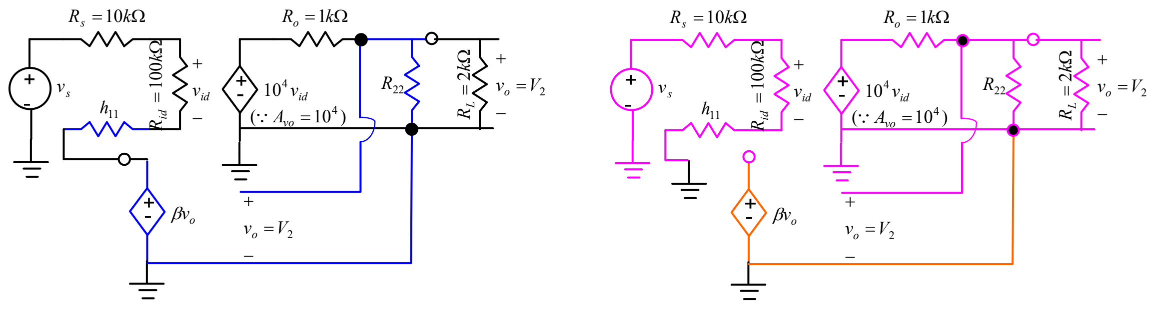


Figure 4.6: Construction of the *loaded* amplifier (a) complete equivalent circuit, (b) positioning of the loading elements inside the basic amplifier (pink lines).

Considering the equivalent circuit of the loaded amplifier, we can determine:

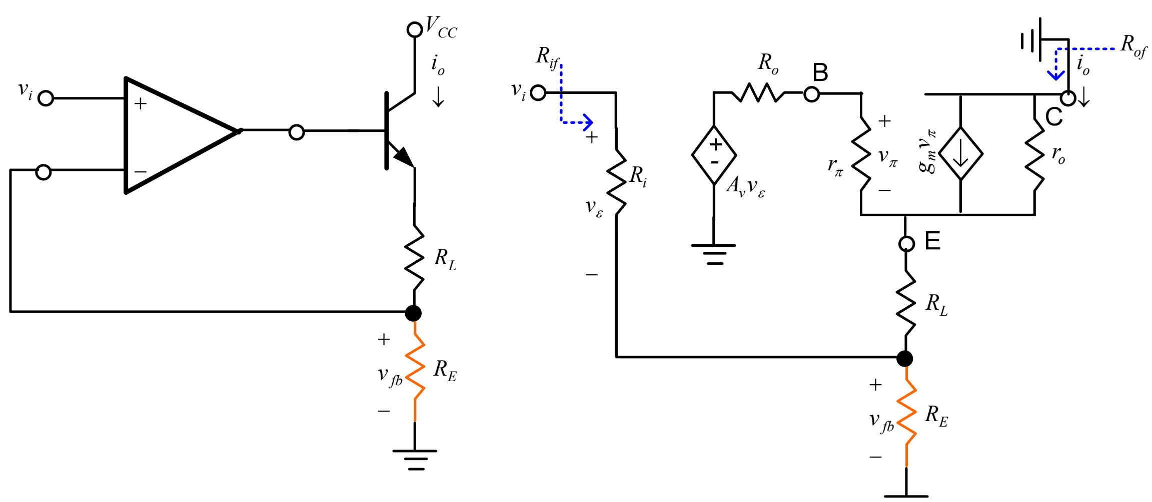
$1+A\beta$ = 1+6006*1E-3=7.006. Thus A_f =A/7.006 = 858 V/V, R_{if} =R_{in} (1+Aβ) = 111*7.006=777k, and R_{of} =($R_L || R_o || R_{22}$)/($1+A\beta$) =667/7.006=95.3Ω.

Now, $R_{in} = R_{if} - R_s$ =777k –10k =767k. But $R_{out} || R_L = R_{of}$ giving R_{out} =100Ω.

4.3.2 Series-in series-out feedback (basic amplifier is VCCS)

- *OP-AMP based system example*

Figure 4.7(a) presents an OP-AMP followed by a BJT common collector amplifier



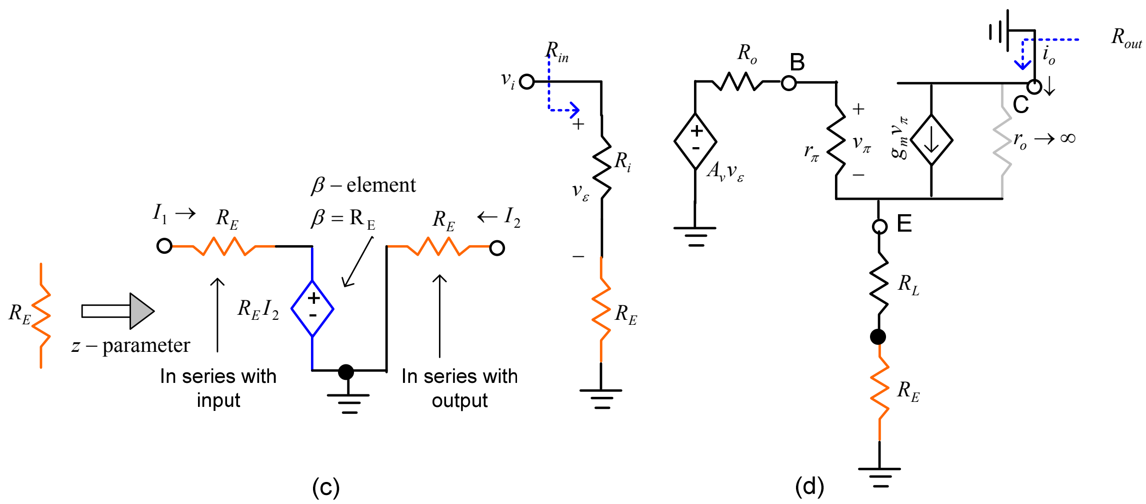


Figure 4.7:

connected in a series (in)-series (out) feedback system. The resistance R_E provides the negative feedback. Given $R_i = 50\,k\Omega, A_v = 1000, R_o = 100\,\Omega, I_C = 2\,mA, h_{FE} = 100$. Further, $R_E = 100\,\Omega, R_L = 1\,k\Omega$. Figure 4.7(b) is the ac equivalent circuit of the connected system. We want to calculate the gain i_o/v_i, R_{if}, and R_{of} as shown in Fig.4.7(b).

Because the feedback is *series-in, series-out*, the feedback resistance R_E need be modeled using *z-parameter* equivalent circuit. Figure 4.7(c) shows the pertinent *z*-parameters, i.e., z_{11} (=R_E), z_{22} (=R_E), and z_{12} (=β=R_E). Figure 4.7(d) presents the *loaded* amplifier.

***Analysis with the loaded amplifier circuit*

Since the *Early* voltage of the BJT is not given, we can assume $r_o \rightarrow$ *infinity*. Then,

$$i_o = g_m v_\pi = g_m A_v v_\varepsilon \frac{r_\pi}{R_o + r_\pi + (h_{FE}+1)\times(R_L + R_E)} = g_m A_v \frac{r_\pi}{R_o + r_\pi + (h_{FE}+1)\times(R_L + R_E)} \frac{R_i}{R_i + R_E} v_i$$

From the given data, g_m =0.08 mho, r_π =h_{FE}/g_m=1250Ω., we can get the *loaded* gain A as $\frac{i_o}{v_i}$ = 0.8875. Then $A\beta$=AR_E =88.75.

The *transconductance* gain with feedback is then $A_f = A/(1+A\beta) = 0.8875/89.75 = 0.01$ mho or 10 *milli* mhos (watch! it is approximately $1/\beta$)

For the loaded amplifier R_{in} =R_i+R_E =50.1 kΩ. Hence, with *series-in* feedback, R_{if} =(1+$A\beta$)R_{in} =4.496 MΩ.

For the location of R_{out} shown in Fig.4.7(b), clearly R_{out} is infinity, since we assumed r_o as infinity. Hence R_{of} is also *infinity*. However, if we intend to evaluate R_{of} at the emitter node of the BJT (Fig.4.7(b)), we can first find out, by inspection on Fig.4.7(d) that R_{out} is

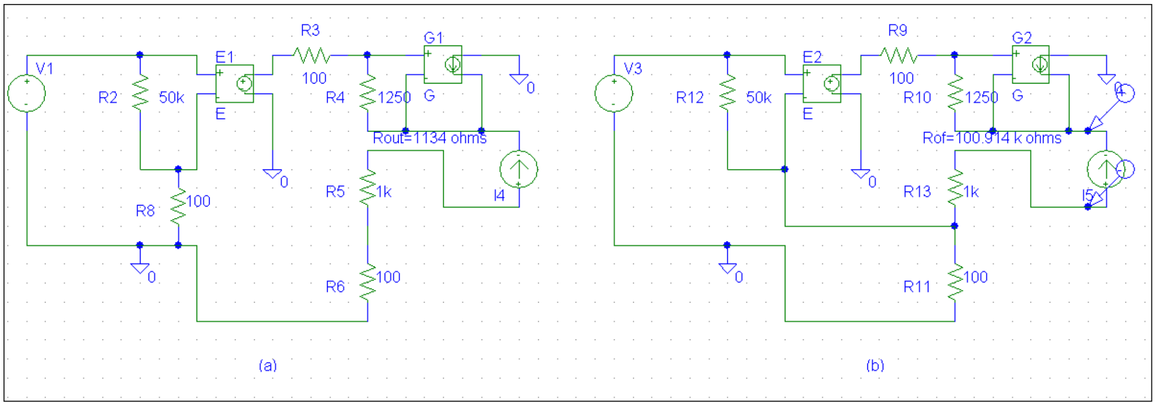


Figure 4.8:

1113.5 ohms. PSPICE simulation (Fig. 4.8(a)) gives a value of 1134 ohms.

Then R_{of} at this location will be 1113.5(1+$A\beta$)= 99936.62 ohms Simulation by SPICE produces a value of 100.914 k ohms (see Figure 4.8(b)), which is very close to the value derived using the *loaded* amplifier equivalent circuit of Fig.4.7(d).

Conclusion: A question may arise about the utility of such a series-in, series-out system. The student may observe that neither an OA or a CC-BJT amplifier affords to *high* input and *high* output impedance as stand- alone devices. An OA has high input, but very low output impedance. So also is the case with a CC-BJT amplifier. Combining the OA and a CC-BJT amplifier in series-in, series-out negative feedback one can achieve the goal.

- *BJT based system example*

A simple example of a BJT based series-in, series-out feedback system is a CE-BJT amplifier with an emitter resistance R_E as shown in figure 4.9(a). The *loaded* amplifier ac equivalent circuit is depicted in Fig.4.9(b). The feedback gain is β=R_E.

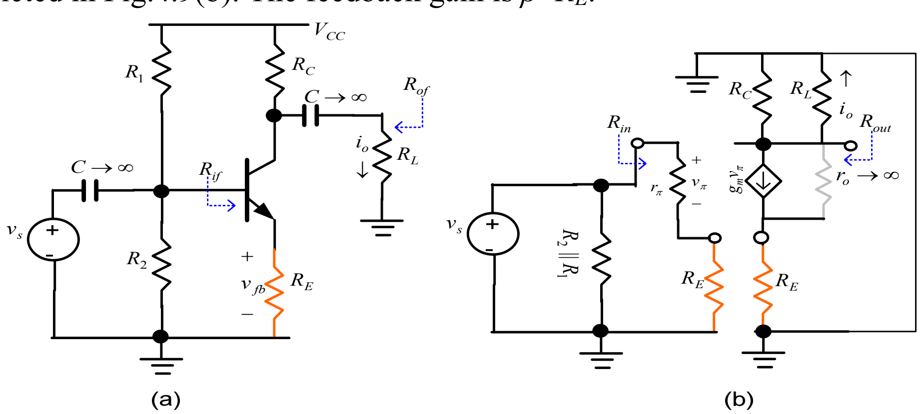


Figure 4.9:

The student is encouraged to work out the details to derive an expression for the voltage gain v_o/v_s where v_o is the signal voltage developed across the load resistance R_L. Note that the voltage gain will be A_f times R_L where A_f is the transconductance gain under feedback.

4.3.3 Shunt-series feedback

- *OP-AMP based system example*

Figure 4.10 depicts a shunt-series negative feedback system using an OP-AMP.The resistances R_1 and R_2 form the feedback elements. We will analyze the system using the

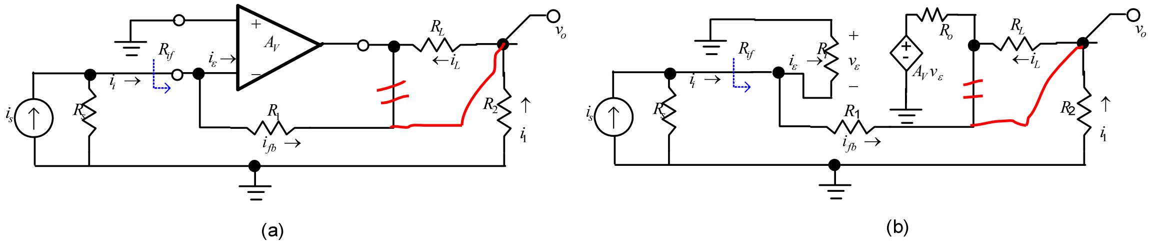


Figure 4.10: (a) The shunt-series feedback circuit schematic, (b) the associated ac equivalent circuit.

technique of *loaded* amplifier circuit.

A shunt series feedback needs the input source to be modeled as a current source. The two-port model of the feedback circuit elements will be the *g*- parameters. Figure 4.11(a)-(c) show the sub-circuits for the calculations of the *g*-parameter coefficients.

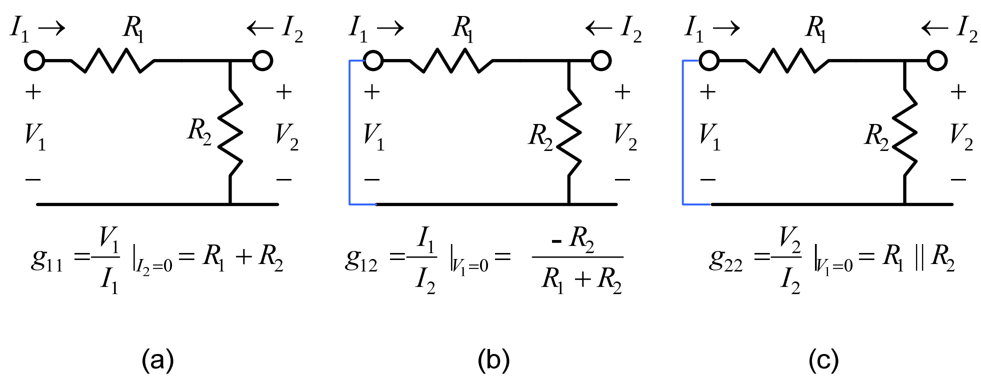


Figure 4.11: Illustration of calculation of the two-port model parameters

We now have to construct the *loaded* amplifier circuit by including g_{11} in shunt at the input, g_{22} in series with the output, and save g_{12} as the feedback gain β. For shunt in-series out feedback, the basic amplifier is to be modeled as a CCCS, i.e., the *loaded* gain will be of the form i_o/i_s. The equivalent circuit for the loaded amplifier is shown in Fig. 4.12.

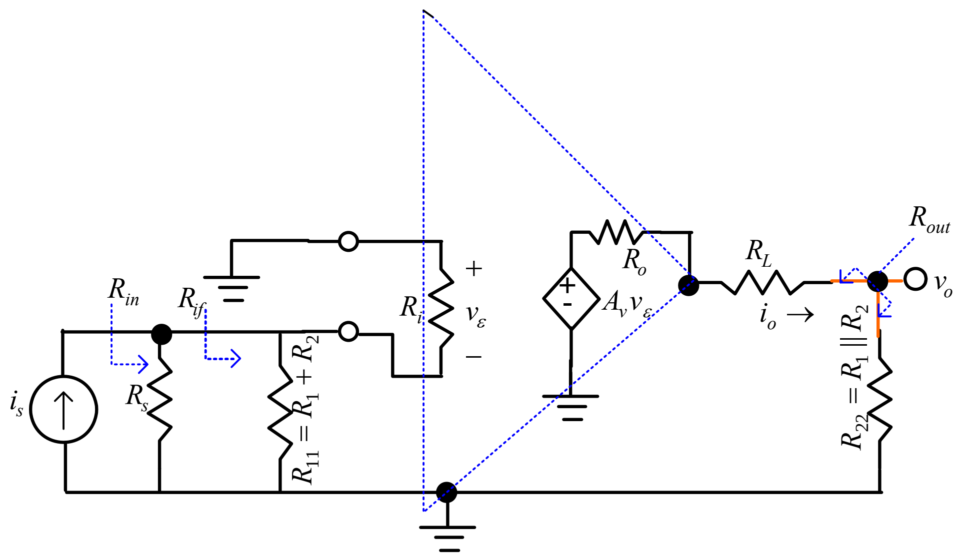


Figure 4.12: Loaded amplifier circuit after including the g_{11} and g_{22} parameters

***Analysis with the loaded amplifier*

From the equivalent circuit $A_i = \frac{i_o}{i_s} = -A_v \times \frac{R_s \| R_{11} \| R_i}{R_o + R_L + R_{22}}$; $R_{in} = R_s \| R_{11} \| R_i$, and

$R_{out} = R_o + R_L + R_{22}$. The feedback factor is $1 + A_i\beta = 1 + A_v \frac{R_s \| R_{11} \| R_i}{R_o + R_L + R_{22}} \times \frac{R_2}{R_1 + R_2}$ =F (say).

Then, the current gain with feedback is $A_{if} = \frac{A_i}{F}$. The input resistance at the location of R_{in} is

$R_{\text{inf}} = \frac{R_{in}}{F}$, and the output resistance at the location of the series (out) feedback will be

$R_{outf} = R_{out} F$.

From Fig.4.12, we can see that $R_{if} \| R_s = R_{inf}$. With R_s, R_{inf} known, R_{if} can be found out.

4.3.4 Shunt-shunt feedback

- *OP-AMP based system example*

Figure 4.13 presents the case of an OP-AMP based shunt-shunt negative feedback circuit with R_f as the feedback resistor. Shunt-shunt feedback will require the feedback resistor R_f to be modeled as a two-port Y-parameter circuit. Further, the basic amplifier has to be modeled as a shunt-series device, i.e., a trans-resistance amplifier. Thus, the loaded amplifier gain need be calculated as v_o/i_s.

The ac equivalent circuit of the loaded amplifier is shown in Fig.4.14(a), while the β circuit is shown in Fig.4.14(b).

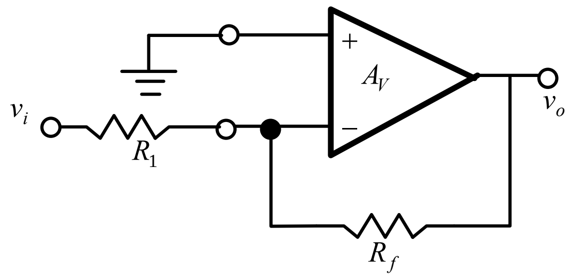


Figure 4.13: OP-AMP based inverting amplifier as a case for shunt-shunt feedback.

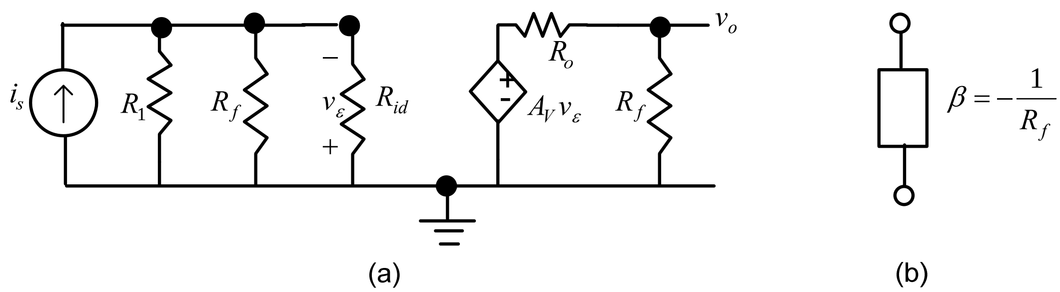


Figure 4.14: (a) ac equivalent circuit of the loaded amplifier, (b) the β circuit.

** *Analysis with the loaded amplifier*

The loaded amplifier gain is $A = \frac{v_o}{i_s} = -A_v R_1 \| R_{id} \| R_f \times \frac{R_f}{R_o + R_f}$.

The feedback factor is $F = 1 + A\beta = \frac{R_o + R_f + A_v R_X}{R_o + R_f}$, $R_X = R_1 \| R_{id} \| R_f$

The feedback gain (as trans-resistance) is $A_f = \frac{A}{F} = -A_v R_X \frac{R_f}{R_o + R_f + A_v R_X} \approx -R_f$, when $A_v R_X >> R_o + R_f$ holds. Remember that A_f is the trans-resistance gain v_o/i_s under shunt-shunt (negative) feedback.

Quiz: Considering that $i_s = v_i/R_1$, what is the voltage gain v_o/v_i in the present case?

4.4 Negative feedback and Stability

In the subject of the previous sections we assumed that the amplifier gain *A* and the feedback gain *β* are fixed numbers, independent of the frequency of the signal. This is far from truth. Since the amplifiers are built using BJT and/or MOS devices, the inherent parasitic capacitances in these devices render the amplifier gain *A* a function of frequency, i.e., *A(s)* with *s=jω*. Similarly, the feedback circuit may have the presence of reactive circuit elements (inductance, capacitance) either by design or as parasitic elements. Hence, in general, the feedback gain function $A_f = A/(1+A\beta)$ will be a function of frequency.

An interesting (but disturbing) result of such frequency dependence is that the system under negative feedback could become *unstable*, i.e., generate oscillations that are not expected. For a *stable* feedback system, it is necessary to investigate its stability characteristic before finalizing the design of the system. Specifically, we need to know if the designed system will be *potentially unstable* in the intended frequency range of application for signal processing. A procedure, originally introduced by H. Nyquist, is very convenient to determine the stability/potential instability scenario of a negative feedback system. In the following, we will assume that *only* the amplifier has a frequency dependent gain while the feedback gain *β* is a fixed number (less than 1) independent of frequency.

4.4.1 Nyquist plot

The Nyquist plot is the graph of *A(jω)β* as a function of frequency, plotted in polar co-ordinates. Consider the gain expression $A_f(j\omega) = \dfrac{A(j\omega)}{1 + A(j\omega)\beta} = \dfrac{x_o(j\omega)}{x_s(j\omega)}$ for a negative feedback system. If the denominator $1 + A(j\omega)\beta = 0$, we meet with the situation of $A_f(j\omega) \to \infty$, or, equivalently, $x_o(j\omega)$ is *finite* while $x_s(j\omega) = 0$. When a *finite* output signal exists with *zero* input signal, the system is said to be *unstable*. The system will produce oscillations at a frequency ω_o which satisfies the equation $1 + A(j\omega_o)\beta = 0$.

Nyquist criterion says that a negative feedback system will be *potentially* unstable when the polar plot of *A(jω)β* has a value of -1 (i.e, 1+ *A(jω)β*=0). Since we have assumed that *β* is a constant, the criterion leads to the following two conditions.

$$|A(j\omega)\beta| = 1, \text{ and } \arctan[A(j\omega)] = -180^{\circ}$$

The negative sign before the phase angle is used to imply a causal system where the output signal *always* lags relative to the input signal. According to usual scientific convention a *negative* phase angle is drawn in a *clockwise* direction.

When the amplifier gain function is known, it is possible to find the frequency ω_o at which the angle $\varphi = arctan[A(j\omega_o)]$=-180°. If at the same frequency, the relation $|A(j\omega_o)\beta| = 1$ holds then the system will be unstable/*potentially* unstable at the frequency ω_o.

An indication about the potential instability can be obtained easily by using the polar plot for the *loop-gain* function $A(j\omega)\beta$ and applying Nyquist criterion. Consider the plots shown in figure 4.15(a)-(c).

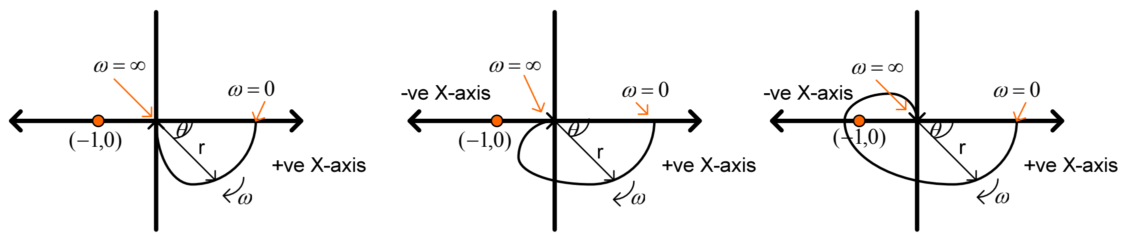


Figure 4.15: Polar plots of $A(j\omega)\beta$ with phase angles (at *infinite* frequency) of (a) 90°, (b) 180°, and (c) 270°.

In these plots the radius vector *r* is $= |A(j\omega_o)\beta|$, while the angle $\arctan[A(j\omega)]$ (described *clockwise*) is labeled as θ . The curves present successive frequency points ω from *zero* (i.e., dc) to *infinity* described *clockwise*.

In the graphs the point (-1,0) represents the radius *r* of value 1, i.e., $|A(j\omega_o)\beta|$=1, with the angle $\arctan[A(j\omega)]$=-180°. On close examination we can understand that out of the three plots in Fig.4.15(a)-(c), only the curve in (c) with a phase angle $\theta = -270^o$ at $\omega = \infty$ can cross the *negative* X-axis and hence has the possibility of *enclosing* the point (-1,0). One must realize that the *negative* X-axis represents a phase angle of -180°. At the point where curve in (c) intersects the *negative* X-axis, we have $|A(j\omega_o)\beta|$>1, and $\arctan[A(j\omega)]$=-180°. In terms of the polar plots in Fig.4.15(a)-(c), we can re-state Nyquist criterion as follows.

A negative feedback system with a forward gain *A(s)* and a feedback gain β will be *potentially* unstable if the polar plot of $A(j\omega)\beta$ *encloses* or *passes through* the point (-1,0) on the X-axis. The word *encloses* implies that the point in question remains on the *right* of the curve which is described *clockwise* as the frequency ω spans the values from *zero* to *infinity*.

When the plot *passes* through (-1,0), we have the conditions $|A(j\omega_o)\beta|$=1, and $\arctan[A(j\omega)]$=180° satisfied exactly. When the plot *enclose* (-1,0), we have $|A(j\omega_o)\beta|$

>1, while $\arctan[A(j\omega)]$=-180°. In the both the cases, the system will be *potentially* unstable.

4.4.2 Applications of Nyquist criterion

4.4.2.1 First order transfer function

Consider $A(s) = K/(s+a)$, where K and a are constants.

The phase angle of $A(j\omega)$ is –arctan(ω/a). As the frequency ω spans the values from *zero* to *infinity*, the angle will span the value from *zero* to -π/2 (i.e., -90°). Figure 4.15(a) represents this case.

Thus, for a first order (i.e., order of denominator *minus* the order of the numerator=1) gain function (or, transfer function) the condition arctan($A(j\omega)$) =-180° will never arise.

Hence a *first* order gain (or transfer) function will remain stable according to Nyquist criterion.

4.4.2.2 Third order function

Consider $A(s) = \dfrac{K}{(s+a)(s+b)(s+c)}, c > b > a$.

The phase angle is: –arctan(ω/a)-arctan(ω/b)-arctan(ω/c). Each term can contribute up to -90° as the frequency to *infinity*. Hence, total phase angle can be -270°. Thus, for a third order gain function, the phase angle can exceed -180° at some *finite* value of frequency. Figure 4.15(c) represents this case.

One can intuitively see that the gain function $A(j\omega)$ must be of order *two* for the phase angle to become 180° in magnitude at *inifinite* frequency.

Conclusion: In general if a transfer function *A(s)* has m_z number of simple (i.e., first order factors as ($s + z_i$), $i=1,2,..$) zeros and n_p number of simple (i.e., first order factors as ($s + p_i$), $i=1,2,..$) poles, the phase angle at *infinite* frequency will approach (m_z-n_p) times 90°.

Example 4.4.2.1: Consider the gain function $A(s) = (\dfrac{10}{1+s/10^3})^3$. If the feedback ratio β is independent of frequency, what value it should have so that the feedback system with the loop gain $A(s)\beta$ remains stable?

Solution/hint: First find the frequency where *arctan*(*A(jω)*) is -π radian, i.e., -180°. For the given function *arctan* [*A(jω)*]$= -3\tan^{-1}(\omega/10^3)$. If this is to be -180° (i.e., π radians) we can solve for $\tan^{-1}(\omega/10^3) = \pi/3$ i.e., 60°. The result is ω=1.732×10^3 rad/sec.

So at ω_o=1.732×10^3 rad/sec, *arctan*[1] [*A(jω)*] becomes -180°.

Now, we need to check for $|A(j\omega_o)\beta|$<1, i.e., β <$1/|A(j\omega_o)|$.

From the given expression for *A(s)*, we find $|A(j\omega_o)| = \left|\frac{10}{1+j\frac{1.732\times10^3}{10^3}}\right|^3 = 5^3 = 125$.

Hence, for the feedback system to be stable, *β<1/125* ,i.e., <0.008

4.4.3 Gain margin and Phase margin

From the above it is becoming clear that if at the frequency *ω* where *A(jω)* has a phase angle equal to -180°, the quantity | *A(jω)β*| remains <1, the possibility of instability can be avoided. Alternatively, if | *A(jω)β*| becomes =1 at certain frequency *ω*, but *ATan*[*A(jω)*] remains <-180° , the instability can be avoided.

The above inequalities are given more quantitative implications by the definitions of *gain margin* and *phase margin.*

The *gain margin* (GM) is the difference between the number 1 and the magnitude of *A(jω)* on the logarithmic scale, i.e., decibels (dB), when *ATan*[*A(jω)*] =-180°. It is given by:

GM=0-$20\log_{10}|A(j\omega)|$, i.e., -$|A(j\omega)|_{dB}$

The *phase margin* (PM) is the difference between 180° and the magnitude of *ATan*[*A(jω)*] in degrees, when | *A(jω)β*| =1. It is given by:

PM=180° - |*ATan*[*A(jω)*]|$_{\text{degrees}}$

A rule of thumb for a stable negative feedback system design is to maintain a GM ≥ 20 dB, and/or a PM $\geq 45°$.

Example 4.4.3.1: Consider an amplifier with an open loop gain of A_o =10^5. At high frequencies it is modeled as a single time-constant system with a pole frequency f_p =10

[1] Henceforth we will use *ATan* for *arctan*

Hz. The amplifier is connected in a negative feedback with a closed loop gain of +100 at low frequencies.

At what frequency will $|A(jf)\beta| = 1$? What is the phase margin?

Solution/hint:

The high-frequency model of the amplifier is $A(jf) = \dfrac{A_o}{1 + j\dfrac{f}{10}}$.

The feedback gain at *low-frequency* is $\dfrac{A_o}{1 + A_o\beta}$, which is given as +100, with A_o=10^5.

Solving for $\dfrac{10^5}{1 + 10^5\beta} = 100$, one gets β =0.01.

Now we set up the equation $\dfrac{10^5 \times 0.01}{|1 + j\dfrac{f}{10}|} = 1$, for $|A(jf)\beta| = 1$. On solving, we get f =10^4 Hz.

For phase margin, we need to find *ATan*[*A(jf)*] at f =10^4 Hz. That is $-ATan(10^4/10)$ which is $\approx -90^o$. Hence the PM is $180^o - |-90^o| = 90^o$,i.e., 90 degrees.

4.4.4 Frequency compensation

Frequency compensation modifies the frequency response of the feedback system so that the system remains stable. In particular, the gain function of the amplifier is modified such that at a frequency where the phase shift of the amplifier is equal or close to -180^o, the gain magnitude is reduced to a value below unity by a good margin.

A high gain amplifier, such as an OP-AMP, containing several devices (i.e., transistors) the gain function will in general be of order three or more. From the discussion in section 4.4.2 we can infer that the phase angle will reach the value of 180^o at some (*finite*) frequency in between the second and the third pole of the frequency response function. Thus, if the gain of the amplifier is reduced to *unity* at the frequency of the *second pole*, the gain magnitude will assume a value less than *unity* at the frequency of -180^o phase shift. Figure 4.16 clarifies the concept.

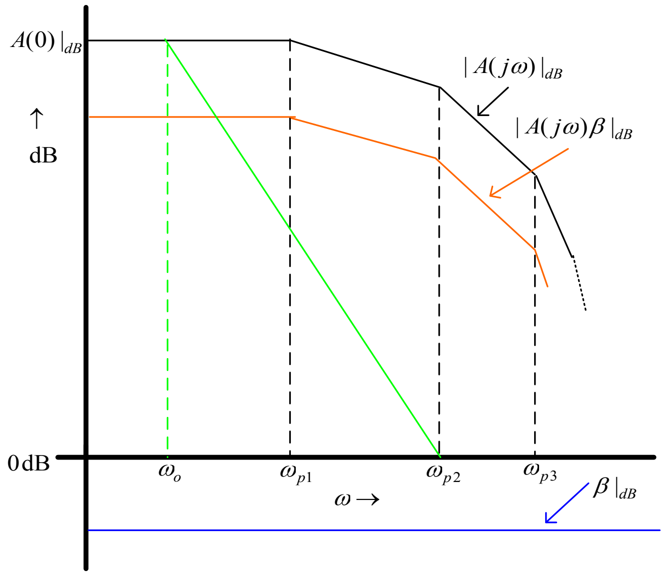


Figure 4.16: An amplifier gain plot and construction of the compensated gain curve

In a practical feedback system, the quantity β is always < 1, so that the loop gain $|A(j\omega)\beta|$ will become <<1 when $|A(j\omega)|$ is rendered <1. Hence a substantial GM will be achieved which, in turn, will render the system *stable*.

In Figure 4.16 we can erect a line at the second pole (ω_{p2}) with a slope of +6 dB/octve (i.e., +20 dB/decade) and extend it upwards until it intersects the mid-band segment (see *green* line in Fig.4.16) of the gain curve. If the corresponding frequency is ω_o, the system can now be viewed as a *first order* system with the pole at ω_o. In other words, the gain function of the frequency compensated can now be formulated as $\frac{A(0)}{1+ s/\omega_o}$. Since this represents a *first order* system, it will be stable under negative feedback.

In terms of the compensated gain curve (*green* line in Fig.4.16), at frequencies >> ω_o, the gain will change as $A(0)\omega_o / s$. This will become equal to *unity* (i.e., 0 dB) in magnitude at $\omega = \omega_t = A(0)\omega_o$. The frequency ω_t is the *unity gain-bandwidth* of the frequency compensated amplifier. In terms of Fig.4.16, since the compensated gain curve starts off from zero dB (=*unity* in value) at ω_{p2}, the ω_t equals ω_{p2}, the second pole of the frequency uncompensated amplifier. Thus $A(0)\omega_o=\omega_{p2}$.

Viewed from another angle, since the compensated gain curve falls off at 20dB/decade from the value $A(0)|_{\mathrm{dB}}$, an alternative relation between the -3dB frequency (i.e., ω_o) of the compensated curve and the second pole ω_{p2} of the uncompensated curve is $\omega_o = 10^{-m}\omega_{p2}, m = A(0)|_{dB}/20$ .Thus,

$\omega_o = \omega_{p2} / A(0) = 10^{-m}\,\omega_{p2}$, where $m = A(0)|_{dB} / 20$.

Example 4.4.4.1: Consider a multi-stage voltage amplifier with a low-frequency gain of 100 dB and poles at 10,000 rad/sec, 30,000 rad/sec and 10^5 rad/sec.

(a) Write an expression for the gain of the amplifier as a function of frequency.

(b) Find the frequency at which the phase angle of the gain function reaches -180°.

(c) Assuming that we need to maintain a gain margin of 20 dB, suggest the procedure to accomplish the task and calculate the new *compensated* pole of the compensated first order system.

(d) Assuming that we adopt a frequency compensation scheme to fix the *unity gain* bandwidth at the second pole of the uncompensated amplifier, what will be the realized gain margin relative to the *uncompensated* gain response curve?

Solution/hint:

(a) Since 100 dB is equivalent to the ratio 10^5, the gain function can be expressed as

$$A(j\omega) = \frac{10^5}{(1+j\frac{\omega}{10000})(1+j\frac{\omega}{30000})(1+j\frac{\omega}{10^5})}$$

(b) The phase angle (in *radians*) of the gain function is

$$\varphi = -ATan(\frac{\omega}{10000}) - ATan(\frac{\omega}{30000}) - ATan(\frac{\omega}{10^5})$$

From the analysis presented in section 4.4.2, it is understood that the phase angle will be -180° (i.e. –π radians) at a frequency in between the second and the third poles. At this frequency, the contribution due to the first pole will be close to -90°.

Thus we need to solve for $\frac{\pi}{2} = ATan(\frac{\omega}{30000}) + ATan(\frac{\omega}{10^5}) = ATan\frac{\frac{\omega}{30000}+\frac{\omega}{10^5}}{1-\frac{\omega^2}{3\times10^9}}$

Since *tan*(π/2) is *infinity*, the solution is $1 - \omega^2 / 3\times10^9$ =0, or ***ω*=54,772.26** rad/sec.

(c) According to the principle of frequency compensation discussed in section 4.4.4, the gain magnitude (in dB) at 54,772.26 rad/sec will be forced to -20 dB. The response curve will be a straight line rising upward at +20 dB/decade of frequency (characteristic of a first order pole) beginning from -20 dB at 54,772.26 rad/sec.

Since the low-frequency (i.e., mid-band) gain is 100 dB, the straight line will have to rise by 120 dB, i.e., by 120/20=6 decades of frequency, to intersect the mid-band gain line (see Fig.4.16, the *green* line). Hence, the new *compensated pole* frequency will be located at 54,772.26/10^6, i.e., **0.05477** rad/sec.

(d) If the compensated curve is set to *zero* dB (i.e., *unity* value) at ω=30,000 rad/sec, the *compensated* pole will be at $30{,}000\times10^{-100/20}$=0.3 rad/sec. The *compensating* straight line of slope 20db/decade will now glide down from 0.3 rad/sec., to 54,772.26 rad/sec which is the frequency of -180^o phase angle of the *uncompensated* frequency response curve.
The frequency 54,772.26 is $\log_{10}$ (54772.26/0.3)=5.26 decades above the *compensated* pole of the system. The compensated gain curve will drop by 105.23 dB (i.e., @ 20 dB/decade) over 5.26 decades. So the gain will reach a value of 100-105.23=-5.23 dB.
The gain margin relative to the *uncompensated* gain response curve will be 5.23 dB.

Quiz? Will the system be stable with only 5.23 dB of gain margin? Justify your answer.
Example 4.4.4.2: Repeat *example* 4.4.4.1 for an amplifier gain of 60 dB and the pole frequencies of 20,000, 60,000, and 200,000 rad/sec.

Hint: Following similar procedure as in *example* 4.4.4.1, the frequency for -180^o phase shift will be $\sqrt{6\times2\times10^9}$ rad/sec. Proceed on.

4.4.5: A technique to implement frequency compensation

The simplest technique to implement the frequency compensation in a multi-stage voltage amplifier system (i.e., an OP-AMP) is to locate two output nodes in the chain of amplifiers with a known *low-frequency* gain. Suppose these nodes are labeled as *m* and *p* respectively with an inter-stage gain of $-K_{mp}$ (take note of the *negative* sign). If we connect a capacitor *C* between nodes *m* and *n*, this will be reflected as a capacitor $(1+K_{mp})C$ (by Miller's theorem) between node *m* and ground. If R_{om} is the output resistance at the node *m*, the Miller magnified capacitor and the resistor forms a time constant of $(1+K_{mp})CR_{om}$.

The frequency compensation technique involves setting the *compensated pole* of the amplifier (i.e., 0.05477 radian/sec, in part (c) of *example 4.4.4.1*) equal to inverse of the time constant, i.e., equal to $1/(1+K_{mp})C$. We can then determine the value of the necessary compensating capacitor *C*.

In practice, however, we include a *lead* compensation by inserting a resistance in series with the capacitor *C*. This resistance can be realized, in an integrated circuit technology, by diode connected transistor(s). The interested student may refer to more advanced text books on this subject.[2]

4.5 Additional practice exercises

Q.1: Consider a feedback amplifier with open-loop gain of 10^4 at low frequencies. It has a -3 dB frequency of 100 Hz. Under negative feedback the low-frequency (closed loop) gain becomes 50.
(a) Write an expression for the frequency-dependent gain of the amplifier.
(b)What will be the -3 dB frequency under negative feedback?
(c) What is the loop gain (i.e., $A\beta$) at low-frequency?
(d) What is the feedback factor (i.e., $1+A\beta$) at low frequency?

Q.2: You need to amplify a signal from a microphone and deliver to a speaker. The mike produces 10 mV ac signal and has an output resistance of 5 kΩ. The signal input to the speaker pre-amplifier must be 0.5 V ac and the input resistance of the pre-amplifier is 50 Ω.
You are given an OP-AMP with R_i =10 kΩ, R_o =100 Ω, and a low-frequency open loop gain of 10^4. Use negative feedback principle to design the driver stage between the mike and the pre-amplifier. Neglect frequency dependence of gain.

(hint: *The OP-AMP output resistance has be lot smaller than pre-amplifier input resistance. So a negative feedback with shunt connection at output will be required. Similarly, the input resistance of the OP-AMP should be lot higher than just 10 kΩ. This can be achieved with … ? feedback connection that at the input*. Proceed further.)

Q.3: Design a feedback amplifier producing a closed loop current gain of 10. The current source has R_S =10 kΩ, and the load is R_L =50 Ω. An OP-AMP with R_i =10 kΩ, R_o =100 Ω, A_{vo} =10^4 can be used as the active device.
(hint: *a shunt-in, series-out feedback will be required around the OP-AMP (why?). Proceed further)*

Q.4: deleted
Q.5: Consider the single stage BJT amplifier in figure P.5(a) with a shunt-shunt feedback applied via R_F= 82 kΩ. The bias current is I_C= 0.5 mA. Given that h_{fe}=99, V_{BE} (ON)=0.7 V, V_A = infinity. Find the trans-resistance gain of the system under negative feedback.

[2] *Analog Integrated Circuit Design* (ch.5) by David A. Johns and Ken Martin, John Wiley & Sons Inc.,© 1997, ISBN 0-471-14448-7.

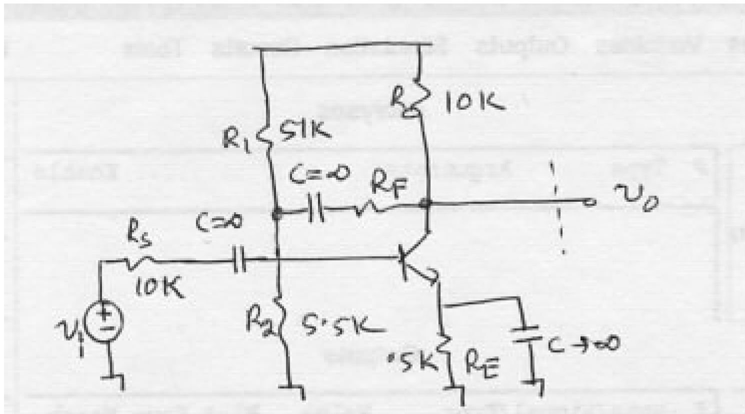


Figure P.5(a):

(hints: $r_\pi = 5\,k\Omega$, $g_m = 18.9\ mA/V$, $A_f = \dfrac{v_o}{i_s} = -65.8\ k\Omega$. *Use the ac equivalent circuit given in Fig.P.5(b).*)

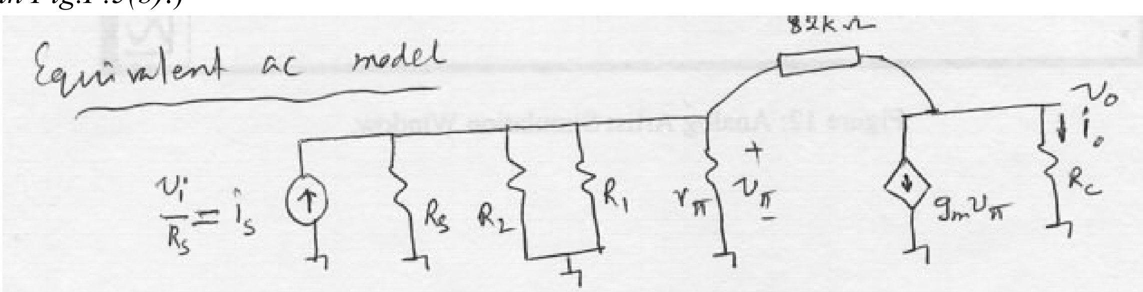


Figure P.5(b):

Q.6: A three-pole amplifier has a loop-gain function $T(f) = \dfrac{100\beta}{(1+j\dfrac{f}{10^5})^3}$. Determine the stability condition of this function for (i) β=0.2, and β=0.02. (hint: Determine the frequency where phase angle of *T(f)* is 180 degrees. Then find |T(f)| at this frequency for the different values of β. Then comment on these results.)

Q.7: Consider a three-pole feedback amplifier with a *loop gain* function given by:

$$T(f) = \frac{100\beta}{(1+j\frac{f}{10^3})(1+j\frac{f}{5\times10^4})(1+j\frac{f}{10^6})}$$

. Determine the value of β that yields a phase margin of 45 degrees. (hint: Find *f* where *T(f)* has -135° of phase angle. Use this *f* and set $|T(f)|=1$. You will find $\beta = 0.705$ (approx.))

Q.8: For the loop gain function in Q.6 above, find β (i) at which the system becomes unstable, (ii) at which the system has a phase margin of 60°. (ans: 0.08, 0.022 respectively).

Q.9: Given a 3-pole feedback amplifier with a *loop gain* function $T(f) = \dfrac{5\times 10^5}{(1+j\frac{f}{10^6})(1+j\frac{f}{10^7})(1+j\frac{f}{10^8})}$, find a compensating dominant pole frequency (<< 10^6 Hz) which will ensure stability of the system by maintaining a phase margin of 45^o at $f = 10^6$ Hz. (hint: *the new dominant pole can be M decades below 10^6 where* $M = 20\log_{10}(5\times 10^5)/20 = 5.7$. *Then proceed.*)

Q.10: Consider Figures P.10(a)-(b) which are examples of series-series negative feedback using BJT and MOSFET devices respectively. The feedback elements are R_{E2}, and R_{S2} respectively.

(a) Draw the ac equivalent circuit for the loaded amplifier corresponding to Fig.P.10(a).

(b) Draw the ac equivalent circuit for the loaded amplifier corresponding to Fig.P.10(b).

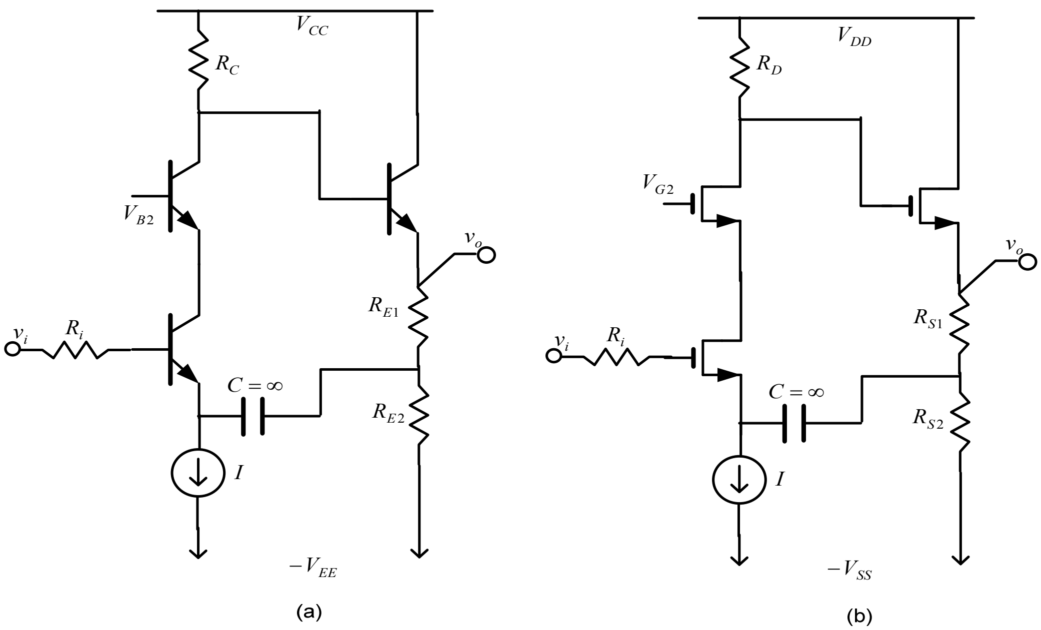


Figure P.10:

Q.11: Consider Figures P.11(a)-(b) which are examples of series-shunt negative feedback using BJT and MOSFET devices respectively. The feedback elements are R_{E1}, and R_{E2} in (a), and R_{S1}, and R_{S2} in (b).

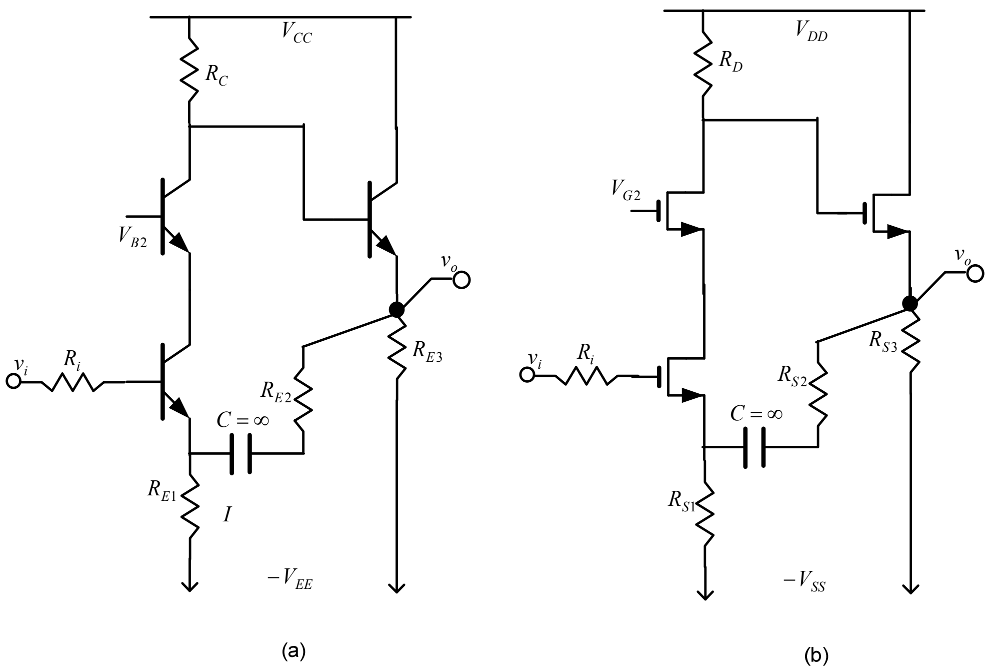


Figure P.11:

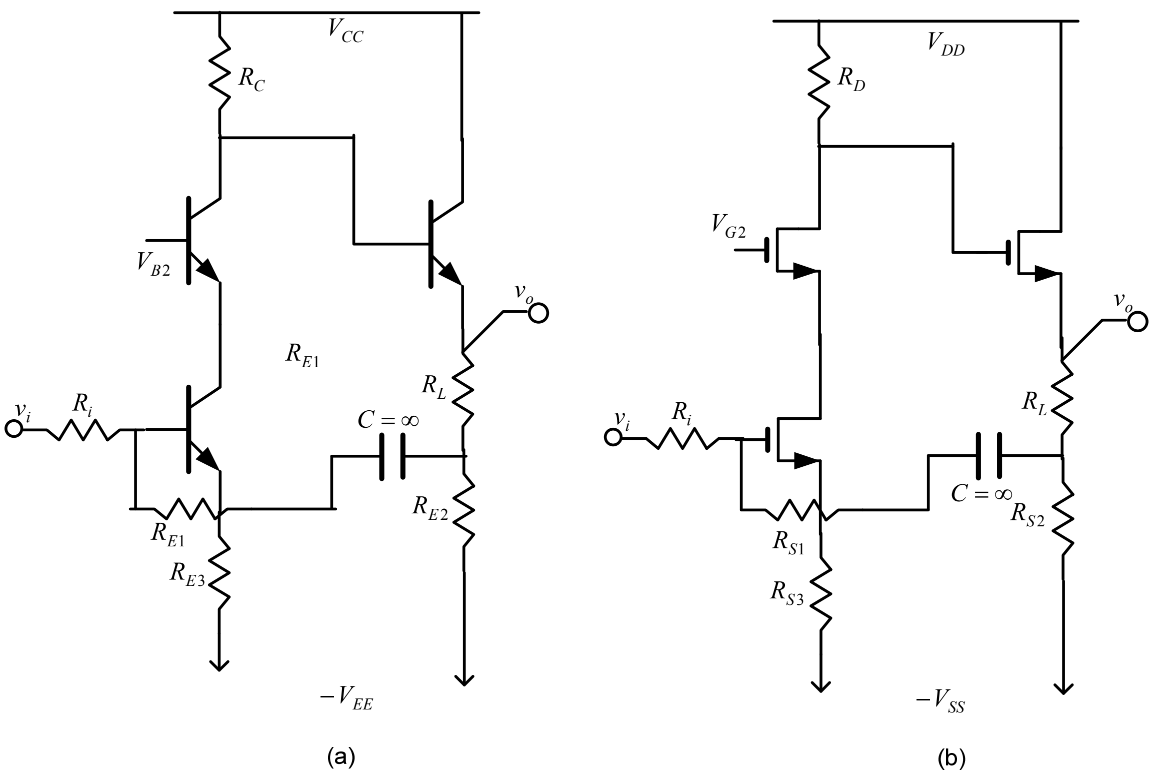
(a) Draw the ac equivalent circuit for the loaded amplifier corresponding to Fig.P.11(a).

(b) Draw the ac equivalent circuit for the loaded amplifier corresponding to Fig.P.11(b).

Q.12: Consider Figures P.12(a)-(b) which are examples of shunt-series negative feedback using BJT and MOSFET devices respectively. The feedback elements are R_{E1}, and R_{E2} in (a), and R_{S1}, and R_{S2} in (b).

(a) Draw the ac equivalent circuit for the loaded amplifier corresponding to Fig.P.12(a).

(b) Draw the ac equivalent circuit for the loaded amplifier corresponding to Fig.P.12(b).



V_{CC}
R_C
V_{B2}
R_{E1}
v_o
R_L
v_i
R_i
$C=\infty$
R_{E1}
R_{E2}
R_{E3}
$-V_{EE}$
(a)
V_{DD}
R_D
V_{G2}
v_o
R_L
v_i
R_i
$C=\infty$
R_{S1}
R_{S2}
R_{S3}
$-V_{SS}$
(b)

Figure P.12: