Liquid Impingement Erosion: Modeling Droplet Impacts onto Elastic Solids

Numerical Modeling Team:

Dr. Ali Dolatabadi, Mohsen N. Marzbali

October 3, 2013

Outline

- Milestones
- Deliverables
- Introduction
- Methodology
- Results
- Conclusions

Key milestones

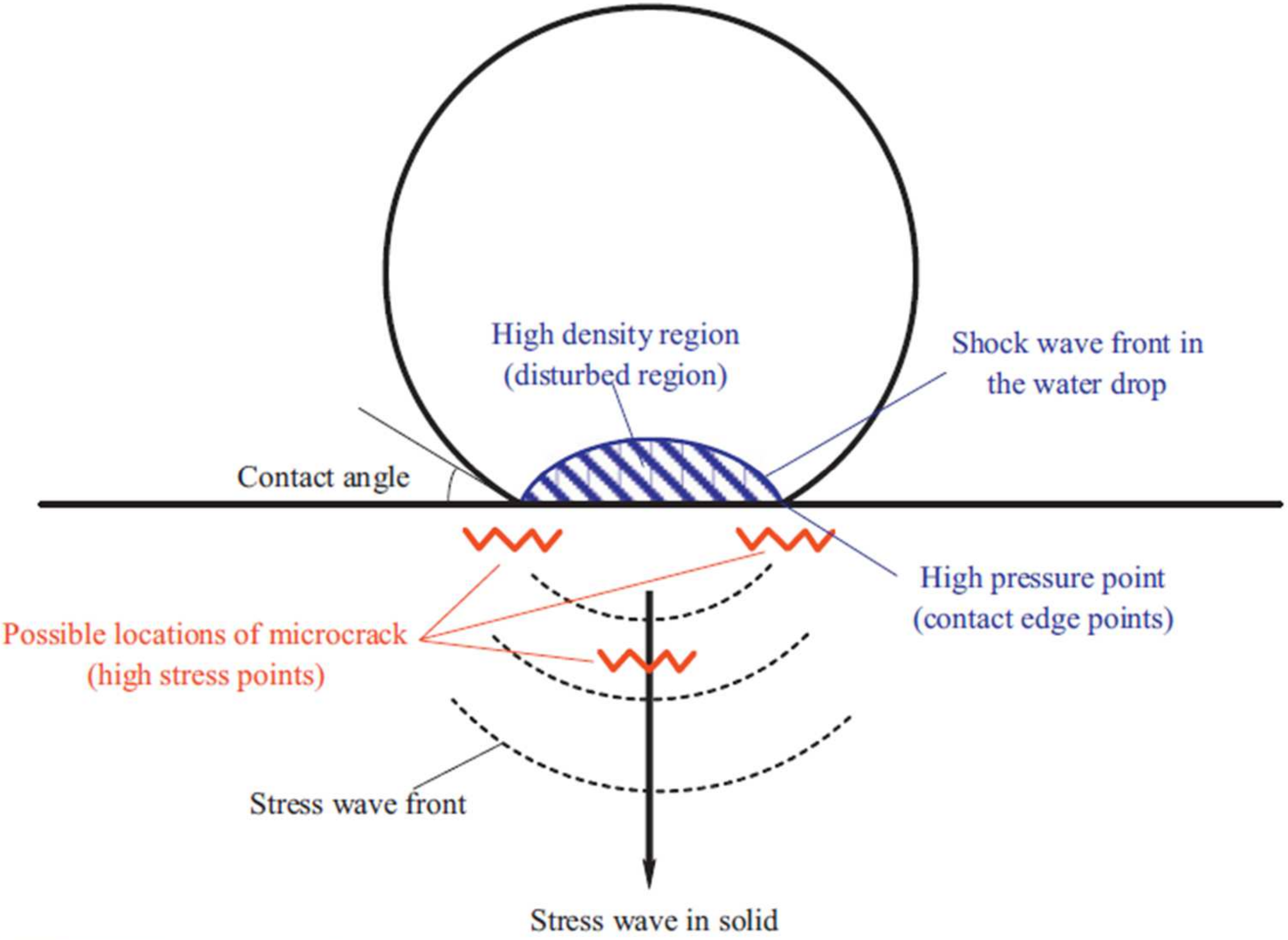
- **Incompressible FSI Model**:
  - → Incompressible VOF coupled with elastic solid solver
  - → 2-way coupling approach
- **Compressible FSI Model**:
  - → Compressible VOF solver with a rigid substrate
  - → 1-way coupling approach
- **Elastic Solid Model**:
  - → Isotropic Ti64 substrate

Deliverables

- Compressible fluid modeling:
  - ✓ 3D simulation using compressible VOF model
  - ✓ Impact velocity up to 500 m/s
  - ✓ Droplet size range of 50-1000 µm

- Elastic solid modeling:
  - ✓ Ti64 substrate (isotropic material)
  - ✓ Response to the impact pressure generated by the droplet impingement
  - ✓ Resolving the stress components in the solid substrate in elastic mode
  - ✓ Compare the peak stress component with critical material threshold

High speed drop impact



Li et al., 2008

Governing equations, compressible fluid

Continuity:
$$\frac{\partial \rho_f}{\partial t} + \nabla \cdot \left(\rho_f V_f\right) = 0$$

Momentum:
$$\frac{\partial\left(\rho_f V_f\right)}{\partial t} + \nabla \cdot \left(\rho_f V_f \otimes V_f\right) = \nabla \cdot \sigma_f + \rho_f g$$

Equation of state:
$$\rho_f = \rho_{f_0} + p_f \psi$$

Fluid stress tensor:
$$\sigma_f = -p_f I + \mu_f \left(\nabla V_f + \nabla V_f^{\,T}\right)$$

Equation of state

- Air: Ideal gas law @ isothermal condition

$$\rho_f = \rho_{f_0} + p_f \psi, \quad \psi : compressibility\ factor$$

$$\rho_{f_0} = 0, \psi = \frac{1}{RT} \approx 10^{-5} \Rightarrow p = \rho RT$$

- Water: Tait's equation of state

$$\frac{p_f + B}{p_a + B} = \left(\frac{\rho_f}{\rho_{f_0}} \right)^N$$

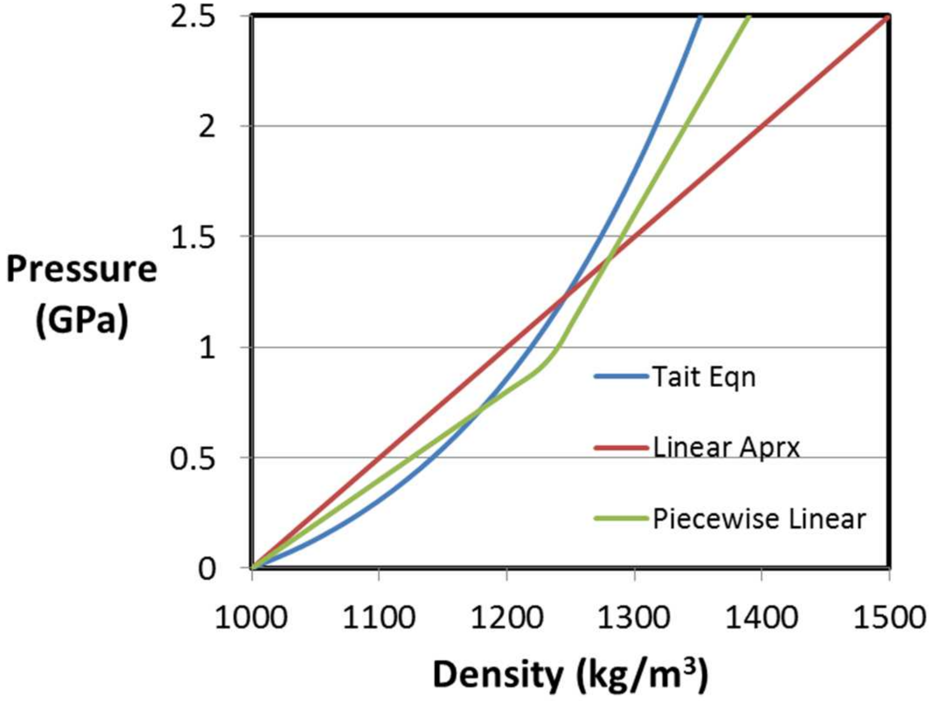
$$B = 300MPa,\ p_a = 0.1MPa,\ N = 7.415$$

- Linear approximation:

$$\rho_f = \rho_{f_0} + p_f \psi$$

$$\rho_{f_0} = 1000, \psi = 10^{-7}$$

$$\rho_f = \rho_{f_0} + p_f \psi$$



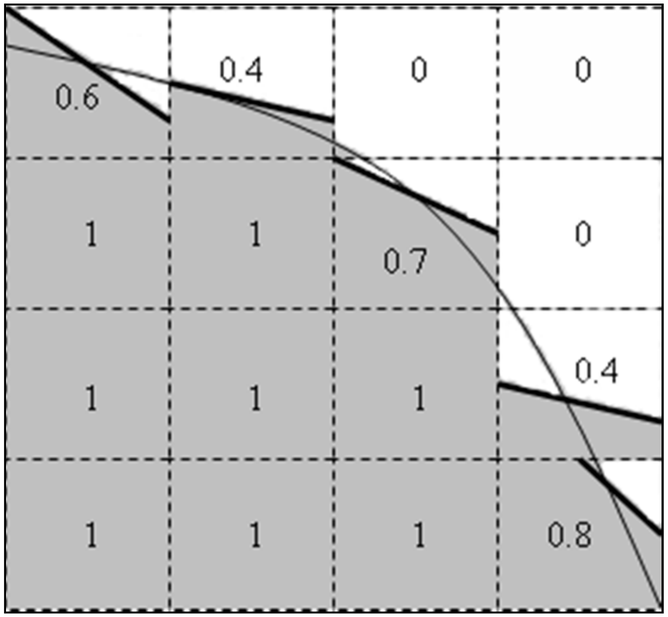
Volume of Fluid method

- Liquid volume fraction:

$$\begin{cases} \alpha_l = 0 & \text{Gas phase} \\ 0 < \alpha_l < 1 & \text{Interface} \\ \alpha_l = 1 & \text{Liquid phase} \end{cases}$$

- VOF Advection:

$$\frac{\partial \alpha_l}{\partial t} + \nabla.(V_f \alpha_l) = 0$$



- Interface Reconstruction method:

Piecewise Linear Interface Calculation (PLIC) of Youngs (1982)

Numerical scheme

- Solver: compressible VOF
- Segregated solver and fixed system of grids
- 2nd order accuracy in space and time
- Pressure-velocity coupling → Pressure-Implicit with Splitting of Operators (PISO) method
- Adaptive time step based on CFL initially set to 0.1

Governing equations, elastic solid

Elastic deformation: $$\rho_s \frac{\partial V_s}{\partial t} + \rho_s \nabla \cdot \left(V_s \otimes V_s \right) = \nabla \cdot \sigma_s + \rho_s g$$

Solid stress tensor: $$\sigma_s = \frac{1}{J} F\left(\lambda_s \left(trS \right) I + 2\mu_s S \right) F^T$$

Deform. grad. tensor: $$F = I + \nabla U_s$$

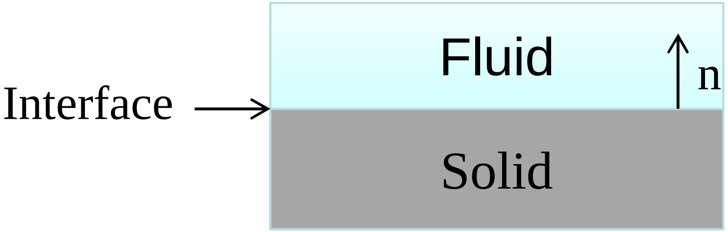
St. Venant-Kirchhoff: $$S = \frac{1}{2}\left(F^T F - I \right)$$

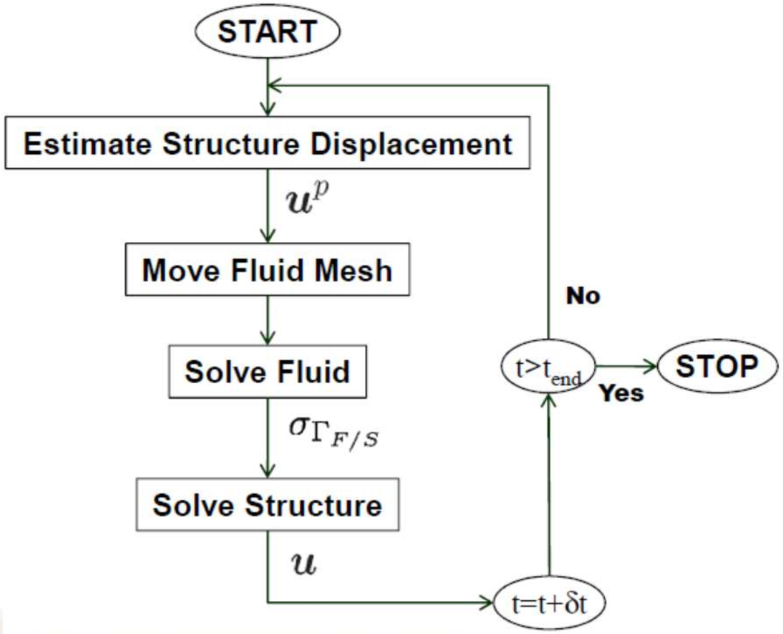
Lamé coefficients: $$\lambda_s = \frac{\nu_s E}{\left(1+\nu_s\right)\left(1-2\nu_s\right)} \qquad \mu_s = \frac{E}{2\left(1+\nu_s\right)}$$

Fluid-solid coupling at interface

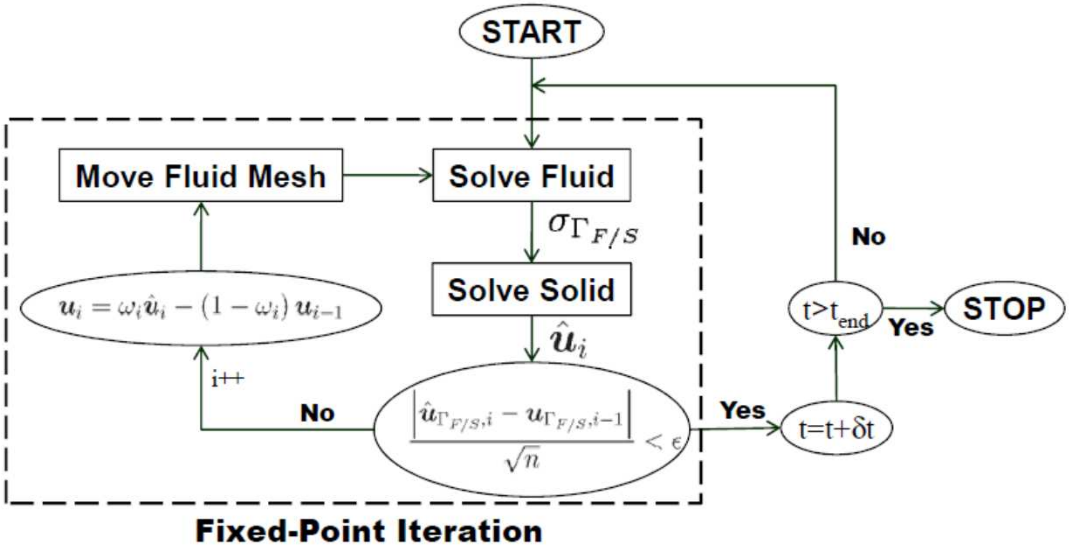
Force balance: $\sigma_s n = \sigma_f n$

No-slip condition: $V_s = V_f$


Interface
Fluid
n
Solid


START
Estimate Structure Displacement
u^p
Move Fluid Mesh
Solve Fluid
$\sigma\Gamma_{F/S}$
Solve Structure
u
t=t+δt
$t>t_{end}$
No
Yes
STOP

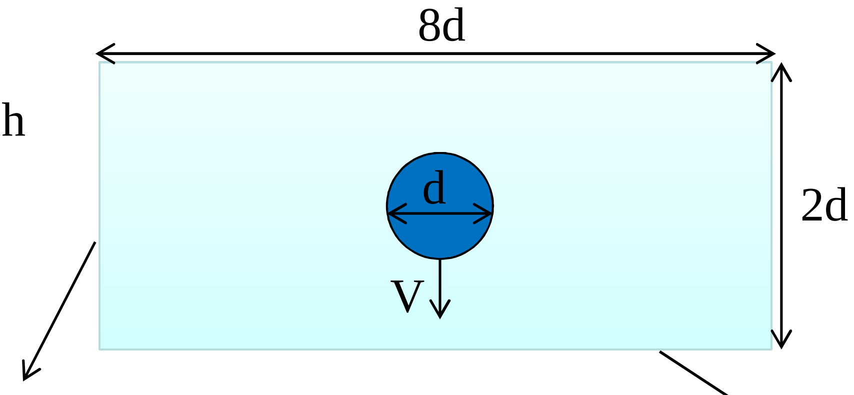
1-way coupling


START
Move Fluid Mesh
Solve Fluid
$\sigma\Gamma_{F/S}$
Solve Solid
$\hat{u}_i$
$u_i = \omega_i \hat{u}_i - (1-\omega_i)\, u_{i-1}$
i++
No
$\frac{\left|\hat{u}_{\Gamma_{F/S},i} - u_{\Gamma_{F/S},i-1}\right|}{\sqrt{n}} < \epsilon$
Yes
t=t+δt
$t>t_{end}$
No
Yes
STOP
Fixed-Point Iteration

2-way coupling

Domain & boundary conditions (fluid)

Uniform hexahedral mesh
Grid size = d/20



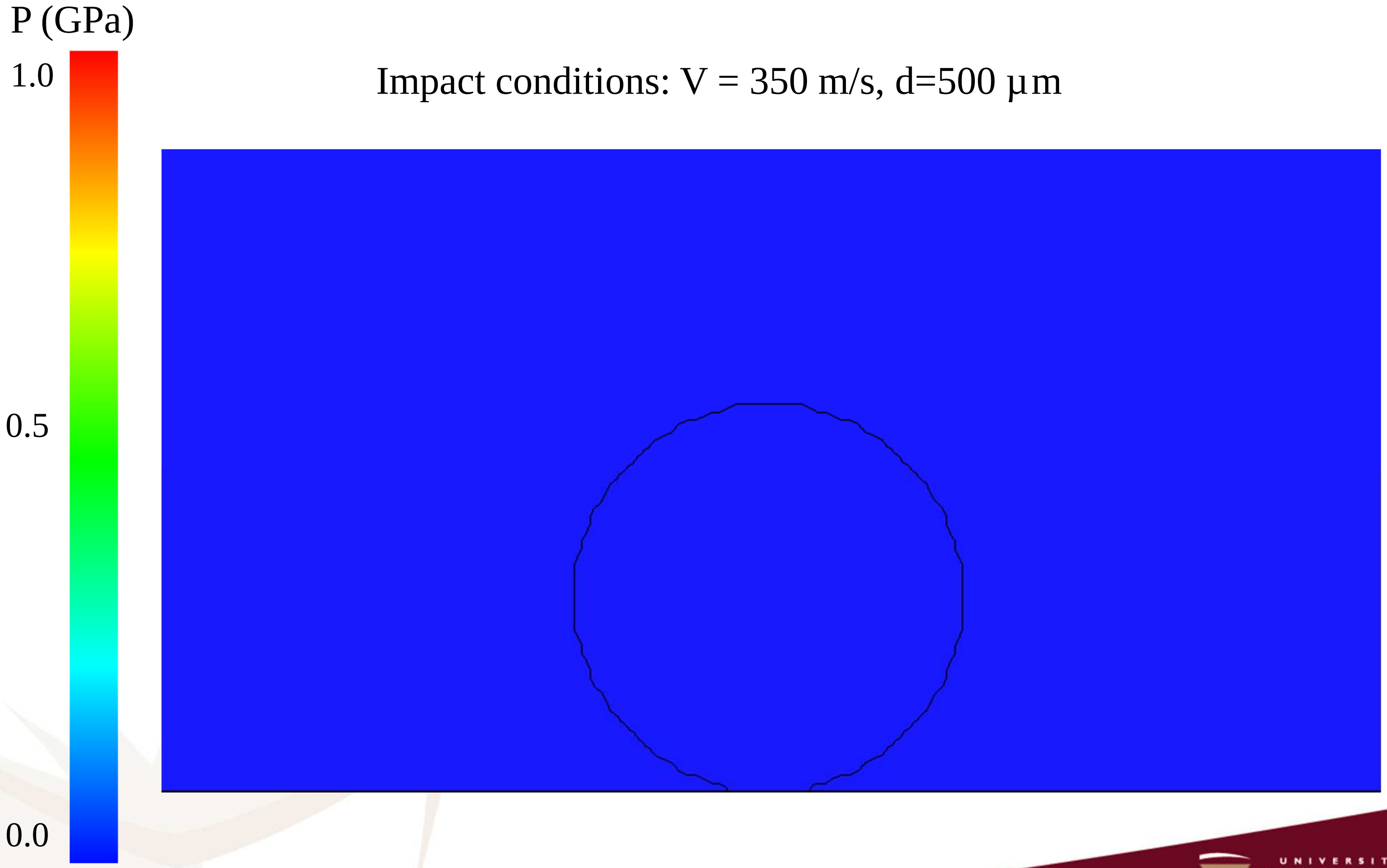
Note: not to scale

Pressure outlet

Rigid wall

Fluid initial properties	Air	Water
Density (kg/m^3)	1	1000
Kinematic viscosity (m^2/s)	1.48e-05	1e-06
Surface tension (N/m)	-	0.07

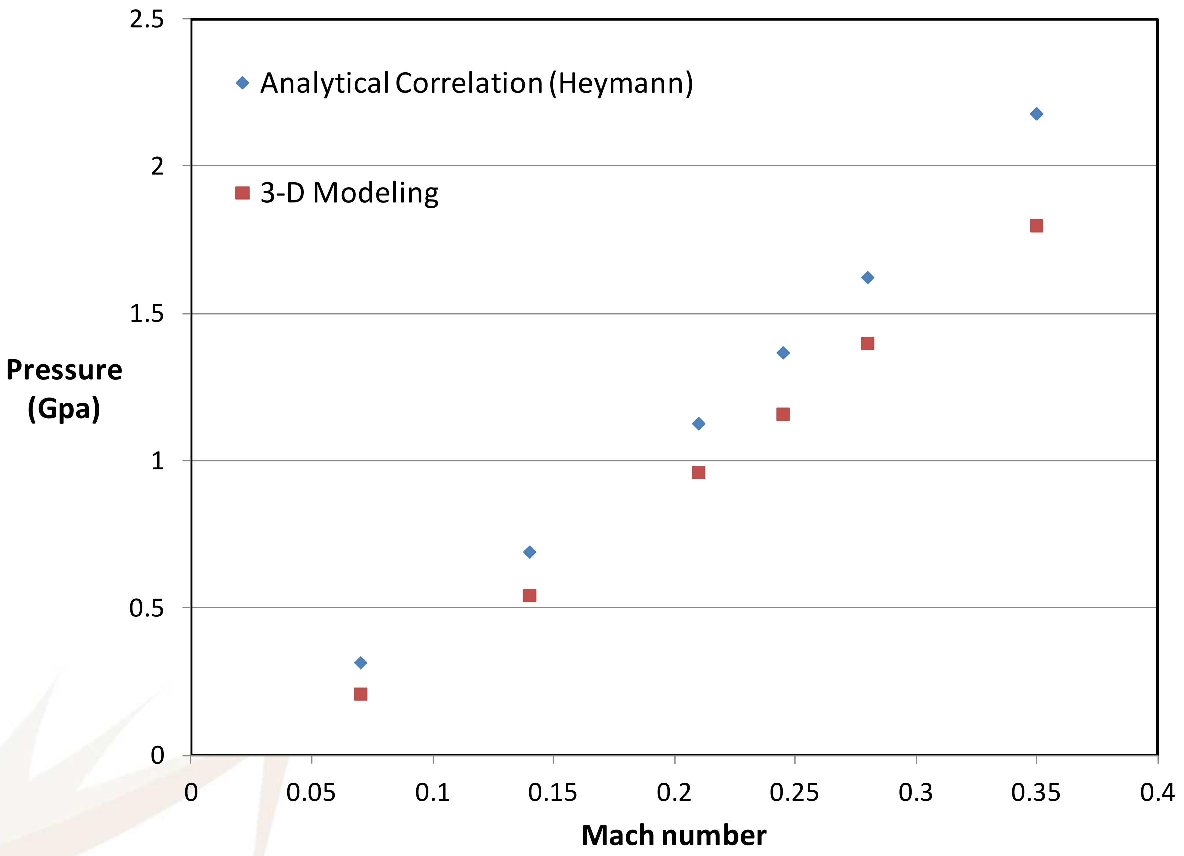
Generated pressure upon impact



Impact conditions: V = 350 m/s, d=500 µm

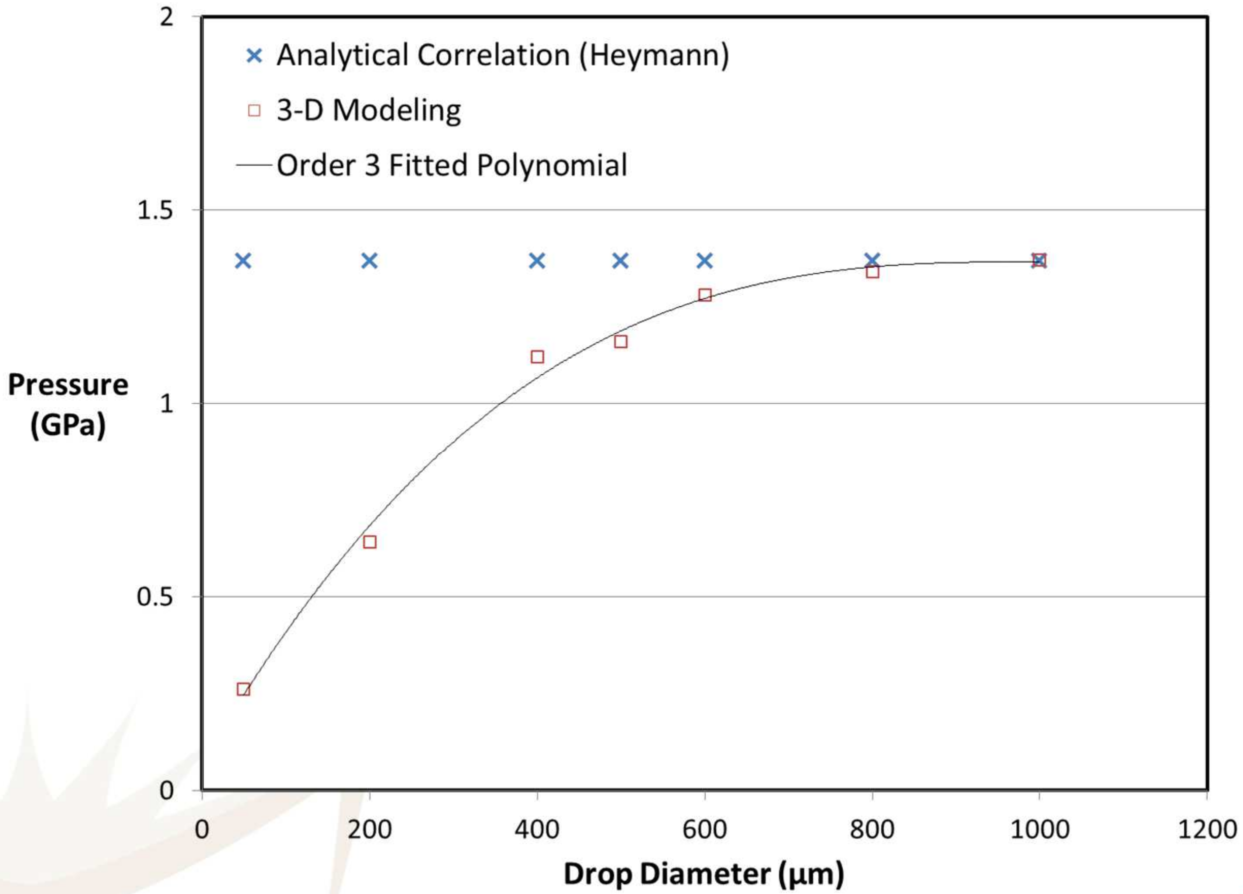
Results, effect of impact velocity

Impact conditions: d=500 µm, V varies
Ma=V/C, C is sound speed in liquid

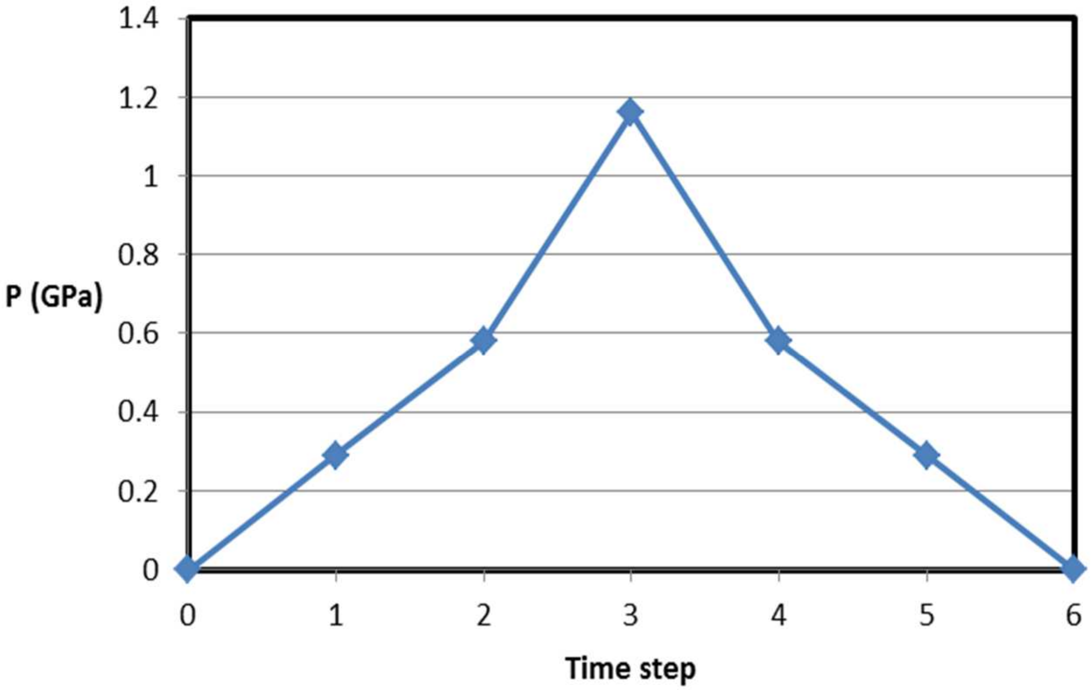


Results, effect of droplet diameter

Impact conditions: V = 350 m/s, d varies

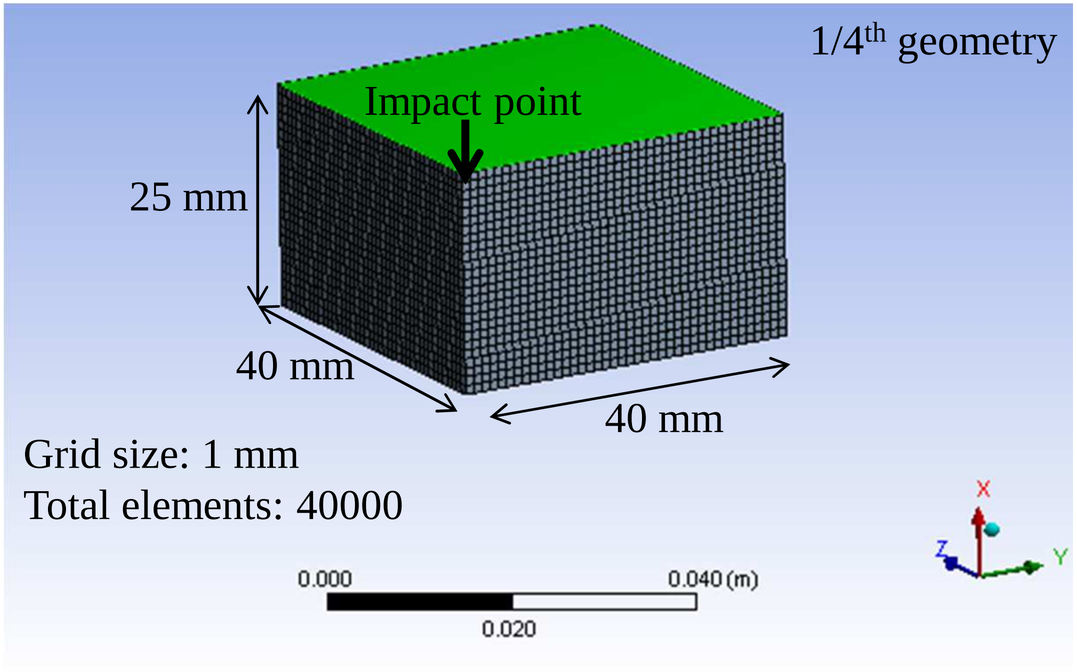


Imposed pressure history



Time step: 1 μs

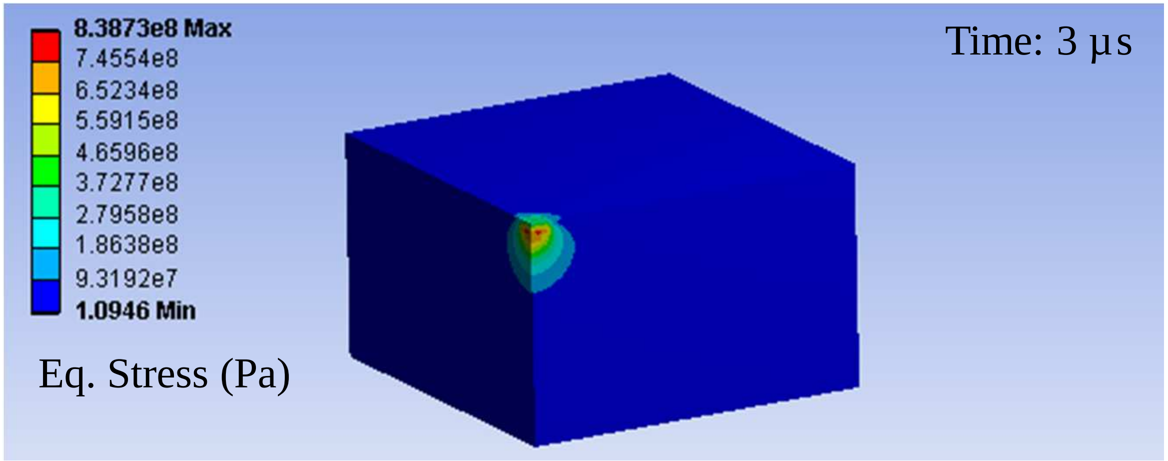
FEM model

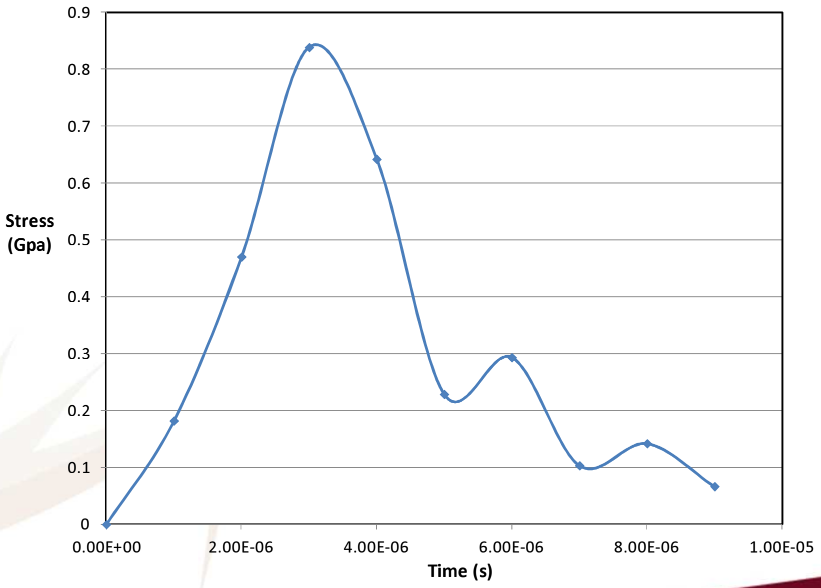


Grid size: 1 mm
Total elements: 40000

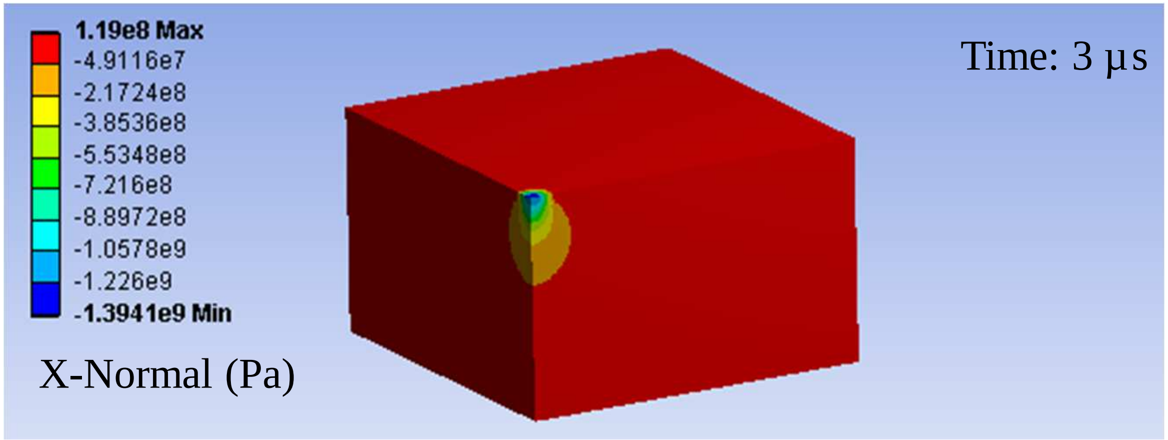
Substrate material: isotropic Ti64		
Young's Modulus	**Poisson's Ratio**	**Bulk Modulus**
1.138e+011 Pa	0.342	1.2004e+011 Pa
Tensile Yield Strength	**Tensile Ultimate Strength**	**Shear Modulus**
8.8e+008 Pa	9.5e+008 Pa	4.2399e+010 Pa

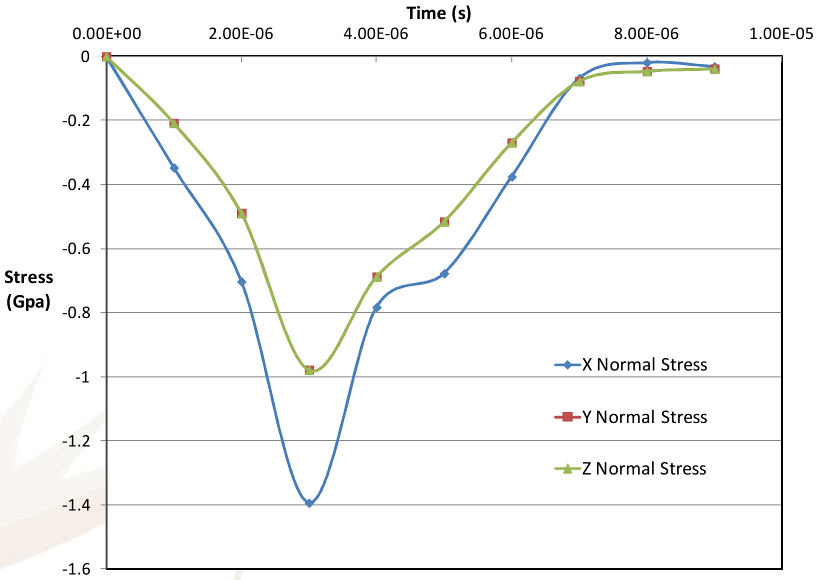
Equivalent Stress



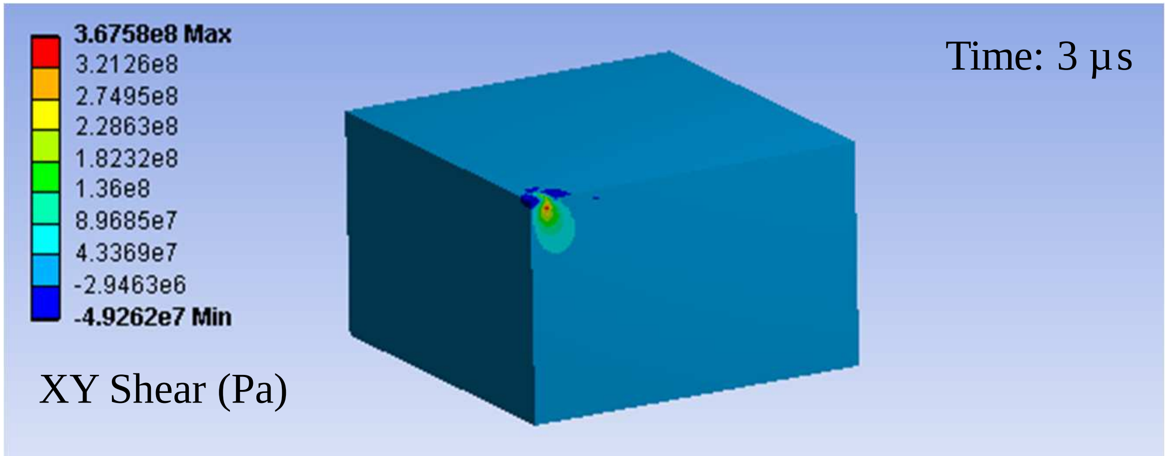


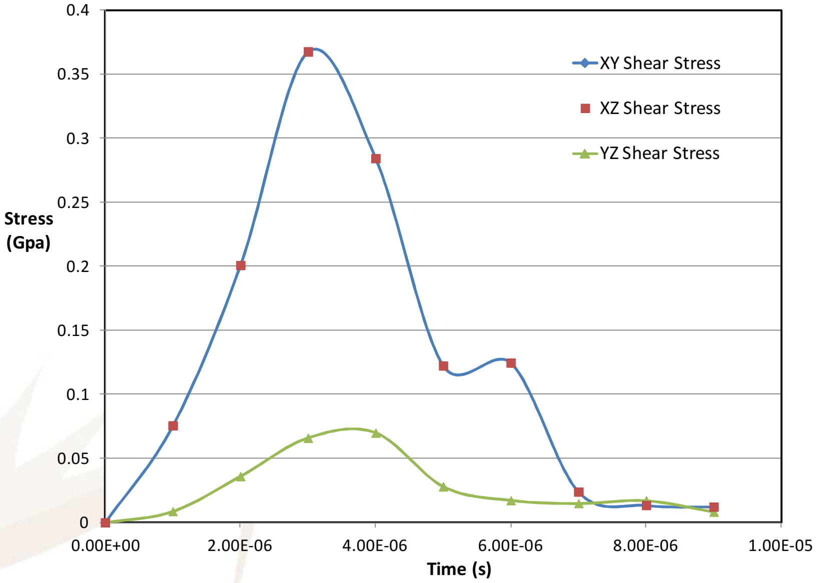
Normal Stress



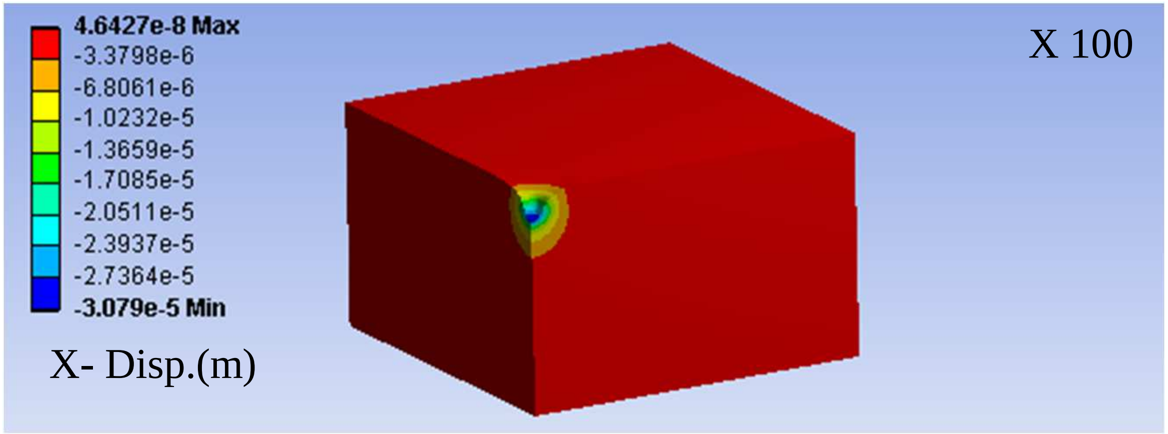


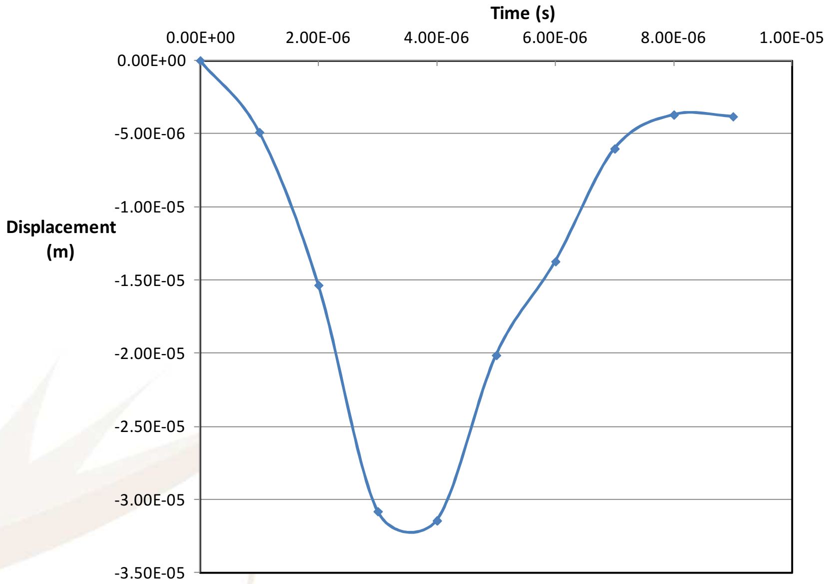
Shear Stress





Axial displacement





Conclusions

- Typical compressible droplet impact:
    - Diameter=0.5 mm, velocity=350 m/s
    - Generated peak pressure: 1.2 Gpa

- Elastic Ti64 response to the impact:
    - Max. equivalent stress: 0.8 GPa
    - Max. normal stress: 1.4 GPa
    - Max. shear stress: 0.3 Gpa
    - Max. deformation @ center point : 30 microns

Tensile Yield Strength	**Tensile Ultimate Strength**
0.88 Gpa	0.95 GPa

Thank you!

Questions?

UNIVERSITÉ
Concordia
UNIVERSITY

www.concordia.ca