F. J. Heymann[1]

On the Time Dependence of the Rate of Erosion Due to Impingement or Cavitation

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ABSTRACT: Increasing attention is being given to the observed variations in the time rate of weight loss during impingement and cavitation erosion testing. It is generally recognized that a fuller understanding of these variations is needed for more meaningful use of the test results and more insight into erosion mechanisms. A pattern often observed is that of an initial period of little or no weight loss, successively followed by a period of high rate of loss and one of diminished rate of loss. Some results, however, follow other patterns, such as a continuously declining rate or a continuing sequence of fluctuations. There has been no full agreement concerning the causes for, and the significance of, the various phases.

Part I of this paper reviews the pertinent findings in the literature and discusses each of the following influences in some detail:

(*a*) The effect of the geometrical changes in the eroded surface on the macroscopic flow, cavitation, or impingement conditions which determine impact severity.

(*b*) The effect of the metallurgical and geometrical changes in the surface on the resistance of the surface to the impacts.

(*c*) The relative significance of the various material-removal mechanisms, such as single-impact failure, fatigue failure, and corrosion.

Part II develops a tentative statistical model of the erosion process for the case where fatigue is the predominant failure mechanism. The analysis predicts rate-time patterns similar to many of those observed, and it leads to the conclusion that the instantaneous erosion rates during nonsteady periods are strongly dependent on the scatter associated with finite fatigue life and with test parameters such as bubble or drop diameters. While the analysis predicts the eventual attainment of a steady-state rate independent of that scatter, this state is probably often unattainable in practice because the increasing roughness of the surface itself affects the erosion rate.

KEY WORDS: erosion, impingement, cavitation, work hardening, roughness, surfaces, mathematical models, statistical analysis, cracking, fatigue (materials)

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The recent literature dealing with the resistance of materials to impingement and cavitation erosion has become increasingly concerned with the fact that the rate of material loss is not uniform in time. While this, as a fact, had been noted for many years, some of its consequences have only lately been emphasized. Thus, as Thiruvengadam and Preiser [*1*][2] have pointed out, the comparison of test results can be very misleading if not based on corresponding phases of the rate-time curve, and therefore the rather common practice in the earlier literature, of testing all specimens for the same length of time, is subject to criticism.

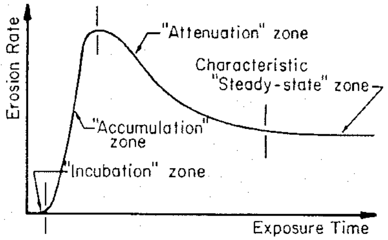

FIG. 1—*Characteristic rate-time curve according to Thiruvengadam* [*1*].

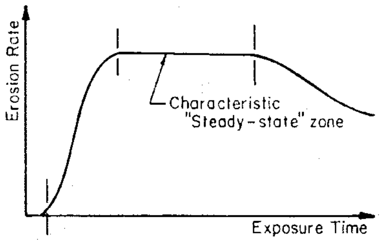

FIG. 2—*Characteristic rate-time curve according to Plesset and Devine* [2], *Hobbs* [*3*], *and Pearson* [*4*].

The authors of Ref *1* proposed that characteristic erosion-time curves could be described in terms of four zones: (1) "incubation zone" with no weight loss, (2) "accumulation zone" with loss rate increasing to a peak, (3) "attenuation zone" with decreasing loss rate, and finally (4) "steady-state zone" with constant loss rate (Fig. 1). They do not attempt any detailed explanation for these zones, but suggest that the first three zones are influenced by the initial condition of the surface and that only the final zone is truly characteristic of the material itself and that it should be used for comparison or correlation purposes.

This particular suggestion was disputed by Plesset and Devine [*2*],

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[2] The italic numbers in brackets refer to the list of references appended to this paper.

who showed photographically that in a magnetostrictive oscillator the attenuation zone is associated with a cavitation cloud of much reduced intensity, and attributed this to hydrodynamic damping effects due to the heavily roughened specimen surface. Moreover, the authors of Ref *2* held that the maximum erosion rate persists as a steady-state rate for some time rather than forming a narrow peak, as described by Thiruvengadam and Preiser [*1*], and that there is no real indication of any final steady-state zone. A characteristic curve of this type is shown by Fig. 2.

Similar statements have been made by a number of recent investigators. Thus, both Hobbs [*3*], using a magnetostrictive oscillator cavitation test, and Pearson [*4*], using a drop impingement erosion rig, have called the region of maximum erosion rate the "steady-state" period, and have based their correlations of erosion with material properties and test conditions (such as oscillation amplitude or impingement velocity) on this maximum loss rate. Both have associated the declining loss rate of the final period with heavy surface damage, as did Plesset and Devine [*2*], and feel that it is not a practicable measure of the erosion resistance. This point of view is also supported by Hammitt [*5*], whereas, on the other hand, a point of view similar to that of Thiruvengadam and Preiser [*1*] has been adopted by Evans and Robinson [*6*].

The object in Part I of this paper is to review in some detail the observations concerning erosion rate-time patterns which can be found in the literature and to discuss these in relation to the possible influences of surface-characteristics and material-removal mechanisms. The object in Part II is to show that a simple statistical model of the erosion process—which regards erosion as a multiplicity of fatigue failures—can predict characteristic rate-time curves of most of the observed types, and to discuss some of the implications of this model in relation to the measurement and correlation problem.

Part I—Review of Factors Influencing Erosion Rate

Observed Erosion Rate-Time Patterns

As early as 1928, Honegger [7] presented his impingement erosion results in terms of "specific erosion" (erosion rate) curves, which generally exhibited a rising and then falling pattern, although not enough readings were taken to establish the curves very precisely. His explanation was entirely a geometric one: "As long as the surface of the specimen is smooth, it offers an unfavorable surface for the impinging water to attack; hence, the water flows off to either side. Erosion does not take place therefore for some time. As soon as any roughness is formed, however, the erosion proceeds rapidly as the jet impinges with great force in the unevennesses. If, finally, the unevenness has attained a considerable depth, a layer of water adheres to the now completely roughened surface. This water absorbs part of the impact of succeeding drops of water so that the force of the jet is not so effective as formerly. The specific erosion consequently decreases after a certain depth has been attained." This explanation, especially of the reduction of erosion rate as the surface gets roughened, is essentially the same as that advanced in current papers on impingement erosion.

Curves of weight loss versus time given by Kerr [*8*], obtained in a magnetostriction cavitation device, do not show a distinct zero-loss incubation time, but do exhibit a slope which at first increases and later decreases. In tabulating the data, he acknowledged this by listing "initial erosion" based on the first 30 min of test and "relative resistance" based on the subsequent 60 min of test. By contrast, many of the loss-time curves obtained with wheel-and-jet type of impingement devices, during the 1930's and 1940's, exhibit a very marked incubation stage with zero or exceedingly small slope, followed by a stage of very much steeper slopes, but often of rather irregular and unpredictable shape.[3] Such curves were given, for example, by von Schwarz et al [*9*]

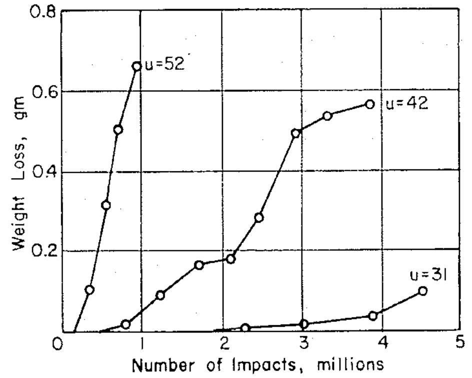

FIG. 3—*Typical cumulative erosion-time curves from jet impingement tests, adapted from Fig. 4 of Ref 10. (u = impact velocity, m/sec.)*

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[3] A possible explanation for this irregularity lies in the fact that these results were obtained at relatively low impingement velocities (generally less than 250 ft/sec) against the side of a relatively large diameter jet (up to 10 mm diameter). Thus, if we accept a fatigue mechanism of failure, the low impact stresses should result in a pronounced incubation time, and the large jet size—that is, large stress field applied on each impact—could be expected to result in large erosion fragments when failure does occur. Thus, the loss rate should exhibit a larger random fluctuation than if erosion took place by the formation of many smaller fragments, as it might in accelerated cavitation tests where the size of the individual bubbles is small but the impact stresses are much higher and occur at much greater frequency.

and Brandenberger and de Haller [*10*] (Fig. 3). However, even in these data one sees a definite tendency for the loss rate to decrease eventually. Nevertheless, many authors over the years have smoothed their results in such a manner as to show the erosion as progressing from an incubation stage, through an acceleration stage, to a presumed linear or steady-state stage, and have ignored the eventual diminution of the erosion rate.

Cavitation erosion curves of what might be called the conventional pattern were shown by Leith and Thompson [*11*] (Fig. 4). In the discussion to that paper, Plesset held that even during the incubation stage there is a small but increasing rate of material loss and that surface damage will eventually result in a nonlinearity of the erosion rate.

Most authors have presented their results in terms of cumulative loss-time curves, rather than by rate-time curves which require the differentiation of empirical data and will magnify all the scatter and uncertainty of those data. Whether, from a curve such as the upper one in Fig. 4, one would draw a rate-time curve such as Fig. 1 or one such as Fig. 2 will depend very much on one's belief of what the rate-time curve *should* look like. From the practical point of view of quantifying the results in terms of one or two numbers, it is convenient and not too inaccurate to draw a straight line through the general region of maximum steepness, and to specify its slope as a rate parameter and its intercept as an incubation parameter as done, for instance, by Mathieson and Hobbs [*12*]. This empirical procedure is not much affected by whether the "true" shape is a straight line leading to the rate curve of Fig. 2 or an S-shaped line leading to that of Fig. 1. (From a theoretical point of view the distinction *is* of interest, and this will be discussed in Part II.) The major criticism which can be leveled at some of the earlier investigators is that they have not even followed the aforementioned procedure, but they have tabulated and correlated their results on the basis of cumulative loss or of loss rates measured at one arbitrarily chosen point in time which may fall within completely different stages of the erosion-time pattern for different materials or conditions. The author has discussed some of the foregoing points more fully in Ref *13*.

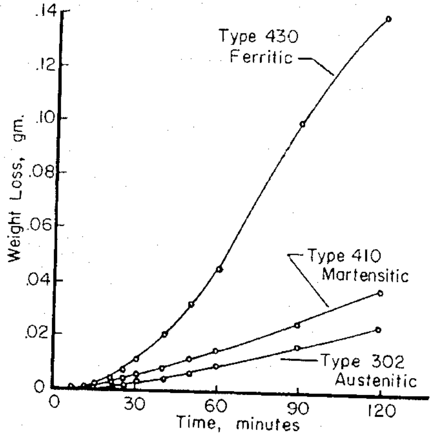

FIG. 4—*Typical cumulative erosion-time curves from cavitation tests, adapted from Fig. 7 of Ref 11. (Magnetostriction device, in distilled water.)*

Less Frequently Observed Rate-Time Patterns

All the previously mentioned results exhibited what may be called the "conventional" pattern or some minor variation thereof. However, there are erosion results which do not follow this pattern at all. Thus, Lichtman et al [*14*] presented loss-time curves of which many exhibit no apparent incubation or acceleration stages, but rather begin with a maximum rate which declines thereafter (Fig. 5). These results were obtained in a rotating-disk cavitation device.

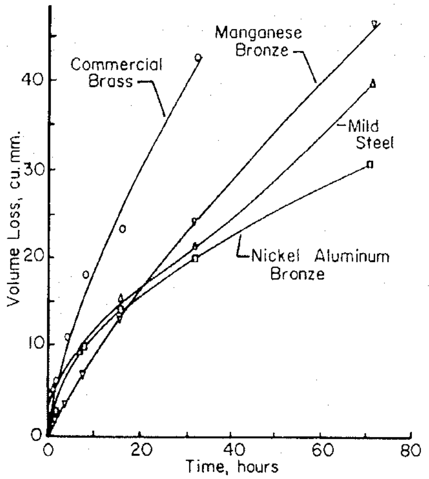

FIG. 5—*Cumulative cavitation erosion-time curves which begin at maximum rate, adapted from Fig. 24 of Ref 14. (Rotating-disk device at 150 ft/sec)*

Exactly the same type of result has been obtained in the spray impingement erosion test facility at the author's laboratory. Erosion rates invariably seem to begin at a maximum value and then decrease—rapidly at first, and then more gradually leading into or approaching a lower steady-state value. Figure 6 shows some "characteristic erosion-rate curves," obtained by curve fitting through points obtained from several specimens for each material. One might suspect that incubation and acceleration stages lie in the region to the left of the curves, as shown, and were simply missed because initial weight-loss readings were gen-

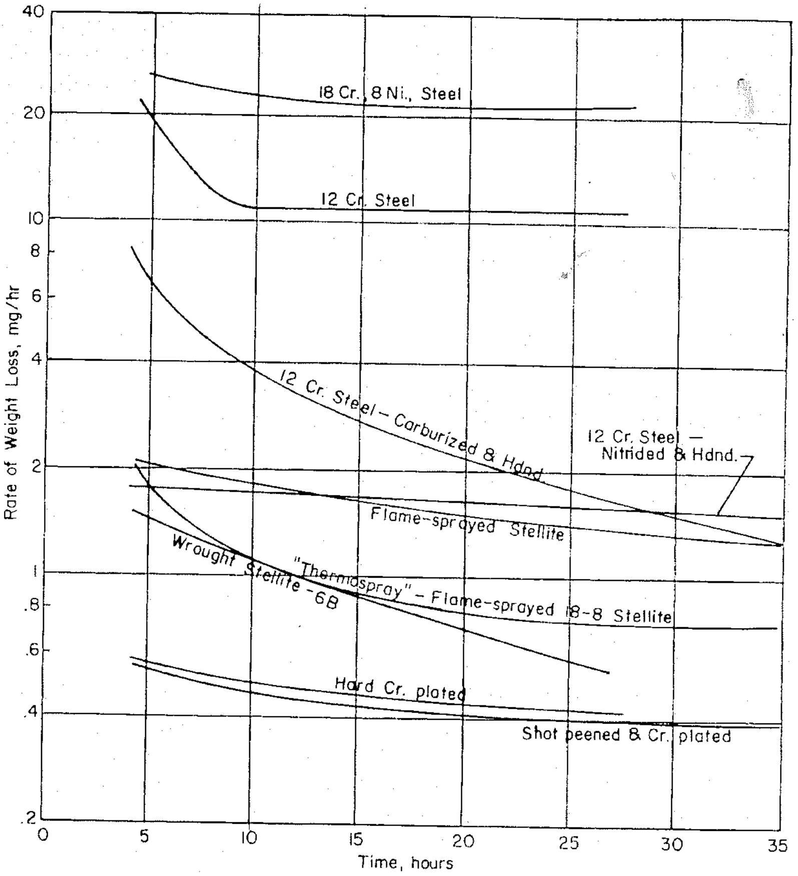

FIG. 6—*Typical erosion rate-time curves obtained in Westinghouse Steam Division spray impingement facility during 1956–1959.*

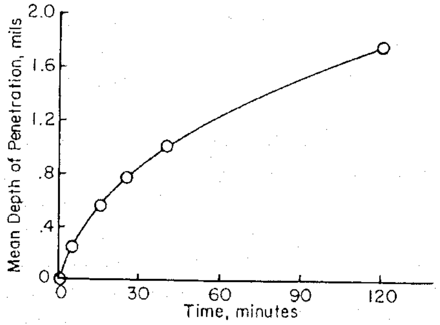

FIG. 7—*Early loss measurements for a titanium (6Al, 4V) alloy tested in the Westinghouse Steam Div. Facility.*

erally not taken until about 2 hr of exposure. In order to check this, we made measurements on one specimen—a titanium alloy of fairly good erosion resistance—after 5 min of exposure and several more times during the first hour of testing. (In all of the titanium specimens which we have tested, the erosion rate has continued to decrease for at least 30 hr.) The result is shown in Fig. 7 and gives every appearance that the erosion rate does in fact begin at a maximum value, or that if there is an incubation stage it occurred within the first minute. The latter alternative is supported by some of the analytical results to be discussed in Part II. However, it may be worth noting that Thiruvengadam [*15*] has shown the rotating disk to be the most intensive cavitation damage device and that our test facility produces impingement of probably rather small droplets at a high velocity, probably exceeding 2000 ft/sec. Thus, some single-impact damage may be occurring in both cases, contributing to the lack or deemphasis of an incubation period.

At the other extreme of a rather low-intensity cavitation damage device, deviations from the "conventional" rate-time pattern are also found. Thus, Hammitt [*16,17*] has described results obtained in a cavitating venturi, in which a brief initial stage of high loss rate is followed by a sort of "displaced" incubation period of little additional loss, again followed by a stage of significantly higher rate of erosion which may take on a variety of patterns. In some cases, for example, there is a series of increasing fluctuations. It should be mentioned, however, that the initial "high" loss rate represents a total material loss which is too small to be weighed and must be determined by pit-counting methods. It is possible that this stage—said to represent the removal of isolated very weak spots on the surface—occurs so rapidly in accelerated tests that it simply cannot be measured, and that the whole extent of these results corresponds to what would be the incubation and acceleration stages of an accelerated test.

In summary, erosion rate-time patterns such as described in Refs *1* and *2* have long been noted in both impingement and cavitation erosion test results. Any final explanation of this pattern, with a view to establishing which characteristic of it should be used for reporting the data and making correlations, should also be able to account for those results which deviate from the conventional pattern. Parenthetically, the comparison between different erosion results and the interpretation of the variations found in the erosion-time patterns would be made easier if these results were expressed in a rationalized form, such as "mean depth of penetration" (MDP), wherever this is feasible. This is becoming more common but is unfortunately not yet standard practice.

Factors Influencing the Erosion Rate

The erosion-time pattern can be influenced by at least three kinds of effects:

(*a*) The effect of the geometric condition of the surface on the cavitation or impingement severity and thus possibly on (*c*).

(*b*) The effect of the physical and geometric condition of the original surface, and of the changes in that condition, on the resistance of the surface to impingement.

(*c*) The relative significance or interaction of the various material-removal mechanisms, such as single-impact failure, fatigue failure, and sometimes corrosion, as determined by the impact severity and fluid properties, etc.

The desirability of expressing the material loss by a rationalized parameter was mentioned earlier. Of no lesser desirability is that the test duration be expressed in some rationalized manner, by which results may be more readily compared between different tests and extrapolated to operating conditions. In impingement erosion tests, the number of impacts has often been used, though for many correlation purposes the volume of water impinged per unit area, as used by Pearson [*4*], is preferable and results in a nondimensional erosion rate when combined with the MDP as a measure of loss. In cavitation erosion, the problem is somewhat more difficult, though Thiruvengadam [*18*] has made an attempt at formulating such a parameter.

For the purpose of investigating the nature of the erosion rate-time relationship, however, the most fundamental independent variable is surely some measure of the damage to the surface itself—the cumulative erosion or MDP, or the surface roughness, or perhaps some other measure. Clearly, any time variation of erosion rate (presuming the constancy of the gross environment) must somehow or other be related to changes in the material surface, and the discovery of which surface property provides the best correlation would in itself help the understanding of the causality involved. (Part II of this paper implies that the median lifetime of erosion fragments provides a significant time scale.)

Let us briefly discuss each of the three previously mentioned effects, with primary reference to ductile, work-hardening materials.

Effect of Surface Geometry on Impingement Severity

The manner in which the roughness of an eroded surface can reduce the intensity of cavitation attack has been referred to earlier and is discussed in detail by Plesset and Devine [*2*] for magnetostriction devices and by Hammitt et al [*17*] for cavitating venturi and rotating-disk devices. In impingement erosion devices, a protective liquid layer held in the depressions of a roughened surface has been supposed to account for a diminution of erosion rate with time, by Honegger [*7*] and many subsequent investigators. One might argue, however, that if this were indeed so, then while the valleys of the rough surface were protected the peaks would not be; thus, one would expect this process to converge rapidly to an equilibrium condition in which both the roughness and the erosion rate remain constant. This does not generally occur.

Some authors (for example, Hammitt et al [*17*]) have also supposed that in the earlier stages of roughening—before the previously mentioned effects become operative—the increased surface area due to the roughening exposes more surface to erosion attack and may therefore partially account for an initially increasing erosion rate. This seems unlikely, since if the number of droplets impinging, or number of bubbles collapsing, against the projected surface area remains constant, an increase of actual surface area would only result in an effective reduction of the concentration of impacts on the exposed surface.

In impingement erosion one may postulate other reasons why slight roughness may tend to increase the erosion rate and heavy roughness may tend to decrease it. Bowden and Brunton [*19,20*] have demonstrated that the high-speed impact of liquid on a ductile metal produces a ring of roughening, due to plastic deformation, around the impact point. They have stated that actual material removal in a single such event is by shear failure of these or previously existing minute surface steps, due to the high-speed radial outflow of the drop after impact. Even at much lower impact velocities, one may suppose that small surface irregularities of whatever source can provide stress raisers for the impact itself and may help to initiate fatigue cracks due to the radial flow-induced shear forces on the "peaks," which in turn could result in tensile stresses combined with stress concentrations in the "valleys." (See Fig. 8*a*.)

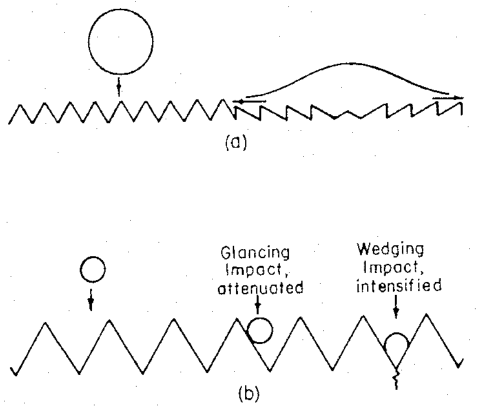

FIG. 8—*Schematic illustration of possible effects of small and large surface roughness.*

On the other hand, when the surface damage is gross enough so that the irregularities exceed the drop size, then the impact in many cases may be effectively a glancing one with a much reduced velocity component normal to the local surface, and it has been shown by a number of investigators recently [*4,21*] that the normal component of impact velocity is of paramount significance in determining erosion damage. However, a droplet hitting directly into the bottom of a depression or pit (if this is not protected by retained liquid within it)[4] may produce a multiplied or prolonged impact force, due to the lack of a free boundary from which pressure-release waves can immediately begin to propagate inward to relieve the water-hammer pressure in the impacting drop, as is believed to occur (see Ref *20*) in impact on a flat surface. Thus, the foregoing may help explain why erosion tends to take the form of deep pits rather than a general wearing away of the surface. (See Fig. 8*b*.) Pohl [*22*] has presented excellent photomicrographs of crack formations at the bottom of erosion pits.

Another geometric effect leading to a reduced impingement severity and erosion rate is believed to occur in steam turbines, where it has long been observed that the erosion of low-pressure blades may proceed to a rather severe stage in the first few months of operation and thereafter may slow down greatly or even cease completely. Here erosion is due to large liquid drops, which are swept slowly off the trailing edge of the previous stationary blade, and do not have time to be fragmented or to be accelerated to full steam velocity before they are impacted by the moving blades at a high relative speed and a negative angle of attack. As the blade is eroded, its "mean" leading edge recedes, and the path traveled by the drops may be sufficiently lengthened so that the additional fragmentation and acceleration of drops allows them to better follow the steam path through the blades. This effect, of course, is difficult or impossible to reproduce in conventional test facilities.

It is clear that some of the factors discussed in the foregoing influence the effective impingement velocity and thereby the relative significance of single-impact and multiple-impact or fatigue-type damage. There is also an interaction with the drop-size distribution, since a certain roughness may, according to the previous arguments, increase the damage potential of large drops and decrease that of small drops.

Effect of Surface Condition on Erosion Resistance

The surface conditions which affect the resistance of the material itself may include conditions of prestress, the degree and extent of work hardening, the prevalence of surface or immediate subsurface flaws which can act as stress raisers and crack-initiation points, and the manner in which the surface geometry facilitates local stress patterns conducive to rapid formation of detached fragments.

Work hardening of the surface material could be imagined to have two contradictory influences: on the one hand, a higher hardness is generally associated with higher erosion resistance; on the other hand, in ductile materials, erosion due to multiple impacts is generally held to involve repeated plastic deformation causing work hardening, which leads to eventual brittle failure or to formation of cracks which propagate like fatigue fractures. (See, for instance, Refs *20* and *23*.) By the first criterion, initial work hardening of the surface should retard the onset of erosion; by the second, it should promote it. Hobbs [*3*] has observed that in specimens work hardened by machining or grinding the incubation period is lengthened, and it is his practice to remove the work-hardened layer by hand polishing prior to testing. We have seen evidence of similar behavior. The apparent contradiction can probably be resolved by recognizing that in the foregoing case a cohesive work-hardened layer is formed over the whole surface, which reduces the amount of plastic deformation due to the impacts, and thereby retards the rate at which the amount of work hardening can be increased to the point where work hardenability is exhausted and fractures begin. The work hardening induced by erosion itself will, on the other hand, be localized at certain points, which for various reasons are susceptible to the greatest plastic deformation, and will not significantly inhibit the continuation of this process to the failure condition. Plesset and Devine [*2*] have shown by X-ray analysis that the plastic deformation in a surface subjected to cavitation damage reaches a stable depth almost immediately after the beginning of exposure and that this depth remains essentially constant thereafter as erosion progresses. They therefore conclude that subsequent changes in erosion rate are not attributable to changes in the metallurgical condition of the surface.

Most investigations of the effect of various surface treatments have shown an influence on the early stages only. This includes the effect of shotpeening [*12*] and etching [*9,24*]. Etching increased the erosion rate initially when compared to polished specimens. It was observed that, in the latter, one of the early effects of erosion attack is a "bringing out" of the grain boundaries and the creation of something like an etched appearance, due to minute failures in the weaker intercrystalline material. This observation has also been made by others, for example, Wheeler [*25*]. Keller [*26*] has shown that by periodic polishing of the surface the erosion rate can be maintained indefinitely at a value corresponding to that of the incubation period.

As far as the effect of the surface geometry on erosion resistance is concerned, it seems evident that the irregular shape of an eroded surface not only brings more inclusions, discontinuities, and other possible

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[4] Nor am I fully convinced that the presence of a small "pool" of liquid in an existing pit would necessarily mitigate the stress or force felt by the surface when a drop impinged therein. Some analytical work in this direction would be enlightening.

stress raisers and crack-initiation points to the surface, but also provides a geometry which makes it easier for the impacting drops to apply local bending or tensile stresses which, in combination with the aforementioned stress raisers, will induce failure. This is well exemplified by Fig. 9 which shows a section through an eroded surface.

Effect of Material-Removal Mechanisms on Rate-Time Pattern

For the sake of argument, the spectrum of erosion mechanisms in a ductile material may be divided into several regimes as a function of impact intensity, or, in the case of droplet impingement, as a function of impact velocity if drop size is held constant. These regimes obviously merge one into the other; there are no sudden transitions between them.

FIG. 9—*Photomicrograph of a section through an eroded stellite specimen (×250).*

For very low velocities below some "first threshold" value, no measurable damage or material loss will occur during any practical exposure time, or material loss is confined to isolated weak spots. Such threshold velocities, empirically deduced from test or operating experience or arbitrarily derived from the endurance limit of the material by some safety factor, have been used as design guides in some phases of steam turbine and condenser design. It is not fully established whether there actually is a velocity below which erosion will *never* occur. Honegger [*7*] doubted it; and Vater [*27*], who suggested that the dependence of erosion on velocity could be regarded and plotted analogously to the dependence of fatigue life on applied stress, regarded the erosion process as one somewhat similar to corrosion fatigue (in which there is no endurance limit). He therefore stated that the "threshold velocity" had to be defined as that velocity below which no measurable weight loss occurred after some specified number of impacts. In any case, one might say that in this first regime the erosion, if any, corresponds to that in the incubation stage of the conventional rate-time pattern; that is, it will be low, possibly gradually increasing with some random fluctuations, and will be highly influenced by the initial surface conditions, as previously discussed, and by the possibility of simultaneous corrosion as shown by Wheeler [*25*].

As the velocity exceeds the first threshold, something akin to fatigue failure becomes the predominant failure mechanism. Metallurgical observations substantiating this, and descriptions of the probable sequence of events leading to failure and the formation of loose fragments, have been provided by many investigators including Mousson [*28*], Vater [*27*], Pohl [*22*], von Schwarz et al [*9*], Plesset [*23*], Wheeler [*25*], Brunton [*20*], Thomas [*29*], and Marriott and Rowden [*30*]. Some investigators have found more plastic deformation in the surface than might be expected. Thomas [*29*] noted small plastic depressions in the surface during the early stages of exposure at velocities whose presumed impact pressures were less than the yield point of the material. Brandenberger and de Haller [*10*], on the basis of extensive radiographic studies, concluded that fracture in erosion is neither like static fracture nor like fatigue fracture, but is accompanied by a degree of damage to the crystal structure which is intermediate between that associated with those failure modes. It must be remembered, though, that the stress-geometry condition—at least when the surface is still relatively smooth—is not of such a nature as to make "static" rupture easily possible. Thus the general regime of predominant fatigue or repeated-impact rupture will extend well into the velocity range where each drop could be expected to produce noticeable plastic deformation.

In this regime one may expect to find rate-time curves exhibiting the "conventional pattern;" that is, an incubation stage related to the fact that a certain number of impacts are required before fatigue failures occur, an acceleration stage, possibly a steady-state stage, an attenuation stage, and possibly a final steady-state stage though probably no generalizations should be made about the behavior when gross surface damage has set in. The possibility of relating these phases in the erosion rate-time curve more specifically to the fatigue properties of the material will be explored in Part II of this paper.

A second threshold velocity may be associated with that velocity at which the material loss due to single-impact damage process becomes significant. This is probably related to the "visible damage threshold" described by DeCorso and Kothmann [*31,32*], above which a single impact leaves a distinct crater in a smooth material surface. This regime eventually must merge into the regime of hypervelocity impact. The exact determination of the second threshold velocity from the point of view of material removal is difficult, because in single-impact experiments—such as those performed by DeCorso, and also by Brunton [*20*], Engel [*33*], and others—the actual material removed from the surface could not be reliably established, although crater depths or crater profiles were measured. From two curves reproduced in Ref *34*, one can deduce that, for hypervelocity impact of $\frac{1}{16}$-in.-diameter aluminum spheres on an aluminum surface, the ratio of target volume loss to crater volume is approximately 0.15 at a velocity of 7 km/sec (23,000 ft/sec), reducing to about 0.09 at 4 km/sec (13,000 ft/sec). One may cautiously infer from this that at the velocities of interest to us, say 1000 to 4000 ft/sec, the corresponding ratio will be very much smaller yet.[5] Of course, this must be balanced by the fact that such loss occurs with each impinging drop, whereas many repeated impacts over some finite area are required to generate one erosion fragment by the fatigue failure mechanism. For any quantitative estimate of the relative significance of the two mechanisms, more data are needed on each.

Qualitatively, one may say that as single-impact erosion becomes significant the incubation period can no longer be a zero-weight-loss period, but rather will begin by exhibiting an erosion rate corresponding to the single-impact erosion, this rate increasing in time as additional fatigue-type erosion sets in. Fatigue in this instance probably corresponds more to low-cycle fatigue due to strain cycling than to high-cycle fatigue due to stress cycling. The geometry of the eroded surface will now be affected by the heavy plastic deformation due to each drop as well as by the breaking away of larger erosion fragments due to fatigue fractures. More work is needed on the relationship between surface geometry and impact damage under these conditions before the rate-time pattern in the regime of single-impact damage can be reliably described.

Even when initial conditions are still in the "fatigue regime," once the surface is slightly eroded it may become susceptible to single-impact failures because of geometrical effects discussed earlier.

Part II—A Statistical Erosion-Rate Model

Qualitative Description of Proposed Model

As we saw in the previous sections, the "conventional" erosion-rate-versus-time pattern is that associated with a predominant fatigue mechanism for material removal. It is in this regime that most of the test data and the practical experience lie. As is well known, fatigue is intrinsically a statistical process exhibiting a considerable scatter, and this fact will be made use of in developing an analytical model for the erosion rate-time pattern applicable to this regime. The qualitative results have interesting implications with reference to the previously reviewed findings and to previously attempted correlations between erosion and fatigue data. The approach to be described, though numerical in nature, can at this time predict no more than qualitative trends and should be considered as exploratory.

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[5] This inference should be valid qualitatively although the actual material-removal mechanism in the hypervelocity regime is a liquid-like flow of the target material accompanied with some "splashing out," whereas that in our regime of interest is related to the shear effect of radial outflow, as described earlier.

The mathematical and logical formulation of the model, in a preliminary form, is given in detail in the Appendix. The basic reasoning of the model is as follows: We assume that each small element of surface is subjected to an impact fatigue environment and that after a certain time (that is, a certain number of impacts) it will be detached from the surface as an erosion fragment, due to subsurface fatigue failure. Further, we assume that when many such surface elements are considered, the individual times required for their removal would be described by some statistical distribution function, much as the number of cycles to failure of a large number of fatigue specimens (stressed to the same level) can be described by a distribution function. When erosion fragments are removed and expose "fresh" surface to impingement attack, the time to remove elements of this new surface will likewise be described by a distribution function and so on. The time-to-failure distribution function for these newly exposed surfaces will probably not be the same as that for the original surface, since they will have been subjected to some subsurface stress condition, even before being exposed to direct impingement, and since the surface geometry will be different.

In the case of conventional fatigue specimens, the distribution occurs primarily as a result of the statistical nature of the fatigue process itself. In the case of erosion fragments, it must ultimately reflect the variations in the concentration and the severity of impacts (that is, droplet velocities and sizes), variations in the local surface geometry and properties, and variations in the size of fragments formed. At present, however, one arbitrary distribution curve is assumed to represent all of these sources of scatter.

Qualitatively, it can be seen that if these distributions had very little scatter or dispersion, that is, if the lifetimes of all surface elements were about equal, then the erosion rate would be zero until that lifetime was reached, at which instant a very high rate would be exhibited while all of the original surface flaked off, to be followed by another interval of zero rate until the second layer flaked off and so on.

If, however, these distributions have a significant dispersion, one can intuitively predict that this will result in a rate-time curve which up to a first peak looks somewhat like the distribution curve, but in which subsequent peaks and valleys are attenuated and a steady-state rate is approached. An "incubation period" will exist if the dispersion is not ex-

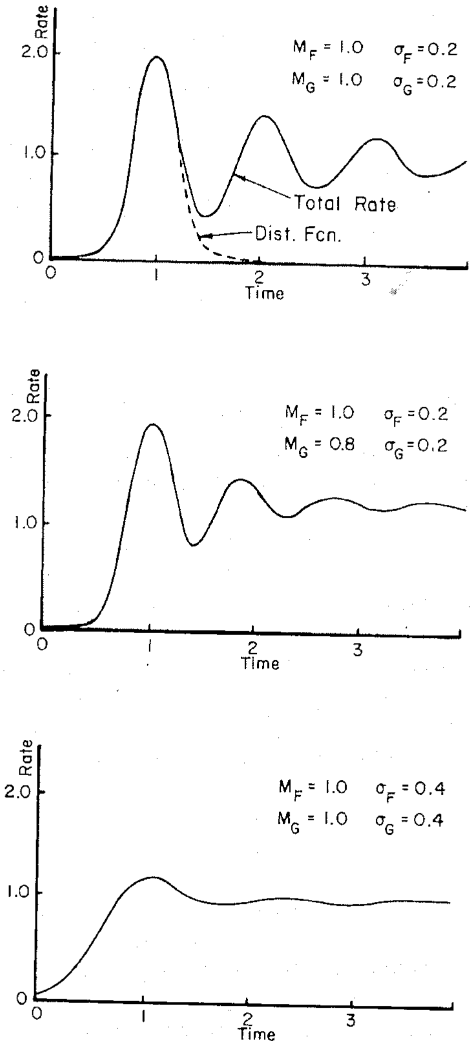

FIG. 10—*Typical computed erosion rate-time curves from preliminary statistical model, using Normal distribution functions.*

cessive. One might think of the variation in the surface-element lifetimes as "dispersing" the periodicity associated with one layer being removed after another.

Results of Preliminary Model

The preliminary mathematical formulation and computer program considered one distribution function applicable to the original surface

and one other applicable to each of the subsequently exposed "surfaces." Both were specified as Normal distributions truncated and normalized over a finite time span. Thus the significant input parameters were the nominal mean lifetime, M_F, and standard deviation, σ_F, for the original surface, and the corresponding values, M_G and σ_G, for the "undersurfaces." Figure 10 shows some rate-time curves obtained by this program, with the distribution parameters as indicated. Note that the attaining of a steady-state rate is hastened both by increasing the dispersion of the functions and by specifying a shorter mean lifetime for the undersurfaces as compared to the original surface.

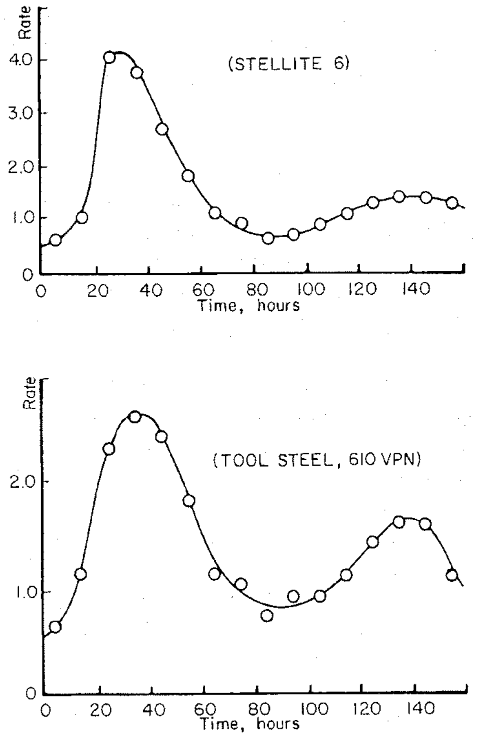

FIG. 11—*Experimental erosion rate-time curves, computed from cumulative erosion curves given in Ref 35.*

Fluctuations such as shown in Fig. 10 have occasionally been observed, as illustrated by Fig. 11 which shows rate-time curves computed from experimental cumulative erosion curves presented by Kent [*35*]. Moreover, fluctuations which would appear quite prominent in rate-time curves are not nearly as evident if the same data are plotted as cumulative erosion versus time—which, after all, is how the data are actually obtained. Therefore, it seems quite conceivable that in many cases such fluctuations would barely have been noted and would have been "smoothed" out of the raw data, or might have been lost entirely through the data points being too far apart in time.

The fluctuations, however, are by no means an inevitable consequence of this model if nonsymmetrical distribution functions are used, as will be seen in the results obtained from the elaborated formulation of the model, described in the following section.

Description of Elaborated Model

The elaborated model permits the specifying of a different distribution function for each "level" below the original surface and of two different functions for the original surface: one for the "unaffected surface," in which erosion takes place by the initiation of new pits, and one for the "affected surface," which is that surrounding existing pits and in which erosion is presumed to take place by the growth of these pits. The program computes the rate of erosion, the cumulative erosion, and the exposed area at each level, from which, in turn, it computes an average surface profile and surface roughness at selected time points. The formulation and programming approach is described in detail in Appendix A of Ref *46*.

For the elaborated analysis, we have adopted the log-Normal distribution; that is, one which would appear as a Normal distribution if the frequency of failures (based on *logarithmic* time increments) were plotted on a logarithmic time scale. This has been shown to provide a reasonable description of fatigue life data by, for instance, Stulen [*36*], Roeloffs and Garofalo [*37*], and Epremian and Mehl [*38*], and is convenient to handle mathematically. It is generally recognized, however, that a more rational and accurate description would be provided by three-parameter distributions such as the Weibull distribution [*39*] or extreme value distribution [*40*]. The use of one of these distributions should be considered in further developments of this model.

A rigorous analysis intended to give quantitative results would also have to take into account the variations in drop sizes and velocities which may be encountered under real impingement conditions. This would bring the theories of cumulative damage into the picture. The physical and statistical aspects of fatigue life under repeated stress cycles of varying amplitude have been discussed, for instance, by Freudenthal [*41*], who presents a number of different possible approaches to the problem. Several of them lead to the conclusion that "high stress amplitudes shorten the fatigue life out of all proportion to their number of application or their cycle ratio." This would seem to argue against the adoption of a simplified approach such as a superposition of the erosion rates due to various droplet size or velocity ranges computed independently.

For our present, explorative purposes, we remain with the log-Normal distribution or loss-rate function. On a logarithmic time scale, the log-Normal is described by its mean, m, and its standard deviation, σ, both m and σ of course being logarithmic magnitudes. For our purposes, we must transform the distribution onto a linear or "real-time" scale. It then becomes a skewed distribution and its *mean, median,* and *mode* values no longer coincide, as they do in a symmetrical distribution.

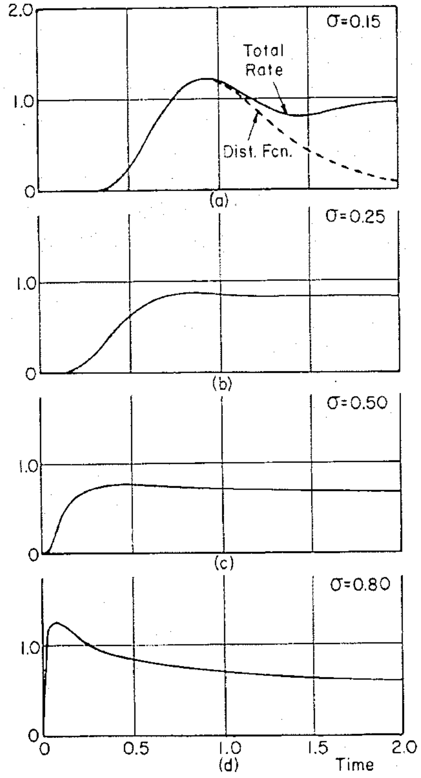

FIG. 12—*Computed rate-time curves based on log-Normal distributions, showing effect of varying dispersion σ while keeping median at constant value $M = 1.0$.*

The real-time value whose logarithm is m, which we shall denote by $M = 10^m$, establishes the *median* value of the log-Normal distribution, that is, that value of t at which half of the specimens (or surface elements) will have failed. This is the value generally used to establish a point of an engineering S-N curve. The *mode,* or peak in the distribution curve, will occur at a value less than M. The *mean* value, E, or arithmetic average of all lifetimes, will occur at a time value greater than M, namely at time $E = M \times 10^{1.15\sigma^2}$. For purpose of discussion, we will characterize all distributions by their values of σ, and either M or E.[6]

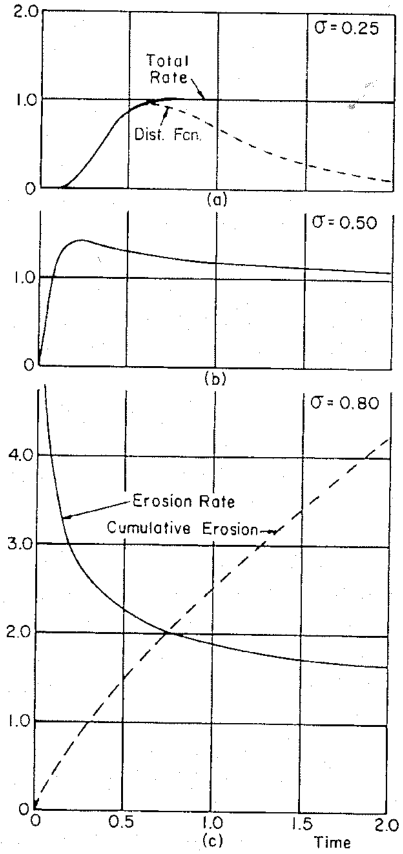

FIG. 13—*Computed curves based on log-Normal distributions, showing effect of varying dispersion σ while keeping mean at constant value* $E = 1.0$.

From Refs *38* and *41* one may infer that, for fatigue data, σ ranges from 0.13 to 0.40, with 0.25 a representative value. For erosion fragments one might well expect even higher dispersions. These statistical aspects are discussed in more detail in Appendix B of Ref *46*.

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[6] The log-Normal distribution function and its properties are discussed on pages 115 and 117–118 of Ref *42*, but the formula given there for the probability density function (that is, our rate function) seems to be incorrect. A correct expression can be found on p. 220 of Ref *43*.

Results of Elaborated Model

The number of variations which could be investigated with this program is unlimited, and all we can demonstrate here are some of the important effects. The most significant of these is the effect of the dispersion parameter σ. Figure 12 shows computed erosion-time curves for various values of σ from 0.15 to 0.80, with the median M held constant. Figure 13 shows a similar set of curves with the mean E held constant. In each case, the same distribution is assumed for all surfaces and levels. Since in such cases the eventual steady-state erosion rate must be proportional to the reciprocal of the mean lifetime, all curves in Fig. 13 approach the same steady-state rate.

Two striking results appear from these curves: (*a*) The maximum erosion rates vary considerably, and (*b*) almost all of the experimentally found rate-time patterns (discussed in Part I of this paper) can be at least qualitatively generated by proper choice of the dispersion parameter σ. When σ is small, the curves exhibit damped fluctuations similar to those of Fig. 10. When σ is increased, the fluctuations die out and the steady-state rate is attained quite quickly. When σ is further increased, a single peak appears in the curve. At very high values of σ, this peak may occur so early that the time resolution is just not fine enough to show the acceleration stage of the rate-time curve, and the curve therefore appears to begin at its maximum value. The same is probably true for experimental data like those of Figs. 5 and 7. It does not seem unreasonable to suppose that erosion due to very small droplets, where each impact stresses only a minute portion of the surface area, would be characterized by a high dispersion in the fragment lifetimes.

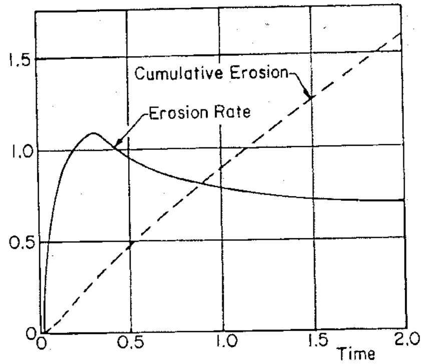
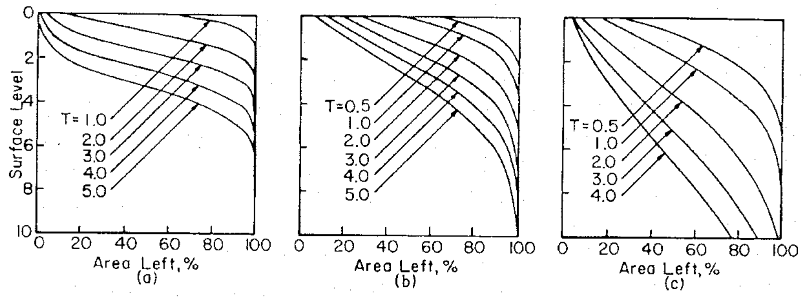


FIG. 14—*Example of the effect of using a higher median value for the "unaffected" surface* ($M_u = 3.0$) *than for all other surfaces* ($M = 1.0$). *Compare with Fig. 12d, but note difference in vertical scale.*

In many of the curves of Figs. 12 and 13 the ratio of the erosion-rate peak to the expected steady-state value is not as great as sometimes found in practice. But it should be recognized that at time values greater than the median, the surface has suffered heavy erosion damage, and one may therefore expect that geometric effects such as described in Part I may have set in by this time and have caused an additional diminution of the erosion rate and possibly suppression of further fluctuations. Certainly one would expect the results predicted by this analysis to be at least modified by the geometric effects. Thus, Figs. 12*d* and 13*b* may correspond to experimental results of the type of Fig. 1 and Figs. 12*b* and 13*a* to results of the type of Fig. 2. It is possible, however, that some appropriate combination of distribution functions for the different surfaces could result in an elongated hump such as in Fig. 2, which then again would not constitute a steady-state value.



(*a*) The case of Fig. 12*b*.
(*b*) The case of Fig. 14.
(*c*) The case of Fig. 13*c*.

FIG. 15—*Examples of computer "surface profile" curves, showing the uneroded area as a function of level below the original surface, at various values of time T.*

Figure 14 shows an example of slowing down the loss rate from the "unaffected" surface as compared to that of all other surfaces—which are presumed to be more susceptible to erosion because of the irregular geometry, as discussed in Part I. This case is identical to that of Fig. 12*d* except that for the unaffected surface the median lifetime has been increased to 3.0. Note that the shape of the rate curve has been made more similar to that typified by Fig. 1; the cumulative loss rate is also shown and is quite similar to typical curves such as Fig. 4.

Figure 15 shows "surface profile" curves at various values of the time *T*, computed for some of the previous cases. The ordinates indicate the surface "level," with 0 representing the original surface. The abscissas represent the area not yet eroded at each level. (Thus the difference in abscissa between adjacent levels represents the area "exposed" at the lower of the two levels.) Note that in Fig. 15*a*, a case of low dispersion value ($\sigma = 0.25$), the erosion is shallower and more evenly distributed than in the other two cases which represent high dispersion values ($\sigma = 0.8$). This suggests that the geometric effects due to severe roughness—which tend to reduce the erosion rate—are delayed in the former case, which may explain why the maximum erosion rate in such a case may persist for some time and give rise to rate curves typified by Fig. 2. Figure 16 shows the computed surface roughness, versus computed mean depth of penetration, for the same three cases, confirming the lower roughness associated with a lower dispersion value.

Discussion and Conclusions

Let us now examine the implications of this model with respect to correlations of incubation times and erosion rates. Since the incubation time seems obviously related to the fatigue nature of erosion, several investigators have attempted correlations reflecting this. Thus Leith and Thompson [*11*] correlated the incubation times of several materials with the corrosion fatigue limit for 10^7 cycles of these materials. Mathieson and Hobbs [*12*] made a similar correlation with the conventional endurance limit for several aluminum alloys. In both cases, the results were reasonably consistent, but the approach is hardly logical since the incubation time in erosion surely should be related to a finite lifetime to failure rather than to a stress value at which no failure occurs. Thus the success of these correlations surely depended on a second, implicit correlation between the finite fatigue lives at the test stress, and the endurance limits, valid for the group of materials compared. Ripken et al [*44*] have used a more logical approach and have correlated the number of impacts corresponding to the incubation time at a given impact velocity, with the number of cycles to failure in bending fatigue at an equivalent stress level.

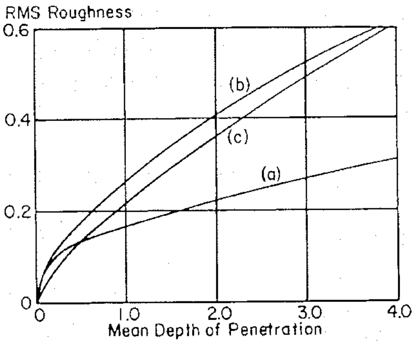


FIG. 16—*Computer rms surface roughness versus mean depth of penetration (cumulative erosion) for the examples of Fig. 15. The letters* (*a*), (*b*), *and* (*c*) *correspond to the similarly designated cases in Fig. 15.*

The stress level was assumed to be given by the well-known water-hammer pressure, $p = \rho CV$. The incubation period was defined by the intercept on the time axis of the straight-line tangent to the steepest slope of the cumulative weight loss curve.

If the previously developed model is valid, this procedure is still not quite correct. The statistical model implies that the apparent incubation period depends not only on the mean lifetime of the erosion fragments but also on the scatter in these lifetimes. The erosion rate becomes nonzero when the first "element" fails and continues to increase until approximately the mode or most probable value of the lifetime is reached on the top surface. But it is the median value—which may occur later yet if the distribution is skewed—which corresponds to the nominal lifetime at the appropriate stress as obtained from a conventional *S-N* fatigue curve. Whether either the median lifetime or the associated scatter in erosion fragments corresponds to that of full-scale bending or pull-type fatigue specimens is at present a moot question. However, the discrepancies in the correlation of Ref *44* are in the direction which the foregoing argument would predict.

If one stipulates a steady-state erosion process, then the erosion *rate* would certainly be inversely proportional to the *mean* lifetime of erosion fragments (provided their size distribution remained constant). This is the basis from which one can draw the analogy between the loss-rate reciprocal versus impact velocity in erosion, and cycles to failure versus stress level in fatigue, as proposed by Ref *27*. This appears to provide a rational basis for attempting to predict an erosion-speed relationship on the basis of known fatigue data for the material, although to my knowledge this attempt has not been made. But here, again, the statistical model suggests that the "obvious" approach is not quite correct. It implies that the maximum erosion rate—by which many investigators have, for practical reasons, reported and correlated their results—does not necessarily represent a steady-state erosion process at all, but rather the "deluge" of erosion fragments from the top surface layer which takes place in the vicinity of the "most probable" fragment lifetime from the beginning of exposure. Thus the maximum instantaneous erosion rate is, again, not merely a function of the average fatigue life of the surface elements but also of the scatter in lifetimes. Consequently, anything which influences that scatter will influence the maximum erosion rate, even though it may not affect the eventual hypothetical steady-state rate.

What can this model contribute toward the resolution of the dispute referred to in the beginning of this paper? It implies that Thiruvengadam and Preiser [*1*] are correct in claiming that the erosion rates during the stages encompassing the first peak in the rate-time curve are not characteristic merely of the material under test, since, as we have seen, the shape of this curve depends on the shape of distribution functions which, in turn, depend in part on characteristics of the test method such as the distribution of bubble or droplet sizes, etc. It implies, also, that while the erosion rate would, in the absence of other influences, tend toward a steady-state value as postulated in Ref *1*, this generally occurs only after most of the original surface had been eroded, at which time the surface damage will be so severe as to make the erosion conditions susceptible to geometry effects such as described in Ref *2*. In short, the instantaneous erosion rate may never be characteristic of only the material, and for meaningful correlations it will become necessary to standardize the test method very carefully, or to use properly chosen cumulative erosion measurements (for example, time required to attain some specified mean depth of penetration, of significance to the intended application).

The view of erosion as a fatigue process is not new, and yet it carries with it a number of other implications which perhaps have not been sufficiently emphasized in the past and to which I should like to draw attention, even though they are only peripheral to the present topic. These include:

1. There is little likelihood of finding one specific independently measurable material property which will predict erosion resistance, since none has been found to predict fatigue strength uniquely although far more research has been done on fatigue than on erosion.
2. In fatigue, the relation between stress and endurance is determined by test for each material, and is not expressible in simple analytical form.[7] Similarly, the relation between impact velocity and erosion very likely does not follow any specific law but must be established uniquely for each material.
3. Although erosion is the result of many failures, and some of the statistical scatter found in fatigue data may well average out in an erosion test, yet to obtain valid results (or results with calculable confidence limits) many more data points must be taken and many more replications must be run than has been customary to date. Related to this is the need to establish accurately the erosion-versus-exposure curve, and to carry out all tests to corresponding degrees of cumulative erosion if one wants to draw any quantitative comparisons from them.

Finally, for further development of the present approach, one would have to learn more about the actual sizes or size distributions of erosion fragments and how this affects their individual and statistical fatigue lives under given impact stresses, and to learn more about the distribution of impact intensities and impact areas and how these can be accounted for by cumulative damage theories.

[7] Empirical formulas to represent the fatigue *S-N* relationship have been proposed and are reviewed by Weibull [*45*], pages 174–183. These, however, are curve-fitting attempts rather than expressions of a physical law.

Acknowledgments

The author wishes to acknowledge the contribution of R. I. Shrager, who was responsible for the mathematical formulation and computer programming of the preliminary model, and of L. B. Godio, who assisted in the formulation and programming of the elaborated model. W. D. Pouchot of the Westinghouse Astronuclear Laboratory, F. G. Hammitt of the University of Michigan, and J. C. Freche and S. G. Young of NASA-Lewis Research Center all contributed helpful discussions and criticisms. A part of this work was supported by NASA under contract NAS 7-390, with technical supervision through Jet Propulsion Laboratory.

APPENDIX

Mathematical Formulation of Model

Let any surface exposed to erosion be thought of consisting of elementary areas (or volumes, if their thickness is considered) whose lifetimes under the erosion attack can be described by a normalized distribution function $f(t)$. Thus, by definition

$$\int_{-\infty}^{\infty} f(t)\, dt = 1.0 \qquad (1)$$

and the distribution function for a specific surface area A, exposed to erosion from time $t = 0$, is therefore

$$F_A(t) = A \cdot f(t) \qquad (2)$$

Since a surface element is lost from the surface when its lifetime is reached, Eq 2 can equally well be regarded as a loss-rate function for the area A.

Equation 2 may be further generalized by stating that the loss rate from an area A_1, first exposed to erosion at time $t = T_1$, is thereafter given by

$$F_1(t) = A_1 \cdot f(t - T_1) \qquad (3)$$

Let us now consider the original or "top" surface of a body exposed to erosion. We may take its area to be unity, and every portion of its area is simultaneously exposed to erosion at time $t = 0$. Thus $f(t)$ adequately describes the loss rate from the top surface. As surface area is eroded, or lost, from the top surface, an equal area is created, or exposed, at the "second" level, located a distance h below the surface where h is assumed the thickness of erosion fragments. For convenience, the thickness h will also be assigned a numerical value of unity on some appropriate scale. In turn, the second-level surface will be eroded to expose a third-level surface, and so on. But, in computing the actual loss rates from all of the "undersurfaces," one must recognize that the lifetimes of surface elements must be measured from the time they were first exposed, and the total loss rate from all surface elements which were first exposed during a time increment dT at time T depends on the total area which was first exposed during that time interval.

Let $Y(t)$ be the total rate of erosion, from all levels, at time t. This is what we desire to compute. But $Y(t)$ is, *ipso facto,* also equal to the rate at which new surface area is exposed, at all levels below the top surface, at time t. (Strictly speaking, it is proportional to it, but with $h = 1.0$ it is numerically equal.)

Thus, the total surface area first exposed during increment dT at time T is $Y(T)\, dT$, and the loss rate from this area at time t is, by Eq 3,

$$F_T(t) = f(t - T) \cdot Y(T) \cdot dT \qquad (4)$$

The total loss rate at time t, from all undersurfaces, is composed of contributions from all undersurface areas first exposed during all time increments from $T = 0$ to $T = t$, or

$$\int_0^t f(t - T) \cdot Y(T) \cdot dT$$

The total loss rate or erosion rate, $Y(t)$, is the sum of that from the top surface and that contributed by all undersurfaces, or

$$Y(t) = f(t) + \int_0^t f(t - T)\; Y(T)\; dT \qquad (5)$$

The fact that the contributions from the undersurfaces and from the top surface from two distinct terms in Eq 5 makes it conveniently possible to assign a different distribution function for the top surfaces as compared to all undersurfaces. This is desirable if one wants to reflect the fact that the top surface has in many ways a different nature and history than the undersurfaces exposed as a result of erosion. We finally, then, state

$$Y(t) = f(t) + \int_0^t g(t - T)\; Y(T)\; dT \qquad (6)$$

where:

$f(t)$ = distribution function for top surface and

$g(t)$ = distribution function for undersurfaces.

For the initial explorations Eq 6 was computer programmed directly, using Normal distributions for functions $f(t)$ and $g(t)$, normalized over specified time spans rather than between the limits of plus and minus infinity as suggested by Eq 1.

The formulation of the elaborated model, which keeps track of the area eroded at each "level," is described in detail in Appendix A of Ref *46*.

The log-Normal frequency distribution function as programmed for the elaborated model is:

$$f(t) = \frac{1}{\sigma(t - T_0)\sqrt{2\pi}} \exp\left\{\frac{-\,[\log_e (t - T_0) - m]^2}{2\sigma^2}\right\}$$

This function has the following properties:

T_0 is a "delay time" which may be introduced to ensure that no failures occur before $t = T_0$. The mean, or expected value, is

$$E = T_0 + e^{m+(1/2)\sigma^2}$$

The median value is

$$M = T_0 + e^m$$

The mode, or most probable value, is

$$P = T_0 + e^{m-\sigma^2}$$

The input may be prescribed in terms of T_0, m, and σ directly; the latter two may also be prescribed in terms of the equivalent logarithms to base 10, or in terms of the equivalent real-time quantities $T_m = e^m$ and $R = e^\sigma$.

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DISCUSSION

Olive G. Engel[1] (*written discussion*)—The picture presented is one of layers of cells whose lifetimes are described by statistical distribution functions. This is a good approach because the general features of the erosion-rate-versus-time curve can be obtained from a model of this kind. In my opinion, however, a model based on the real physical picture is preferable because it is more informative.

Let us consider the same picture of a test specimen consisting of layers of cells. Let us first consider that the test specimen is made of a low-strength brittle material that does not work harden. When drop impingement begins, the cells of the surface layer contain no cracks. We can represent them as c_0-cells, where c represents a cell and the subscript-0 notation indicates that the cell contains no cracks. If the impact energy delivered by a drop is sufficient to initiate a crack in a surface cell, we can represent the event in the following way:

$$c_0 + \epsilon \rightarrow c_1$$

where: ϵ is the energy per unit volume needed to form a crack in the material being considered and c_1 is a cell that contains one crack.

As drop impact is continued, the density of c_1-cells in the surface layer will increase and the chance that a c_1-cell will be struck by another drop and develop a second crack will become increasingly more probable. This event can be described as

$$c_1 + \epsilon \rightarrow c_2$$

The c_2-cells that form contain two cracks; the two cracks may or may not intersect. If the cracks in a c_2-cell do intersect, it is a c_2^*-cell or a critical site. This distinction is made because if just one more intersecting crack should form in a c_2^*-cell in such a way as to complete crack formation around a triangular piece of the solid, this piece will break away as an eroded fragment. The event can be described in the following way:

$$c_2^* + \epsilon \rightarrow c_0 + \text{an eroded fragment}$$

The c_0-cell which is formed in the foregoing event is composed of the underlayer material that is exposed when the eroded fragment breaks away. Consequently, the ejection of an eroded fragment simultaneously regenerates starting material. The newly exposed cell of underlayer material is not exactly the same as a cell of the original surface material, however; the angle that the newly exposed surface presents to the impinging drops differs from that presented by the original surface, and, in addition, the newly exposed material may contain residual crack ends.

The simplified picture of the erosion process presented so far is restricted to consideration of the cracking damage produced by the impact energy; it has not taken into account the damage that results from the radial flow of the liquid contained in the drops. It is applicable to an erosive environment in which flow is minimized, such as cavitation erosion produced with use of a magnetostriction oscillator, and, with a little reflection, it can be seen that this simple model is able to interpret the characteristic features of curves of erosion rate plotted against time for data obtained with the use of a magnetostriction oscillator.

There can be no loss of material until a minimum of three cracks intersect in such a way as to circumscribe a triangular pyramid. Consequently, there must be an initial period during which no erosion takes place (incubation period). During this period, the density of c_1-cells, c_2-cells, and c_2^*-cells in the surface layer will build up.

When the density of c_2^*-cells becomes high enough so that the probability is substantial that one of them will be struck by a drop, there will be a large number of cells that contain one or more cracks. In fact, the density of c_2^*-cells will be greater at this time than at any later time because up to this time there has been no process operating that removes c_2^*-cells. Consequently, as soon as one of them is hit by a drop and a

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fragment of solid is lost, this event will quite suddenly become a frequent occurrence, and the rate of erosion plotted against time will exhibit a sharp rise (accumulation period).

Concomitant with the process of destroying c_2*-cells by the loss of eroded fragments is the process of regeneration of c_0-cells as the underlayer material is exposed. The initial rapid rate of erosion contains a check against itself which slows it down, since not only are the essential c_2*-cells being destroyed but also c_0-cells are being produced, and these must be converted to c_1-cells and then to c_2*-cells before they can again become sites of material loss. Consequently, after the first rapid rise, the rate of erosion will begin to fall off (attenuation period).

The state of a system that is subject to competing processes will pass through a maximum or a minimum. The rate of erosion plotted against time will pass through a maximum and then fall until the rate of producing c_2*-cells and the rate of destroying them become equal. At this time, the rate of erosion should be essentially constant (steady-state period).

If we apply this model to a ductile metal, which work hardens to the point of embrittlement and then cracks, the essential features are the same but there are some modifications. In the case of a ductile metal, the first drops that impinge produce craters (by plastic flow) rather than cracks. As the cratering process continues, the surface becomes covered with craters and new craters are superposed over old craters until the worked metal becomes brittle enough to crack. When this point is reached, the previously described model will operate as for a brittle material until eroded fragments are broken loose. When the underlayer material is exposed, however, it will have to be work hardened to the point of embrittlement before cracks will form in it to produce new c_1-cells, c_2-cells, and c_2*-cells. This new feature will result in an oscillation in the curve of erosion rate plotted against time; it will be particularly evident in the steady-state period.

What has just been described is the simplest form of the model; it is restricted to the case where fluid flow of a drop liquid or of a cavitating liquid is essentially absent. But usually fluid flow occurs, and there are then two damage-producing attributes of an impinging drop. These are (*a*) the impact pressure that it exerts and (*b*) the radial flow of the liquid of the drop.

If the relative impact velocity is sufficiently high, the impact pressure can produce cracks in the surface of a brittle solid. These cracks are circular in isotropic solids and polygonal in anisotropic solids which have preferred cleavage planes. Forces exerted by the radial flow of the drop liquid bearing against the raised edges of these cracks are able to break pieces of solid away. This constitutes an erosion mechanism that operates without the necessity of intersecting cracks and, consequently, occurs before intersecting cracks are formed.

For this reason, the very first drop that strikes the solid with energy sufficient to crack it may produce erosion loss due to the radial flow of its liquid against the raised edge of the crack it produced. In fact, even at impact velocities that are too low to provide sufficient energy to crack the solid, an impinging drop can erode protruding surface irregularities from the solid by the shear stress that its radially flowing liquid exerts when it bears against them. In the light of these considerations, there can be no zero erosion period (incubation period) where rapid fluid flow is present.

When a low-strength brittle material is eroded, the surface is uniformly roughened. For a material of this kind, the progress of long-term erosion will involve the gradual movement of an eroded surface layer through the thickness of the test specimen. The general surface roughness that is produced as erosion progresses will reduce the rate of erosion, but it appears that the rate of erosion should eventually become constant since once the surface roughness has reached a certain degree of coarseness there should be little change in this degree of coarseness.

The case of a high-strength material is different; it will start to fail at weak spots. When eroded fragments are ejected at these spots, residual ends of cracks remain. The residual crack ends will go on propagating and erosion will continue over restricted areas around the separate weak spots until pits are formed at these spots. The pits will deepen until they eventually pierce the test plate. For materials of this kind, the rate of erosion can be expected to decrease progressively with time because there is no evidence to indicate that it should ever reach a steady-state value. It is possible, however, that a nearly steady value may be reached when the test plate is peppered with deep pits or even with holes and the sloping walls of the remaining material between the pits or holes have a roughly similar angle of inclination.

I am currently working on the further development of this model of the erosion process. At this time I am only able to share with you the thoughts about erosion rate and the outline of the model of the erosion process that are given in the foregoing. I hope to be able to complete the model and to present it to you at a future time.

F. G. Hammitt[2] (*written discussion*)—The author is to be congratulated for this clear and comprehensive summary of the present situation relating to damage rate versus test duration effects in cavitation and impingement erosion, as well as for his very original and significant statistical model of the erosion process when fatigue is the predominant failure mechanism. It is very interesting to note that all the various rate-time curves which have been observed experimentally can be explained without reference to the effect of accumulated damage on the flow pattern and hence upon the cavitation regime. In my opinion, however, this latter effect is of sub-

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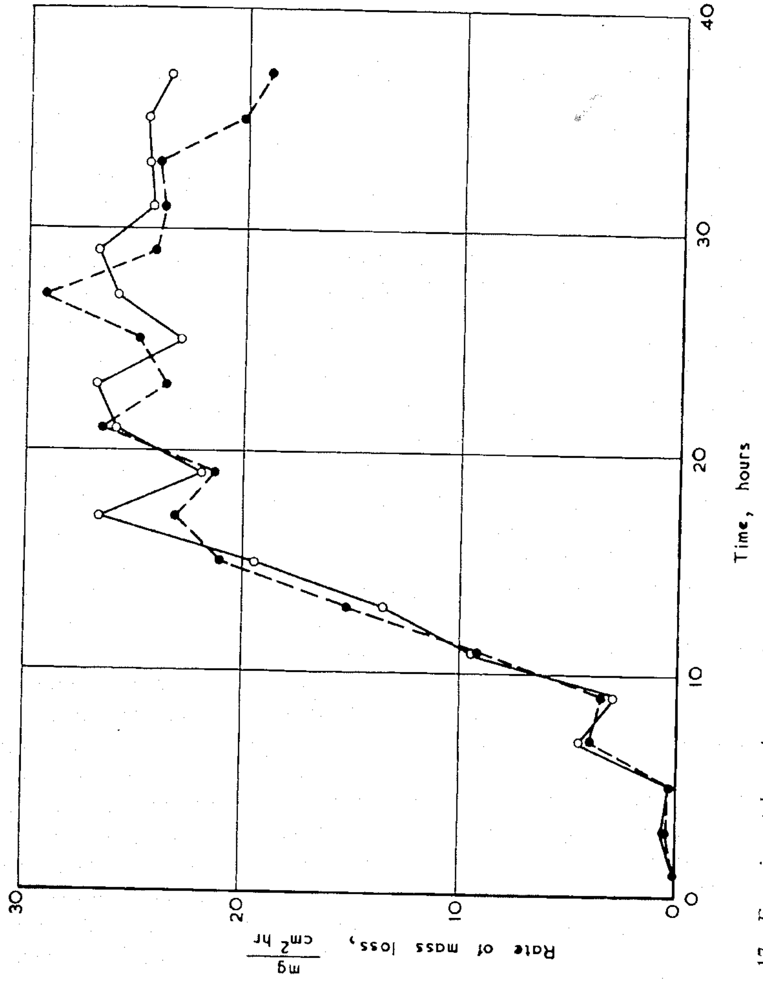

FIG. 17—*Experimental erosion rate-time curves from two identical tests on CEGB impingement erosion rig.*

stantial importance in most cases, and hence should somehow also be included in a predicting model.

Very qualitatively, it seems to me that all the observed rate-time curves, including those showing several maxima and minima, can also be explained simply on the basis of flow-pattern changes through the effect of accumulated damage. In given situations, these changes are capable of both triggering local cavitation which will then spread the damage region and increase damage rates in a generally unpredictable manner and of protecting the surface by attenuating the cavitation shocks imposed on the surface through increasing distance from the cavitating region as the damage in a given region becomes substantial, or both. In various situations, there are also numerous other mechanisms which might be mentioned which are capable of either increasing or decreasing damage. Hence, it would seem to me that the feedback between accumulated damage and the flow pattern is capable of increasing, decreasing, or maintaining constant damage rates, depending upon which of the mechanisms predominates at the moment. Since the dominant mechanism may change as damage proceeds, it is conceivable that maxima and minima may be generated in the rate curve by these flow effects as well as by the fatigue statistics discussed in the paper.

D. Pearson[3] (*written discussion*)—Within Central Electricity and Generating Board (CEGB), erosion results have been reported as cumulative mass loss plotted against time. In the published form, such data cannot be differentiated accurately, and in Fig. 17 I have plotted the mean rate of mass loss between weighings against time for a pair of specimens exposed to the same conditions. The points are not a close fit to any smooth curve, and this must result in the interpretation being influenced by personal opinion, though the results would seem more consistent with Fig. 2, rather than Fig. 1 of the paper.

A major problem in erosion research is to determine the relative importance of the following on the detail shape of the observed erosion mass loss–time curve:

1. inconsistency in test machine performance,
2. the effect of the material being detached in finite-size pieces,
3. inconsistencies in erosion resistance between specimens made from the same batch of material, and
4. genuine variations in erosion rate as erosion progresses.

The author is only interested in the last, but it cannot be distinguished from the other three spurious causes using the test results for a single specimen. Results are required for many specimens of one material tested using the same nominal conditions (however, to permit Effects 1 and 4 to be separated, the specimens should not all be eroded together).

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[3] Central Electricity and Generating Board, Research and Development Dept., Marchwood Engineering Laboratories, Marchwood, Southhampton, Hants, England.

Has the author obtained any such data, and has he been able to analyze these data to determine the shape of the true erosion curve?

J. M. Hobbs[4] (*written discussion*)—Mr. Heymann's excellent paper has gone a long way toward explaining the effects of time on erosion rate but seems to have painted a rather gloomy picture of the present state of the art.

According to Mr. Heymann, it is still apparently a matter of opinion whether any steady-state value of erosion rate has a definite significance. Surely, even though the maximum erosion rate is nothing but "the 'deluge' of erosion fragments from the top surface layer," the same is true under field conditions. Thus in any test, provided that the maximum erosion rate is maintained for sufficient time to indicate that the scatter in the lifetimes of individual particles is evenly distributed, it could be used for comparative rating of materials. This would seem to be preferable to the time required to attain some specified mean depth of penetration, observing that the latter method is sensitive to surface conditions and would therefore necessitate extreme care in specimen preparation.

On the subject of correlations of incubation periods with fatigue limits, I would agree that this approach is hardly logical, but add that in some cases it is unavoidable. In liquid impact tests it is possible to vary the impact velocity and the number of impacts corresponding to the incubation time at each velocity. Thus for different materials it is possible to derive some threshold or endurance limits of velocity which can be correlated with their respective fatigue endurance limits.

It is not possible in any standard vibratory cavitation erosion test to vary the stresses caused by cavity collapse, and hence tests must be conducted with a nominally fixed stress system. To correlate the incubation periods of different materials, fatigue lives of the same set of materials would have to be determined *all at the same stress*. This would be impracticable for more than a very limited range of materials, as a stress equal to the endurance limit of a strong material may well be greater than the yield stress of a weaker one.

Hence, in this type of test for comparison of the behavior of different materials there is little choice but to use fatigue endurance limit on the one hand and some function of either the incubation period or the erosion rate on the other.

R. I. Armstrong[5] (*written discussion*)—Mr. Heymann is to be complimented on his lucid review which provides some clarification of ideas on the reasons for variations in erosion rate with time. His digital analog of the erosion process, which, while it might appear to be oversimplified, does result in similar curves to those obtained from tests.

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[5] Research and Development Div., C. A. Parsons and Co., Ltd., Newcastle upon Tyne, England.

Accepting that Mr. Heymann's first models are tentative only, doubt remains whether his computed rate-time curves (for example, Fig. 10 of the paper) are consistent with experimental observations made at C. A. Parsons and quoted by Mr. Heymann in support of his thesis in Fig. 11. It must be granted that there is a superficial resemblance in the shape of the curves, but this similarity breaks down when an attempt is made to apply the basic premise of the theoretical curves to the experimental ones. Mr. Heymann's premise is that one simple distribution curve should serve for all sources of scatter on the "lifetimes of erosion fragments." In consequence, the time taken to reach the first peak (time = 1 in Fig. 10 of the paper) is given by the mode of the distribution curve. Viewed in this way, the first peak in the erosion rate time curve is, as Mr. Heymann aptly describes, "a deluge of erosion fragments from the top surface layer." This breaking up of the surface is readily observable by microscopic examination. Therefore, to check the applicability of the model, the time taken for significant break up of the surface must be compared with the time taken to reach maximum erosion rate, these being presumed equivalent. From Fig. 11 of the paper, the time to maximum rate for tool steel is about 35 hr, which is typical of the current tests at C. A. Parsons. However, photographs of tool steel specimens eroded in current tests for only 2 hr (Fig. 18) show the top surface to have reached a stage of rapid disintegration, the amount of material lost by this time being only about 2 per cent of that lost in 35-hr test.

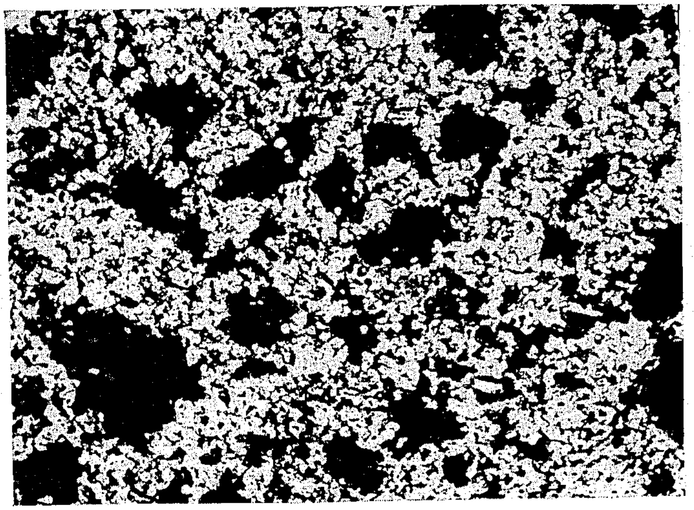

FIG. 18—*Appearance of 18W-6Cr-0.7C tool steel after 2-hr erosion test on the polished surface* (×360).

The observed increasing rate, which follows and which has been shown previously to be by no means coincident with disintegration of the top suface layer, must either be the result of a more rapid production of particles from the subsurface layers or of an increased size of fragment. No evidence for the latter has so far been discovered at C. A. Parsons where stereoscopic microexamination of eroded surfaces with comparison photographs at successive stages have been used to detect such fragmentation. A limited number of actual measurements of pit depth changes and of eroded particles collected by the specimen guard rings have also failed to reveal the presence of materially larger fragments. While this evidence is of a negative nature and the loss of larger particles may still have occurred undetected, it appears more likely that increasing numbers of smaller particles are actually produced. The explanation should then be in terms of factors which can produce equivalent increases in the rate of fatigue damage. These are surely tied up with surface geometry, for example, Mr. Heymann in Part I of his paper suggests that the damage rate would be increased by the development of irregularities smaller than the impinging droplets and reduced where the scale of roughness is larger than the droplet size (Fig. 8 of the paper). The occurrence of considerable undercutting in Parsons' impingement tests, Fig. 19, suggests that secondary water flow effects are responsible, such as Bowden and Brunton have postulated in Refs *19* and *20* of the paper. The tensile loading essential to produce fatigue would be more readily obtained in this manner.

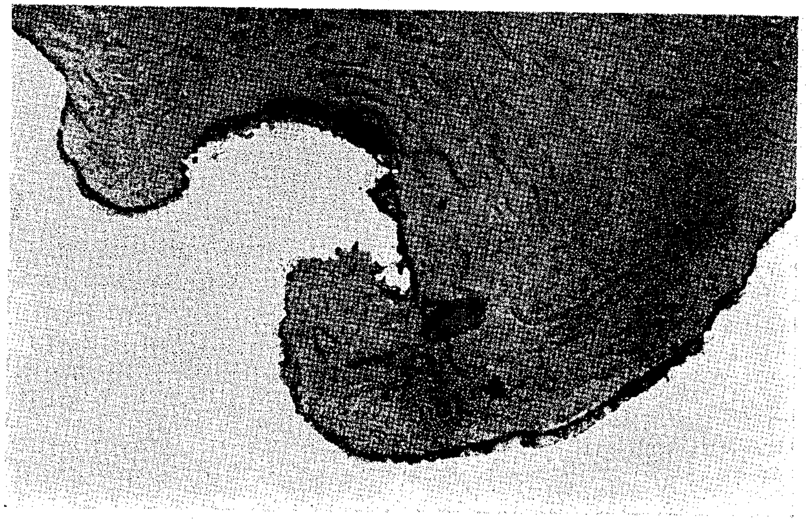

FIG. 19—*Unetched section through an undercut erosion pit in 18W-6Cr-0.7C tool steel after 100-hr test* (×*160*).

F. J. Heymann (*author*)—The discussion is gratifying and has served to put the statistical model into a clearer perspective and to throw more light on some of its limitations.

It is encouraging to see that Dr. Engel has been thinking along somewhat similar lines, and I am grateful that she has given us her early thoughts in some detail. She has examined and modeled the actual physical occurrences more specifically than I have dared to do. What I have proposed is a phenomenological model in the sense that it can predict consequences whose real physical causes are much more intricate than the model explicitly admits. This is useful up to a point, in that it may show what significance statistical properties of the material and of the erosive environment do have for the erosion-time relationship, without requiring a precise or quantitative knowledge of the physical progress of damage leading to material loss. If one seeks to develop a more realistic model based explicitly on the physical processes believed to be going on, one treads on much more dangerous ground. But this, eventually, has to be done and surely there are few people better acquainted with this ground than is Dr. Engel, or more qualified than she to lead the way over it.

I should like to take up Mr. Pearson's comments next. There can be no doubt that the factors which he lists play a role, but I do not agree that I am interested only in the last or that all the others are "spurious causes." It is, in fact, part of my thesis that such factors as random (not, of course, systematic) fluctuations in test machine performance, the size distribution of erosion fragments, and strength variations within the material, all could help mold the shape of the erosion-time curve and could affect the maximum erosion rate, independently of the eventual steady-state rate which may be determined mainly by the average conditions. I do not believe, therefore, that it is meaningful to talk of a "true" erosion curve from which these effects are subtracted, although it would certainly be instructive to be able to separate and evaluate the various influences. To answer Mr. Pearson's question, we have not made any experiments with this objective, and I must admit that at present we have no test facility which is controllable enough to permit it to be done. Mr. Pearson's graph (Fig. 17) is very interesting in that both curves show the same fluctuations. As I understand it, they represent two specimens exposed simultaneously in the same test rig; thus, the fluctuations could be due to slightly different conditions during the various runs, though one might then have expected greater fluctuations during the rising portion of the curve (accumulation or acceleration stage). It may therefore be appropriate to recall Dr. Engel's comment that oscillations "will be particularly evident in the steady-state period."

I certainly cannot dispute Professor Hammitt's point that flow-pattern changes due to accumulated damage will surely affect the erosion rate and could explain many rate-time patterns. In Part I of the paper I have, in fact, referred to a report by Hammitt et al which discusses this in detail. What intrigued me, however, was the similarity between the commonly observed rate-time behavior in impingement erosion and in cavitation erosion, which occur in very different flow regimes. It would seem to be a great coincidence if this behavior were entirely due to feedback of the surface geometry on the flow patterns. I feel sure that both statistical effects and flow effects must play some role in all of the erosion régimes; however, which of these is predominant or what their relative importance is in any particular erosion regime, this is a question which awaits answers.

Mr. Armstrong's discussion is a contribution toward this. His observations are valuable and undeniably constitute evidence for attributing the shape of Fig. 11 more to geometry effects. Another significant difference between Figs. 10 and 11 is that in the former the peaks occur at about equal time intervals from the beginning of exposure, whereas in the latter the time at the second peak is about four times the time at the first peak. Mr. Armstrong's last two sentences corroborate the last paragraph in the section on "Effect of Surface Condition" in Part I of the paper. It would be good if the effects of surface geometry could somehow be incorporated into the mathematical model, but I have not so far been able to devise a convincing way of doing that.

Lastly, as Dr. Hobbs points out, the practical necessity of using erosion test data for comparative or predictive purposes remains and cannot be bypassed. I am not entirely happy with any of the criteria which have been proposed, including the one tentatively suggested in this paper and commented on by Dr. Hobbs. However, if the original surface preparation greatly affects the time to reach a given depth of erosion in a test, then the same would be true in actual service, provided the specified depth of erosion is the same in each case. Thus, it would seem to be more realistic to include this effect than to try to evade it. The problem is that the total testing time to obtain the specified depth of erosion may, of course, be very long for a good material. If, on the other hand, the specified depth of erosion for the test is chosen to be much less than that tolerable in service, then I do not believe that accurate predictions can be made from the test results toward service performance. If that is being "gloomy" then I must confess to it, but corresponding conclusions have had to be accepted in fatigue and creep testing.

With respect to the determination of velocity thresholds or endurance limits and their correlation with fatigue endurance limits, this I agree is sound in principle, but there is probably also a size effect which makes the velocity threshold a function of drop or jet diameter. The results of Refs *4*, *10*, and *32*, among others, suggest this quite strongly.

In summary, I am grateful for the interest evidenced and the thoughts presented by the discussors. I believe that the statistical effects must be present, and even if it can be shown that they are overshadowed by other effects something will have been learned.