

# Vickers indentation fracture toughness test Part 1

## Review of literature and formulation of standardised indentation toughness equations

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*There is considerable interest in determining the fracture toughness of brittle materials by measuring the extent of cracking associated with a Vickers indentation because of the ease of specimen preparation and the simplicity of the test. However, confusion has been engendered by the multitude of models and equations in the literature relating the degree of cracking to the fracture toughness. In Part 1 of this work, nineteen of these equations are reviewed and then modified in a standard manner for both experimental convenience and direct comparison.*

MST/1050a

© 1989 The Institute of Metals. Manuscript received 7 February 1989; in final form 6 June 1989. At the time the work was carried out the authors were in the Department of Materials, Imperial College of Science, Technology and Medicine, London. Dr Ponton is now in the School of Metallurgy and Materials, The University of Birmingham.

### Introduction

#### FEATURES OF VICKERS INDENTATION TOUGHNESS TEST

The application of the Vickers indentation fracture toughness test to brittle materials, particularly glasses and ceramics, has become widespread because (i) it can be used on small samples of material not amenable to other fracture toughness tests, (ii) specimen preparation is relatively simple requiring only the provision of a polished, reflective plane surface, (iii) the Vickers diamond indenter used to produce the hardness indentations is a standard item used on a dedicated hardness tester or on a universal testing machine, (iv) in many instances the crack lengths can be measured optically without undue difficulty, and (v) it is both quick and cost effective.

The undoubted advantages of the technique are, however, offset by a number of complications; namely (i) the accuracy to which the crack lengths can be measured (and hence to which the fracture toughness can be calculated), (ii) all the indentation fracture models given in the literature assume that either one or the other of two idealised crack systems is formed during a Vickers indentation test, which may or may not be the case for the material in question, (iii) the diversity of indentation fracture toughness equations reported in the literature, and (iv) the often reported discrepancy between the indentation fracture toughness of a material and its fracture toughness as measured by conventional methods, such as the single edge notched beam (SENB) test.

These complicating factors form the underlying basis of this two part paper which critically assesses the Vickers indentation fracture toughness test. In Part 1, 19 of the indentation toughness equations to be found in the literature are reviewed and formulated into a standard form for easy comparison and use and in Part 2,<sup>1</sup> these standardised equations are evaluated using Vickers indentation toughness test data for a range of brittle materials.

#### ORIGINS OF VICKERS INDENTATION TOUGHNESS TEST

The extent of the surface radial cracking which generally occurs when non-ductile materials are indented by 'sharp'

indenters such as a Vickers or Knoop indenter was first explicitly recognised as being indicative of the fracture toughness of the material in 1957 by Palmqvist<sup>2-4</sup> who worked exclusively on cermets, e.g. WC-Co. However, the surface radial cracking frequently observed surrounding Vickers indentations (and less often around Knoop indentations) in other more brittle materials, e.g. glasses, ceramics, and glass ceramics, was generally regarded as an unwelcome feature of hardness testing. This view was fundamentally reversed when Lawn and co-workers<sup>5-7</sup> published their seminal work on the principles of indentation fracture in 1975.

It is worth summarising the results of Palmqvist and later workers on cermets before 1975 as they clearly illustrate the need to consider the stress state of the material surface before indentation when interpreting experimental Vickers indentation test data. Palmqvist<sup>2</sup> initially derived the following equation by considering the work done when a Vickers indenter moves a distance into a solid material under the action of a load  $P$

$$A = 0.0649(P_K)(P_K/H_V)^{1/2} \dots \dots \dots (1)$$

where  $A$  is the critical work required to initiate such cracking around a Vickers indentation,  $H_V$  is the Vickers hardness, and  $P_K$  is the critical indenter load required to initiate cracking.

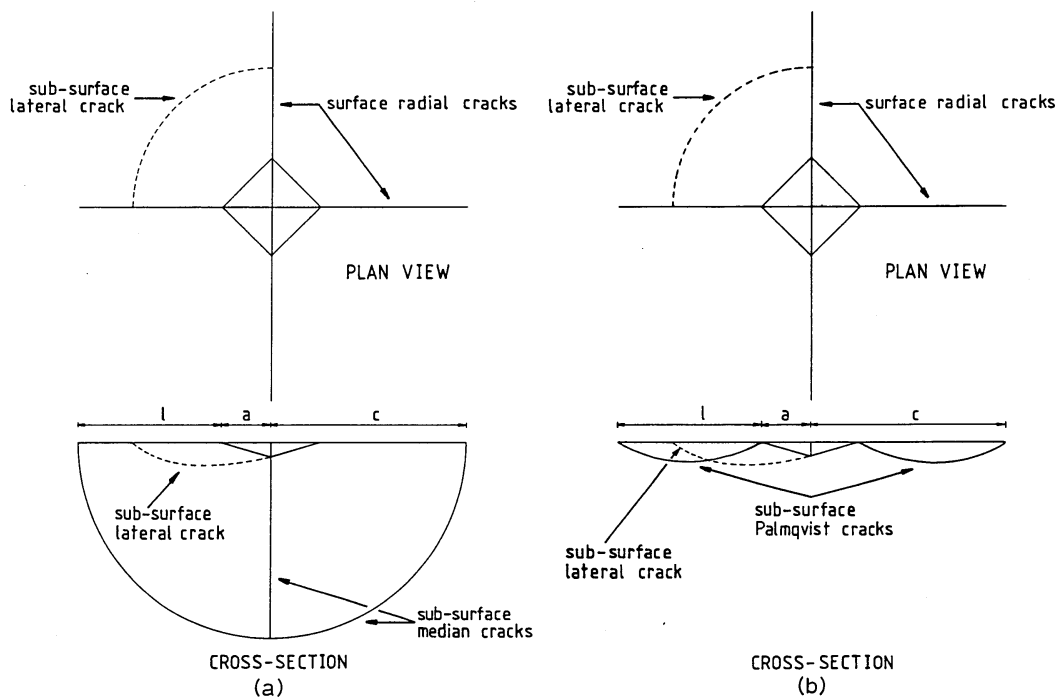
However, it was experimentally difficult to determine  $P_K$  accurately. The problem was solved when Palmqvist<sup>3,4</sup> observed that a plot of the sum of the lengths of the cracks at the four corners of the Vickers indentation  $\Sigma l$  against the indenter load  $P$  was a straight line

$$\Sigma l = a_1 P - a_2 \dots \dots \dots (2)$$

where  $a_1$  is the slope and  $a_2$  is the intercept at  $P=0$ . This linear relationship was confirmed by Dawihl and Altmeyer.<sup>8</sup> Thus, the critical indenter load  $P_K$  is given by the intercept at  $\Sigma l=0$  when  $P=P_K$ . Palmqvist, however, went a step further and redefined equation (1) as

$$A_{300} = 0.0649(P_{300})(P_{300}/H_V)^{1/2} \dots \dots \dots (3)$$

where  $A_{300}$  is the work required to generate cracks of a total summed length of 300  $\mu\text{m}$  around a Vickers indentation in cemented carbides and  $P_{300}$  is the necessary indenter load, which can be calculated for a given cermet by



1 Schematic idealised plan view and cross-sectional view of *a* Vickers indent radial–median or ‘halfpenny’ crack system and *b* Vickers indent radial Palmqvist crack system

using given values for  $a_1$  and  $a_2$  with  $\Sigma l = 300 \mu\text{m}$  in equation (2) or taken graphically from a plot of  $\Sigma l$  versus  $P$ .

To obtain consistent and reproducible results when using equations (1) and (3) the specimen surface conditions had to be highly reproducible and well defined as the crack length was strongly dependent on the surface treatment.<sup>3,4,8</sup> For example, for a given cermet, (i) surface grinding before indentation using a diamond instead of a SiC abrasive wheel resulted in significantly longer crack lengths, (ii) diamond polishing after diamond grinding gave rise to the longest indentation cracks, with the crack length increasing somewhat with increasing polishing time, and (iii) after diamond polishing, the intercept at  $\Sigma l = 0$  occurred at very low values of  $P$ , tending towards  $P = 0$ .

The dependence of the surface radial crack lengths in cermets on the preindentation surface preparation method was critically analysed by Exner<sup>9</sup> in 1969. He clarified the inconsistent and often contradictory explanations in the literature for this dependence by providing experimental evidence that the shorter cracks about Vickers indents in surface ground samples were the result of residual compressive surface stresses produced within the cobalt phase by grinding and that these stresses could be removed by annealing or polishing. Thus, the increase in crack length to a maximum length after annealing or polishing was a result of the removal of the surface layer deformed by grinding and hence the extrinsic residual compressive surface stresses. Exner also concluded that the existence of a significant (i.e. non-zero) critical indenter load for crack initiation was indicative of the presence of superimposed surface compressive stresses; thus,  $A$  in equation (1) was not a valid fracture toughness parameter as it was determined solely by the surface stress state.

Thus, Exner set  $a_2$  in equation (2) to zero, obtaining

$$\Sigma l = a_1 P \dots \dots \dots (4)$$

and rather than use a parameter such as  $A_{300}$  in equation (3), which requires the fixing of an experimental parameter, i.e. constant  $\Sigma l$ , he defined a parameter called the ‘crack

resistance’  $W$ , which is given by

$$W = P/\Sigma l \dots \dots \dots (5)$$

Note that according to equation (4),  $W$  also equals  $1/a_1$  and has the dimensions of force per unit length which corresponds to energy per unit area. In addition, it is independent of the Vickers hardness  $H_V$  and does not specify an experimental critical load, such as  $P_K$  or  $P_{300}$  in equations (1) and (3), respectively.

**Vickers indentation fracture toughness models**

The numerous indentation fracture models reported in the literature are classified into two groups; in one group it is assumed that the cracks which form as a result of Vickers indentation are well developed radial–median, ‘halfpenny’-shaped cracks (see Fig. 1*a*), and in the other group it is assumed that radial Palmqvist cracks are formed (see Fig. 1*b*). There have been two recent and essentially theoretical reviews of indentation fracture dealing with indentation by both blunt and sharp indenters,<sup>10,11</sup> however, neither review covers all the models dealt with chronologically in Part 1 of the present work, nor do they report, as does Part 2,<sup>1</sup> the application of any given model to materials other than those used by the originators of the model.

**MODELS BASED ON A RADIAL–MEDIAN CRACK GEOMETRY**

**Lawn and Swain equation (1974)**

Lawn and Swain<sup>5</sup> used a standard two dimensional linear elastic fracture mechanics approach based on the Boussinesq solutions for the stress field in an isotropic, linear elastic half-space under normal point loading to model the propagation of a ‘well behaved’ median crack associated with the indentation caused by a sharp indenter.

To a first approximation, they regarded a well behaved median crack as a penny-shaped internal crack having a crack diameter  $D$ . The equation they derived can be written in a form applicable to a Vickers indentation as follows

$$K_c = [(1 - 2\nu)/2\pi^{5/2}] (HP/D)^{1/2} \dots \dots \dots (6)$$

where  $K_c$  is the critical stress intensity factor for indentation fracture,  $P$  is the applied indenter load,  $D$  is the major median crack depth,  $\nu$  is Poisson's ratio, and  $H$  is the mean contact or indentation pressure exerted by the Vickers indenter. Note that  $H$  is given by  $P/2a^2$ , where  $a$  is the indentation half-diagonal length, and is independent of the applied indenter load; furthermore, equation (6) is only valid for  $D \geq 2a$ . Although  $D$  can be measured directly in transparent materials, later studies<sup>7,12</sup> on soda-lime glass have shown that  $D \approx c$  (see Fig. 1a, where  $c$  is the more easily measured surface radial crack length), while further work on alumina and opaque glass ceramics<sup>12</sup> indicates that the substitution of  $c$  for  $D$  in equation (6) is also acceptable for opaque materials.

**Lawn and Fuller equation (1975)**

Lawn and Fuller<sup>7</sup> noted that internal penny-shaped median cracks ultimately develop into near ideal halfpenny-shaped, radial–median crack systems during unloading (owing to the residual stresses resulting from the strain mismatch between the plastically deformed indentation zone and the elastic matrix surrounding it). By means of a linear elastic fracture mechanics analysis of the problem of a well developed halfpenny crack loaded at the centre by a conical indenter and primarily propagated by the wedging component (of the indentation force  $P$ ) acting normal to the median crack plane, they derived the following equation, valid for  $D \geq 2a$

$$K_c = [1/(\pi^{3/2} \tan \Phi)] (P/D^{3/2}) \dots \dots \dots (7)$$

where  $\Phi = \theta \pm \arctan(\mu)$ , in which  $\theta$  is the indenter cone half-angle and  $\mu$  is the coefficient of sliding friction between the indenter and the material. Equation (7) can be rewritten for a standard Vickers indenter as

$$K_c = 0.0515P/D^{3/2} \dots \dots \dots (8)$$

assuming  $\mu = 0$  and taking  $\theta$  as  $74^\circ$ , even though the Vickers indenter is not a conical indenter. Furthermore, the substitution of  $c$  for  $D$  is acceptable for opaque material.<sup>12</sup>

**Evans and Wilshaw equation (1976)**

Evans and Wilshaw<sup>13</sup> recognised that the indentation stress field due to a sharp indenter is essentially elastic–plastic in nature. By carrying out a dimensional fracture mechanics analysis of indentation fracture using elastic stress field solutions modified by the presence of the indentation plastic zone they showed that the indentation surface radial crack length  $c$  should be related to the indentation half-diagonal length  $a$ , as follows

$$K_c/(\sigma_Y a^{1/2}) = F_1(c/a)F_2(r_p/a)F_3(\nu)F_4(\mu) \dots \dots \dots (9)$$

where  $\sigma_Y$  is the uniaxial yield stress,  $r_p$  is the indentation plastic zone radius, and  $F_1, F_2, F_3, F_4$  are empirically determined functions. They assumed that  $F_2, F_3$ , and  $F_4$  are approximately constant, in which case  $c/a$  should be a strong function of  $K_c/(\sigma_Y a^{1/2})$  or  $K_c\phi/(H_V a^{1/2})$ , since  $H/\sigma_Y \equiv \phi$ , where  $\phi = f(E/\sigma_Y, \nu)$ ,  $H_V = 0.9272H$ , and  $E$  is Young's modulus.

Evans and Wilshaw carried out a least squares fit of the indentation radial crack data and the  $K_{Ic}$  data for a number of materials plotted as  $K_{Ic}\phi/(H_V a^{1/2})$  against  $c/a$  and obtained an equation apparently valid for  $0.6 \leq c/a < 4.5$ , which, on noting that  $H_V = 0.4636P/a^2$  and assuming

$\phi = 2.7$ , can be written as

$$K_c = 0.1704(H_V a^{1/2}) \log(4.5a/c) = 0.079(P/a^{3/2}) \log(4.5a/c) \dots \dots \dots (10)$$

**Evans and Charles equation (1976)**

The validity of the functional relationships in equation (9) was further investigated by Evans and Charles<sup>14</sup> using conventional double torsion (DT) fracture toughness data, obtained in a dry nitrogen environment for a number of technologically more important ceramic materials, in place of  $K_c$ , i.e. assuming  $K_c \equiv K_{Ic}$  in the general belief that DT specimen fracture effectively occurs by tensile opening or mode I crack propagation to give a  $K_{Ic}$  value. They found that for Vickers indentation data,  $c/a$  correlated with  $K_c\phi/(H_V a^{1/2})$  and increased with decreasing hardness. Their assumption was that the influence of  $\nu$  on  $c/a$  was not significant and that a dependence of  $\mu$  on hardness was not likely, but that the hardness dependence of  $c/a$  was due to  $r_p/a$  increasing with decreasing hardness and  $r_p/a$  being proportional to a power function of  $E/\sigma_Y$ , i.e. of  $E\phi/H$ .

Evans and Charles carried out a dimensional fracture mechanics analysis which gave

$$K_c\phi/(H a^{1/2}) = F_1(c/a)F_2(E\phi/H) \dots \dots \dots (11)$$

The assumption of a power function for  $F_2$  was justified by data fitting, giving  $F_2 = (E\phi/H)^{2/5}$ . The slope of the plot

$$\log \{ [K_{Ic}\phi/(H_V a^{1/2})] (H_V/E\phi)^{2/5} \}$$

versus  $\log(c/a)$  was effectively  $-3/2$  for the high  $c/a$  polycrystalline data; this is also the slope of the elastic solution for a penny-shaped crack wedged at its centre by a load  $P^*$

$$K = P^*(\pi c)^{-3/2} \dots \dots \dots (12)$$

Using  $P^* = P/(2 \tan 74^\circ)$  as given by Ref. 7 together with  $P = H_V a^2/0.4636$ , equation (12) can be written as

$$K = H_V a^2/[0.4636(2 \tan 74^\circ)(\pi c)^{3/2}] \dots \dots \dots (13)$$

By taking  $\phi = 2.7$ , the following equation, after Evans and Charles, can be derived

$$K_c\phi/(H_V a^{1/2}) = k[2.7/(0.4636\pi^{3/2} 2 \tan 74^\circ)](c/a)^{-3/2} = 0.15k(c/a)^{-3/2} \dots \dots \dots (14)$$

where  $k$  is a correction factor to equation (12) required by the presence of the free surface; Evans and Charles found empirically that  $k = 3.2$  at large values of  $c/a$ . Substituting  $0.4636P/a^2$  for  $H_V$  in equation (14) and rearranging gives

$$K_c = 0.1777H_V a^2/c^{3/2} = 0.0824P/c^{3/2} \dots \dots \dots (15)$$

**Evans and Davis equation (1979)**

Evans<sup>15</sup> has reviewed the elastic–plastic stress field approach to indentation fracture, demonstrating that either a generalised stress analysis in conjunction with linear elastic fracture mechanics concepts or a dimensional analysis procedure could be used to obtain the following crack extension relationship

$$K_c/(H a^{1/2}) = g_1(c/a)g_2(E/H) \dots \dots \dots (16)$$

where  $g_1(c/a)$  and  $g_2(E/H)$  are independent dimensionless functions if stresses within the indentation plastic zone exert a negligible influence on the crack extension. Evans treated the indenter as a wedge and applied the elastic solution for a penny-shaped crack wedged by a force  $P^*$  at its centre, i.e. equation (12), to obtain an analogous crack extension relationship for a Vickers indenter

$$K_c/(H a^{1/2}) = f(H/E)(c/a)^{-3/2} \dots \dots \dots (17)$$

where  $f(H/E)$  is a function. Comparing equations (16) and (17) shows that  $g_1(c/a) \equiv (c/a)^{-3/2}$ . Evans found from data analysis that a suitably simple form for  $g_2(E/H)$  was  $(E/H)^{2/5}$ ; a plot of  $\log \{ [K_{Ic}/(H_V a^{1/2})] (H_V/E)^{2/5} \}$  versus

log(*c/a*) produced an unexpectedly good correlation of the Evans and Charles<sup>14</sup> data despite the exclusion of their constraint factor  $\phi$ . Evans also reported a polynomial fit to this plot after Davis (see Ref. 15) of  $\log A \equiv F$ , where

$$A = [K_{Ic}/(H_V a^{1/2})](H_V/E)^{2/5}$$

and

$$F = -1.59 - 0.34B - 2.02B^2 + 11.23B^3 - 24.97B^4 + 16.32B^5 \dots (18)$$

where  $B = \log(c/a)$ . Since  $A = 10^F$ , an equation for  $K_c$  can be written as follows

$$K_c = H_V a^{1/2} (E/H_V)^{2/5} \times 10^F = 0.6305 E^{0.4} P^{0.6} \times 10^F a^{-0.7} \dots (19)$$

However, the fit produced a slope of  $-1.32$  (Ref. 16) rather than  $-3/2$  as indicated by equation (17). Evans attempted to correlate the data in terms of  $g_1(c/a)$  alone, i.e. as if residual stresses and the plastic zone size are not important factors; no satisfactory correlation was found, suggesting their importance via the term  $g_2(E/H)$ .

**Blendell equation (1979)**

Blendell<sup>17</sup> curve-fitted the data of Evans and Charles<sup>14</sup> and obtained the following equation

$$[K_c \phi / (H_V a^{1/2})] (H_V/E\phi)^{2/5} = 0.055 \log(8.4a/c) \dots (20)$$

which, taking  $\phi = 2.7$ , can be written as

$$K_c = 0.0303 (H_V a^{1/2}) (E/H_V)^{2/5} \log(8.4a/c) \dots (21)$$

This equation has the same form as equation (10) due to Evans and Wilshaw<sup>13</sup> except for the presence of the  $E/H_V$  term.

**Lawn, Evans, and Marshall equation (1980)**

The elastic-plastic model of indentation fracture caused by a 'sharp' indenter was extended by Lawn *et al.*<sup>18</sup> who explicitly resolved the complex elastic-plastic stress field beneath the indentation into a reversible elastic stress field component and an irreversible residual stress field component. The contribution of the elastic component to crack propagation is small compared with that of the residual component as a result of its reversible nature.

The elastic component is taken to operate outside the indentation plastic zone; it enhances subsurface (median) crack propagation, but suppresses surface (radial) crack propagation during the loading half cycle, then completely reverses its operation during the unloading half cycle enhancing the latter and suppressing the former. The residual component provides the driving force for continued radial and median (as well as lateral) crack extension during the unloading half cycle resulting in the attainment of a halfpenny equilibrium crack configuration upon complete unloading.

The residual stress component is regarded as being concentrated at a point located at the crack centre at the elastic/plastic interface, acting as a crack mouth opening point-force. Furthermore, it is assumed that the indentation plastic zone volume can be equated to the volume of an internally pressurised spherical cavity, allowing the use of Hill's elastic-plastic solution to the expanding spherical cavity problem.<sup>19</sup> Lawn *et al.* thus derived the following crack extension relationship

$$c = \{ \Phi_r (\cot \theta)^{2/3} [(E/H)^{1/2} / K_c] \}^{2/3} P^{2/3} \dots (22)$$

where  $c$  is the equilibrium surface radial crack length after unloading,  $\Phi_r$  is a dimensionless constant independent of the indenter-specimen system, and  $\theta$  is the characteristic indenter half-angle. The point force approximation used by Lawn *et al.* has been shown to be valid for  $c/a \approx 2$  for Vickers indents with fully developed cracks,<sup>20</sup> in which case

$c$  is proportional to  $P^{2/3}$ , i.e.  $P/c^{3/2}$  is constant. The constant  $\Phi_r$  was evaluated as  $0.032 \pm 0.002$  using  $\theta = 74^\circ$  and the  $E/H$  ratio for soda-lime glass, thus 'calibrating' equation (22), which can be rewritten as

$$K_c = 0.0139 (E/H)^{1/2} (P/c^{3/2}) \dots (23)$$

**Anstis, Chantikul, Lawn, and Marshall equation (1981)**

Anstis *et al.*<sup>21</sup> undertook a critical evaluation of the Vickers indentation fracture toughness measurement technique using equation (23) in the form

$$K_c = \Omega_r (E/H)^{1/2} (P/c^{3/2}) \dots (24)$$

where  $\Omega_r$  is a material independent constant. In order to evaluate  $\Omega_r$ , a number of ceramic materials, e.g. glasses, a glass ceramic, and polycrystalline ceramics, were chosen as 'reference' materials. These materials were tested to determine their  $H$ ,  $E$ , and double cantilever beam (DCB)  $K_{Ic}$  values (or literature values taken), and were also indented with a Vickers indenter to determine  $c$  as a function of  $P$ . From plots of  $P/c^{3/2}$  against  $P$  it was found that  $P/c^{3/2}$  was independent of  $P$  for each material within experimental scatter. Substituting  $K_{Ic}$  for  $K_c$  and using the  $H$ ,  $E$ , and mean  $P/c^{3/2}$  values for each material in the following equation

$$\Omega_r = K_{Ic} / [(E/H)^{1/2} (P/c^{3/2})] \dots (25)$$

a value of  $\Omega_r$  was obtained for each material. Thus, a mean calibration constant  $\Omega_r = 0.016 \pm 0.004$  was obtained, giving the following equation

$$K_c = 0.016 (E/H)^{1/2} (P/c^{3/2}) \dots (26)$$

Anstis *et al.* concluded that an accuracy of better than 30 to 40% should be attainable for those materials which are well behaved in their indentation response.

**Niihara, Morena, and Hasselman equation (1982)**

Niihara *et al.*<sup>22</sup> have proposed that the apparent invalidity of the relationship  $P/c^{3/2} = \text{constant}$  (based on the assumption of a well developed halfpenny crack) for  $c/a \approx 3$  is due to a transition from a radial-median crack system (see Fig. 1a) for  $c/a > \approx 3$  to a Palmqvist crack system (see Fig. 1b) for  $c/a < \approx 3$ . Their proposal is based on the observation that equation (20) does not give a good fit to data for WC-Co materials containing  $> 6 \text{ vol.-%Co}$ , which developed Palmqvist cracks with  $c/a < 3$ .

They plotted the data of Evans and Wilshaw,<sup>13</sup> Evans and Charles,<sup>14</sup> and Dawihl and Altmeyer<sup>8</sup> on axes of  $\log\{[K_c \phi / (H_V a^{1/2})] (H_V/E\phi)^{2/5}\}$  versus  $\log(c/a)$  or  $\log(l/a)$  since  $l/a = c/a - 1$ . Correlation analyses showed that for data with  $c/a \geq \approx 2.5$  the best correlation was in terms of  $c/a$  via the equation

$$[K_c \phi / (H_V a^{1/2})] (H_V/E\phi)^{2/5} = 0.129 (c/a)^{-3/2} \dots (27)$$

Assuming  $\phi = 2.7$ , this equation can be written as

$$K_c = 0.0711 (H_V a^{1/2}) (E/H_V)^{2/5} (c/a)^{-3/2} \dots (28)$$

However, for data with  $c/a \leq \approx 2.5$ , the best correlation was provided via  $l/a$  rather than  $c/a$ , such that for  $l/a$  in the range  $\sim 0.25$  to  $\sim 2.5$ , i.e.  $c/a$  in the range  $\sim 1.25$  to  $\sim 3.5$ , the best fit equation was

$$[K_c \phi / (H_V a^{1/2})] (H_V/E\phi)^{2/5} = 0.035 (l/a)^{-1/2} \dots (29)$$

**Lankford equation (1982)**

Lankford<sup>23</sup> used the WC-Co indentation data of Niihara *et al.*<sup>22</sup> and indentation data for four other materials that form Palmqvist cracks at low indentation loads and well developed radial-median cracks at high loads to generate log-log plots of  $[K_c \phi / (H_V a^{1/2})] (H_V/E\phi)^{2/5}$  against  $l/a$  and  $c/a$ . Representing the individual  $l/a$  dependence of each of

the five material types by  $(l/a)^{-m}$ , he found that  $m$  ranged from 0.5 to 1.05 with the lines diverging at low values of  $l/a$ , where equation (29) should be most appropriate, with a mean slope of  $0.85 \pm 0.22$ . By contrast, when representing the individual  $c/a$  dependence of each material by  $(c/a)^{-n}$ ,  $n$  ranged from 1.45 to 1.66 with a mean of  $1.59 \pm 0.08$  and all the data lines clustered around the 'universal' line of Evans and Charles<sup>14</sup> ( $n = 1.5$ ) in a roughly parallel manner.

Lankford also noted that equation (27) lies close enough to the centroid of the data band such that using it for materials on the band periphery would involve an error of no more than ~35%. He reduced the error by finding the best curve through his data, the WC-Co data of Niihara *et al.*,<sup>22</sup> and the original data of Evans and Charles<sup>14</sup> to give

$$[K_c \phi / (H_V a^{1/2})] (H_V / E \phi)^{2/5} = 0.142 (c/a)^{-1.56} \quad \dots (30)$$

which, assuming  $\phi = 2.7$ , can be written as

$$K_c = 0.0782 (H_V a^{1/2}) (E / H_V)^{2/5} (c/a)^{-1.56} \quad \dots (31)$$

Furthermore, he concluded that the basic Evans and Charles<sup>14</sup> relationship in terms of  $c/a$  is valid for non-ductile materials over the entire indentation fracture morphology spectrum, from Palmqvist to radial–median morphology and capable of giving  $K_c$  to an accuracy of better than 35%, given adequately accurate  $E$  and  $H$  data. Note that  $\phi$  ( $\equiv H/\sigma_V$ ) affects the  $K_c$  value because it depends on  $\nu$  and  $E/\sigma_V$  (e.g. see Refs. 19, 24). Using  $\nu = 0.25$  and  $E/\sigma_V \approx 75$ , which are typical values for ceramics,<sup>15</sup> in the following equation proposed by Marsh<sup>24</sup>

$$H/\sigma_V = 0.28 + (0.60B) \ln Z \quad \dots (32)$$

where

$$B = 3/[3 - (1 - 2\nu)(\sigma_V/E)]$$

and

$$Z = 3(E/\sigma_V)/[4 + \nu - (\sigma_V/E)(1 - 2\nu)(1 + \nu)]$$

gives  $\phi = 2.7$ , which is the value assumed in this paper and explains why in the literature on Vickers indentation toughness,  $\phi$  is quoted as being ~3 and taken as a pseudoconstant.

### Miranzo and Moya equation (1984)

Miranzo and Moya<sup>25</sup> derived an analytical expression using a model after Chiang *et al.*<sup>26</sup> based on the Hill<sup>19</sup> solution to the problem of an internally pressurised spherical cavity in an infinite isotropic elastic–plastic solid, but which takes account of the reduced constraint factor resulting from the stress free surface of the indentation. The expression derived is a continuous function of  $c/a$  valid for  $c/a > \approx 1.3$ , but being analytically complex is not readily usable. However, Miranzo and Moya report that the data of Evans and Charles<sup>14</sup> were in good agreement with the expression for  $c/a > 1.3$ ; thus, they were able to decompose their original unwieldy expression into the following equations

$$[K_c / (H_V a^{1/2})] [f(E/H)]^{-1} = 0.05 (c/a)^{-0.5} \quad \dots (33)$$

valid for  $c/a \leq \approx 2.8$ , and

$$[K_c / (H_V a^{1/2})] [f(E/H)]^{-1} = 0.09 (c/a)^{-1.08} \quad \dots (34)$$

valid for  $c/a \geq \approx 2.8$ .

The function  $f(E/H) = [(\beta_{EXP}^2/\delta) - 1.5]/(1 - \nu)$ , where  $\delta = 2[1 + 3 \ln \beta_{EXP}]/3$  and  $\beta_{EXP}$  is the experimental relative plastic zone size, i.e. the experimental value of  $r_p/a$ . A least squares analysis by Ponton, using the  $E$ ,  $H$ , and  $\beta_{EXP}$  data cited by Chiang *et al.*,<sup>26</sup> of  $\beta_{EXP} = A(E/H)^B$ , where  $A$  and  $B$  are constants gives  $\beta_{EXP} = 0.792(E/H)^{0.408}$  (which is an empirical approximation to equation (37)). Equations (33) and (34) can be rearranged as

$$K_c = 0.05 [f(E/H)] H_V a / c^{0.5} \quad \dots (35)$$

for  $c/a \leq \approx 2.8$  and

$$K_c = 0.09 [f(E/H)] H_V a^{1.58} / c^{1.08} \quad \dots (36)$$

for  $c/a \geq \approx 2.8$ , respectively.

As a consequence of being able to separate their derived expression into two equations about a pivot point of  $c/a \approx 2.8$ , Miranzo and Moya<sup>25</sup> concluded that the apparent discrepancy between experimental data for materials with  $c/a < \approx 3$  and the indentation models based on a fully developed radial–median crack system, depends on the value of  $c/a$  rather than there being a transition in the crack system morphology from Palmqvist morphology at low values of  $c/a$  to radial–median morphology at high values of  $c/a$ .

### Laugier equations (1985)

Laugier<sup>27</sup> derived a modified equation of the form of equations (22) and (24) by employing the analytical approximation  $\beta = (E/H)^{1/3}$  (where  $\beta$  is the relative plastic zone size) to the following function resulting from Hill's elastic–plastic expanding spherical cavity solution<sup>19</sup> (see also Refs. 18, 26)

$$E/H = 9[(1 - \nu)\beta^3 - 2(1 - 2\nu)/3]/[2(1 + \ln \beta^3)] \quad \dots (37)$$

rather than  $\beta \approx (E/H)^{1/2}$  as chosen empirically by Lawn *et al.*<sup>18</sup> He obtained the following equation

$$K_c = 0.010 (E/H)^{2/3} (P/c^{3/2}) \quad \dots (38)$$

in which the calibration constant of  $0.010 \pm 0.002$  was evaluated using the data of Anstis *et al.*<sup>21</sup> excluding data on WC-Co, which does not satisfy the criterion  $c \gg a$ .

Laugier used the same data to calibrate the crack extension relationship of Evans and Charles,<sup>14</sup> i.e.

$$K_c \propto (E\phi/H_V)^{2/5} (P/c^{3/2})$$

using the equation

$$k = K_{Ic} / [(E/H)^{2/5} (P/c^{3/2})]$$

which neglects the constraint factor  $\phi$ , and obtained  $k = 0.022 \pm 0.005$ . Thus,

$$K_c = 0.022 (E/H_V)^{2/5} (P/c^{3/2}) \quad \dots (39)$$

### Tanaka equation (1987)

Tanaka<sup>28</sup> proposed a new model of elastic–plastic indentation fracture termed the 'inclusion core model', in which the pressure due to the indenter is interpreted as a hemispherical compressible, hydrostatic core rather than as an expanding spherical cavity.<sup>18, 26</sup> In the inclusion core model, the material volume displaced by the indenter is accommodated by the compressible, hydrostatic core in conjunction with the hemispherical plastic zone outside the core, whereas in the spherical cavity model the cavity volume is equated with the indentation volume.<sup>18, 26</sup> By applying the Hill elastic–plastic expanding spherical cavity analysis to the plastic zone and the Eshelby spherical inclusion analysis to the core (see Ref. 28), Tanaka ultimately derived the following analytical equation

$$K_c = 0.035 (E/H_V)^{1/4} (P/c^{3/2}) \quad \dots (40)$$

## MODELS BASED ON PALMQVIST CRACK GEOMETRY

### Niihara, Morena, and Hasselman equation (1982)

Niihara *et al.*<sup>22</sup> conducted a data fitting analysis of the data of Evans and Wilshaw,<sup>13</sup> Evans and Charles,<sup>14</sup> and Dawihl and Altmeyer<sup>8</sup> as discussed above. They contend that for Vickers indentation data with  $c/a < \approx 3$ , the crack system is Palmqvist in nature rather than radial–median,<sup>22, 29</sup> thus, the characteristic crack length is  $l$  rather than  $c$ , where  $l/a = c/a - 1$  (compare Figs. 1a and 1b). Niihara *et al.* obtained a best fit equation for  $c/a$  data in the range ~1.25

to ~3.5, i.e.  $l/a$  data in the range ~0.25 to ~2.5 (see equation (29) above), which, on assuming  $\phi = 2.7$ , can be written as

$$K_c = 0.0193(H_v a)(E/H_v)^{2/5}(l^{-1/2}) \dots (41)$$

**Niihara equation (1983)**

Niihara<sup>30</sup> also proposed a model based on theoretical fracture mechanics and elastic-plastic indentation theory in which he modelled Palmqvist cracks as semielliptical surface flaws and derived the following relationship

$$[K_c \phi / (H_v a^{1/2})] (H_v / E \phi)^{2/5} = k_2 (l/a)^{-1/2} \dots (42)$$

where  $k_2$  is a constant equal to  $0.085k_0k_1$ . The term  $k_0$  is the free surface correction factor and  $k_1$  is the crack depth factor, which is assumed to be independent of the crack size. Niihara estimated the value of  $k_2$  by using  $k_0 = 1.12$  and  $k_1 = 0.5$  to give  $k_2 = 0.048$ . When deriving equation (42), Niihara assumed that: (i) typically  $1 \leq l/a \leq 2.5$ , (ii)  $d/l \leq 1$  for typical Palmqvist cracks, where  $d$  is the crack depth, and (iii) the maximum crack depth on unloading is about the order of the indentation depth. Assuming  $\phi = 2.7$ , equation (42) can be rearranged as

$$K_c = 0.0264(H_v a)(E/H_v)^{2/5}(l^{-1/2}) \dots (43)$$

**Shetty, Wright, Mincer, and Clauer equation (1985)**

Using Exner's definition<sup>9</sup> of the crack resistance  $W$  in equation (5), which can also be written as

$$W = P/4l \dots (44)$$

Shetty *et al.*<sup>31</sup> noted that equation (42) after Niihara<sup>30</sup> could be modified to

$$K_c = \tau(H_v W)^{1/2} \dots (45)$$

by using  $H_v = 0.4636P/a^2$  in conjunction with equations (43) and (44). However,  $\tau$ , which is a function of  $E/H_v$ , was not explicitly defined; in fact  $\tau = 0.0360(E/H_v)^{2/5}$ . They carried out an approximate fracture mechanics analysis to explain the nature of equations (44) and (45); each collinear pair of Palmqvist cracks (see Fig. 1b) was represented as a two dimensional through thickness crack, since the crack depth was of a similar magnitude to the indentation plastic zone depth. The residual crack opening force was regarded as a rigid wedge loading the crack at its centre such that the crack is in equilibrium with the wedge; the wedge thickness was taken as being equal to the indentation plastic zone expansion required to accommodate the indentation volume. Shetty *et al.* were thereby able to derive the following equation\*

$$K_c = [1/[3(1 - v^2)(2^{1/2}\pi^{5/2} \tan \theta)^{1/3}]] [HP/(4l)]^{1/2} \dots (46)$$

where  $\theta = 68^\circ$ . Equation (46) can also be written as

$$K_c = \Gamma(HW)^{1/2} \dots (47)$$

where  $\Gamma = 1/[3(1 - v^2)(2^{1/2}\pi^{5/2} \tan \theta)^{1/3}]$  and  $\theta = 68^\circ$ . When  $v = 0.25$ , equation (47) becomes

$$K_c = 0.0902(HW)^{1/2} = 0.0937(H_v W)^{1/2} \dots (48)$$

**Laugier equation (1987)**

Laugier<sup>35</sup> recently derived the following analytical stress intensity factor for Palmqvist cracks  $K^{SC}$ , by representing individual Palmqvist cracks as semicircles

$$K^{SC} = 2[\pi/(2 + \pi)]^{1/2}(l/a)^{-1/2} K^{CLP} \dots (49)$$

\* It has been noted by Ponton<sup>32</sup> that the exponent of  $\pi$  in equation (46), i.e. 5/2, is missing from the same equation given in the original paper of Shetty *et al.*,<sup>31</sup> (equation (12) in Ref. 31), and in two subsequent papers by Shetty *et al.*<sup>33,34</sup> As far as the present authors are aware, the corrected version, i.e. equation (46) above, first appears in Ref. 32.

where  $K^{CLP} = (1/\pi^{3/2})(P_R/c^{3/2})$  is the stress intensity factor for a penny-shaped crack loaded at its centre by a point force  $P_R$ . The force  $P_R$  is the residual plastic crack driving force given by Laugier in a previous paper<sup>27</sup> which varies as  $P(E/H)^{2/3}(\cot \theta)^{2/3}$ , where  $\theta = 68^\circ$ . He calibrated equation (49) using the data of Anstis *et al.*,<sup>21</sup> excluding their WC-Co data to give

$$K_c = 0.015(l/a)^{-1/2}(E/H)^{2/3}(P/c^{3/2}) \dots (50)$$

**Summary of Vickers indentation toughness equations**

**STANDARDISED FORMS OF INDENTATION EQUATIONS**

The following points were noted when reviewing the literature on indentation toughness models in the previous section.

1. The possibility of confusion arising from the use of the Vickers hardness  $H_v$  or the mean contact pressure  $H$ , and an indenter half-angle of  $74^\circ$  or  $68^\circ$ , depending on the indentation fracture model in question.

2. The unfortunate reporting of  $\phi$  as being ~3, which is an unhelpful approximation when one attempts to evaluate the constant term in certain of the equations.

3. The differing functional importance given to Poisson's ratio by the various models, i.e.  $\nu$  is (i) not considered to be a significant parameter (Refs. 7, 15, 18, 21, 27, 35), (ii) incorporated explicitly as a variable (Refs. 5, 25, 31), or (iii) incorporated implicitly as a constant (Refs. 13, 14, 17, 22, 23, 28, 30), e.g. within  $\phi$ , presumably as 0.25.

4. The need for standardised equations that are directly comparable and convenient to use experimentally.

In view of the above points, the authors have rewritten the relevant equations, taking  $\nu$  and  $\phi$  as 0.25 and 2.7, respectively, in terms of the following experimental parameters, and where necessary (see Figs. 1a and 1b): the applied indenter load  $P$ , the mean radial surface crack length  $c$ , the mean Palmqvist (surface) crack length  $l$ , the mean indentation half-diagonal length  $a$ , and the Young's modulus/Vickers hardness ratio  $E/H_v$ . The modified equations are given in Table 1.

**GENERIC FORMS OF INDENTATION EQUATIONS**

It is apparent that the multitude of halfpenny and Palmqvist equations in the literature, the alternative definitions of the indenter half-angle and hardness, and the differing degrees of significance given to Poisson's ratio tend to confuse an already complex subject. This confusion detracts from the stated experimental advantages of the Vickers indentation fracture toughness technique. However, having formulated standardised forms of the 19 equations reviewed, it becomes clear that if the Vickers hardness is independent of  $P$  (i.e.  $a = \alpha P^{1/2}$  where  $\alpha$  is a constant), all halfpenny and Palmqvist equations can be described by one of the generic forms given in Table 2. Furthermore, in these generic equations, the factor  $k$  is an adjustable constant which is different for each form of generic equation and may contain an  $(E/H_v)^n$  term, where  $n$  is a constant less than one, e.g.  $k \equiv k'(E/H_v)^n$  or  $k''\{f[(E/H_v)^n]\}$ , where  $k$ ,  $k'$ , and  $k''$  may be empirical, semiempirical, or theoretical constants.

As a generalisation, equations (51), (52), (55), and (56) indicate that radial-median, i.e. halfpenny cracking, is to be expected if an experimental plot of  $\ln P$  versus  $\ln c$  produces a slope between 1 and 2, whereas if an experimental plot of  $\ln P$  versus  $\ln l$  gives a slope between 1/2 and 1 then Palmqvist cracking is to be expected; the converse is also implied. Alternatively, the constancy of  $P/c^{3/2}$ ,  $P/c$ , and  $P/l$

**Table 1** Modified equations (unless stated otherwise, for halfpenny equations it is implicitly assumed that  $c/a \geq 2$ )

| Equation  | Ref. no. | Designation in Part 2 <sup>1</sup> |
|---|----------|------------------------------------|
| <b>Radial–median (halfpenny-shaped) crack equations</b>                                       |          |                                    |
| $K_c = 0.0101 P/(ac^{1/2})$ . . . . . (6)   | 5        | LS                                 |
| $K_c = 0.0515 P/c^{3/2}$ . . . . . (8)  | 7        | LF                                 |
| $K_c = 0.079(P/a^{3/2}) \log(4.5a/c)$ for $0.6 \leq c/a < 4.5$ . . . . . (10)                 | 13       | EW                                 |
| $K_c = 0.0824 P/c^{3/2}$ . . . . . (15)   | 14       | EC                                 |
| $K_c = 0.4636(P/a^{3/2})(E/H_V)^{2/5}(10^F)^*$ . . . . . (19)                                 | 15       | ED                                 |
| $K_c = 0.0141(P/a^{3/2})(E/H_V)^{2/5} \log(8.4a/c)$ . . . . . (21)                            | 17       | B                                  |
| $K_c = 0.0134(E/H_V)^{1/2}(P/c^{3/2})$ . . . . . (23)   | 18       | LEM                                |
| $K_c = 0.0154(E/H_V)^{1/2}(P/c^{3/2})$ . . . . . (26)   | 21       | ACLM                               |
| $K_c = 0.0330(E/H_V)^{2/5}(P/c^{3/2})$ for $c/a \geq 2.5$ . . . . . (28)                      | 22       | NMH1                               |
| $K_c = 0.0363(E/H_V)^{2/5}(P/a^{1.5})(a/c)^{1.56}$ . . . . . (31)                             | 23       | JL                                 |
| $K_c = 0.0232[f(E/H_V)]P/(ac^{1/2})\dagger$ for $c/a \leq 2.8$ . . . . . (35)                 | 25       | MM1                                |
| $K_c = 0.0417[f(E/H_V)]P/(a^{0.42}c^{1.08})\dagger$ for $c/a \geq 2.8$ . . . . . (36)         | 25       | MM2                                |
| $K_c = 0.0095(E/H_V)^{2/3}(P/c^{3/2})$ . . . . . (38)   | 27       | L1                                 |
| $K_c = 0.022(E/H_V)^{2/5}(P/c^{3/2})$ . . . . . (39)  | 27       | L2                                 |
| $K_c = 0.035(E/H_V)^{1/4}(P/c^{3/2})$ . . . . . (40)  | 28       | T                                  |
| <b>Palmqvist crack equations</b>  |          |                                    |
| $K_c = 0.0089(E/H_V)^{2/5}P/(a^{1/2})$ for $l/a \approx 0.25$ to $\approx 2.5$ . . . . . (41) | 22       | NMH2                               |
| $K_c = 0.0122(E/H_V)^{2/5}P/(a^{1/2})$ for $1 \leq l/a \leq 2.5$ . . . . . (43)               | 30       | N                                  |
| $K_c = 0.0319P/(a^{1/2})$ . . . . . (48)  | 31       | SWMC                               |
| $K_c = 0.0143(E/H_V)^{2/3}(a/l)^{1/2}(P/c^{3/2})$ . . . . . (50)                              | 35       | L3                                 |

\* Where  $F = -1.59 - 0.34B - 2.02B^2 + 11.23B^3 - 24.97B^4 + 16.32B^5$  and  $B = \log(c/a)$ .

† Where  $f(E/H_V) = [(\beta_{EXP}^2/\delta) - 1.5]/0.75$ , in which  $\delta = 2(1 + 3 \ln \beta_{EXP})/3$  and  $\beta_{EXP} = 0.768(E/H_V)^{0.408}$ .

with increasing  $P$  can be investigated, for example, by calculating the coefficient of variation of  $P/c^{3/2}$ ,  $P/c$ , and  $P/l$ , respectively. However, equation (57), which employs both the classic halfpenny and Palmqvist model parameters, i.e.  $P/c^{3/2}$  and  $(P/l)^{1/2}$ , respectively, implies that, on the basis of the invariance of either one of these parameters with  $P$ , one would not be able to predict the morphology of the cracks developed as a result of indentation.

## Conclusions

Modification of the 19 Vickers indentation toughness equations into directly comparable forms has given rise to seven generic equations. The nature of these generic equations implies that an experimentally confirmed functional dependence of  $c$  or  $l$  on  $P$  may not actually indicate a specific crack morphology. The implications of this will be investigated in Part 2<sup>1</sup> together with the relative ability of the standardised equations to predict the fracture toughnesses of a range of brittle ceramics.

## Acknowledgments

The authors thank the SERC for financial support and Professor D. W. Pashley, FRS for the provision of research facilities.

**Table 2** Generic equations

|   |  |
|---|--|
| <b>Radial–median (halfpenny) crack equations</b>  |  |
| $K_c = kP/c^{3/2}$ . . . . . (51)   |  |
| $K_c = kP/(ac^{1/2}) \equiv k(P/a^{3/2})(a/c)^{1/2} \equiv k(P/c)^{1/2}$ . . . . . (52)                   |  |
| $K_c = k(P/a^{3/2}) \log(Ja/c)^*$ . . . . . (53)  |  |
| $K_c = k(P/a^{3/2})10^{F\dagger}$ . . . . . (54)  |  |
| $K_c = k(P/a^{3/2})(a/c)^m \equiv kP/(a^{1.5-m}c^m) \equiv k(P^{0.5+0.25/m}/c)^m \ddagger$ . . . . . (55) |  |
| <b>Palmqvist crack equations</b>  |  |
| $K_c = kP/(a^{1/2}) \equiv k(P/a^{3/2})(a/l)^{1/2} \equiv k(P/l)^{1/2}$ . . . . . (56)                    |  |
| $K_c = k(P/c^{3/2})(a/l)^{1/2} \equiv k(P/c^{3/2})(P/l)^{1/2}$ . . . . . (57)                             |  |

\* Where  $J$  is an empirical constant obtained by data fitting.

† Where  $F \equiv f[\log(c/a)]$  and is determined by data fitting.

‡ Where  $m$  is an empirical constant between 1 and 2, found by data fitting.

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*Announcement and Call for Papers*

**THIRD INTERNATIONAL CONFERENCE ON  
CONSTITUTIVE LAWS FOR ENGINEERING MATERIALS:  
THEORY AND APPLICATIONS  
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WORKSHOP ON INNOVATIVE USE OF MATERIALS IN  
INDUSTRIAL AND INFRASTRUCTURE DESIGN AND  
MANUFACTURING**

7–12 January 1991

University of Arizona, Tucson

Following the overwhelming reception and success of the two previous conferences, the 'Third international conference on constitutive laws for engineering materials: theory and applications' will be held again at the University of Arizona, Tucson, AZ; the *tentative* dates are 7–12 January 1991. It is also planned to organise during the conference a workshop on 'Innovative use of materials in industrial and infrastructure design and manufacturing', which is expected to be of interest to both industry and academics.

The major topics proposed to be covered at the conference and workshop are:

- Basic theory and unified concepts for various models for mechanical, thermal, fluid, and other environmental loadings and effects
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- Implementation in computational procedures; consistency, stability, and robustness of algorithms, parallel processing, optimisation and stochastic methods, and identification techniques through expert systems
- Material systems: composites – metals, alloys, ceramics, and polymers, intermetallic compounds, high performance plastics, directionally solidified alloys; geomaterials, concrete and other porous saturated and unsaturated materials, extraterrestrial (lunar) materials, reinforced earth; biomaterials; electronic and optical devices – silicon wafers, LSI chips, thin-layered and packaging materials, single crystals, foam composites; discontinuities and tribological contacts, interfaces and joints; tailored, smart, and intelligent materials
- Special topics: damage, fracture, softening, non-continuum models; localisation, shear bands, stability, and bifurcation; micro–macro correlation; mesomechanics; initial and induced anisotropy; creep and high temperature effects; high rate of loading; solidification processes
- Innovative use of available and new materials in industrial and public works (infrastructure) applications.

The workshop is planned to run concurrently with the conference. Tentatively, one session each day will be devoted to the workshop. A number of persons from industries from various countries are expected to participate in the workshop and discuss the innovative uses of available and new materials, and future trends and needs for optimal industrial and infrastructure design and manufacturing.

The conference and workshop are expected to be cosponsored by a number of government and private organisations and professional societies.

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