

# Animating Transmission-Line Transients With BOUNCE

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**Abstract**—This paper presents the program BOUNCE for animating transients on transmission lines. BOUNCE is used for classroom demonstrations of fundamental concepts such as traveling wave, reflected wave, reflection from an unmatched load, and transmission through junctions and branches. BOUNCE provides a “laboratory” for students to use in verifying homework exercises solved with a lattice diagram. BOUNCE is used to demonstrate fundamental principles of logic design, such as ringing because of mismatch at high-impedance gate inputs, the effect of gate-input capacitance, and the effect of logic-gate rise time. BOUNCE graphically demonstrates the transition to the sinusoidal steady state and the concept of a standing wave, as the result of an unmatched load, as an introduction to solving transmission lines with phasors. This paper describes the operation of the BOUNCE program and then discusses various classroom demonstrations and the associated homework exercises.

**Index Terms**—Animation, lattice diagram, logic design, teaching software, transients, transmission line, traveling wave.

## I. INTRODUCTION

TRANSMISSION-LINE theory is part of the standard electromagnetic fields-and-waves course in electrical engineering and computer engineering curricula. It is found in textbooks such as [1]–[8]. Transients on lossless transmission lines are studied by deriving the coupled differential equations

$$\frac{\partial V}{\partial z} = -\ell \frac{\partial I}{\partial t} \quad (1)$$

and

$$\frac{\partial I}{\partial z} = -c \frac{\partial V}{\partial t} \quad (2)$$

where  $z$  is distance along the transmission line,  $t$  is time, and  $V(z, t)$  and  $I(z, t)$  are the voltage and current on the transmission line, respectively.  $\ell$  is the inductance per unit length; and  $c$  is the capacitance per unit length. A wave equation is derived from the transmission line equations, and the solution to the wave equation is shown to be

$$V(z, t) = V^+(z, t) + V^-(z, t). \quad (3)$$

The voltage wave is the sum of a wave traveling in the positive direction  $V^+(z, t) = f(t - z/u)$ , and a wave traveling in the

negative direction  $V^-(z, t) = f(t + z/u)$ , where  $u = 1/\sqrt{\ell c}$  is the wave speed. The current is

$$I(z, t) = \frac{V^+(z, t)}{R_c} - \frac{V^-(z, t)}{R_c} \quad (4)$$

where the characteristic resistance is  $R_c = \sqrt{\ell/c} \Omega$ . At this stage in the introduction to transmission lines, students must visualize a wave shape traveling along a transmission line with a constant speed, namely, a “traveling wave.”

This paper presents the program BOUNCE as an aid to visualizing waves traveling on transmission lines. BOUNCE is designed to be very easy to use. With a few mouse clicks, a wave is traveling across the screen. The reader is encouraged to retrieve BOUNCE over the internet, using a link in the User’s Guide [9] to run it in conjunction with the examples presented in this paper. The “snapshots” included here as figures do not replace the animation, which is the main purpose of BOUNCE.

All textbooks derive the formula for the reflection coefficient for the simplest transmission line circuit, consisting of a generator with internal resistance  $R_s$  connected to a line of length  $L$ , propagation velocity  $u$ , and characteristic resistance  $R_c$ , terminated by load  $R_L$ . A “bounce diagram” [3], [5], [7] or “lattice diagram” [4], [6] or “echo diagram” [1] or “reflection diagram” [2] is used to track the reflections from the load and the unmatched source and to find the voltage amplitudes at an observer as a function of time. The BOUNCE diagram is an invaluable aid to students in organizing their calculations to keep track of the waves and to provide some help in visualizing the waves that travel back and forth on the transmission line. BOUNCE is used in the classroom to demonstrate basic “traveling wave” behavior and to illustrate reflection of step functions and pulses from mismatched loads. Then students should run BOUNCE to verify lattice diagram predictions in homework exercises.

Textbooks extend the analysis to more complex problems: series connections of two transmission lines [2]–[5], [7] or three lines [3]; a series [2] or shunt resistor connected between two lines [7]; a transmission line branching to two lines [3], [5]; lines with capacitors and inductors [1]–[3], [5], [7]; and lines with nonlinear loads [3], [5], [7]. These problems get increasingly difficult for students to visualize. BOUNCE includes capacitive and inductive loads and animation aids in understanding the exponential waveforms that these loads generate.

For students in computer engineering, the ever-increasing speed of CPU chips necessitates the use of distributed circuit analysis for even relatively short interconnections on CPU boards. [7] applies transmission line theory to digital logic; [3] discusses interconnections of logic gates including nonlinear effects. [10] discusses digital circuits viewed as transmission

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Click the mouse on any RED text string to an select action in this program.

Choose a transmission line circuit:

- Transmission line with generator and load.
- Two transmission lines in series.
- Three transmission lines in series.
- Two transmission lines in series, with shunt load.
- Transmission line with branch.
- Transmission line with two branches.
- Transmission line with three branches.
- Quarter-wave transformer.
- Power splitter.
- Read a saved circuit from a .bnc data file.

EXIT from the program.

Fig. 1. BOUNCE's entry menu.

lines, particularly valuable to computer engineering students. [11] is a textbook devoted to distributed circuit considerations in high-speed digital design. It gives a rule-of-thumb technique for relating logic signal rise time to the maximum path length for which lumped circuit analysis can be used. BOUNCE provides a reasonably realistic model of simple digital circuits and it can be used to demonstrate the major design considerations.

An initial version of BOUNCE was presented in [12]. This paper presents an enhanced BOUNCE with many features, permitting more realistic simulations. BOUNCE runs on Pentium computers, under Microsoft Windows. A User's Guide is available [9], providing a full description of the program's features. [13] describes a series of basic demonstrations of transmission line behavior using BOUNCE.

Hewlett-Packard (HP) offers a simple program [14] that animates the solution to the simple series generator, transmission line, and load circuit. The program offers 11 simple lessons, each with an animated simulation, dealing with characteristic impedance, terminated lines, standing waves, and impedance measurement. BOUNCE can run all of these animations and can solve much more complex circuits. BOUNCE does not offer the neatly packaged lessons that are part of the HP software.

The following section provides a brief introduction to the BOUNCE software and its menu system. Then classroom demonstrations are described, including some associated homework using BOUNCE to verify answers and to compute solutions to problems too complex to address by hand calculation.

## II. THE BOUNCE PROGRAM

Starting BOUNCE gets the entry menu of Fig. 1, with one-line descriptions of nine "circuit templates." In BOUNCE, text strings written in red on the computer screen are "buttons." Click the mouse on "Transmission Line with Generator and Load" to choose the simplest circuit. The program sets up the circuit with default values for the generator, transmission line, and load parameters, and then shows the main menu of Fig. 2, with the circuit in the center and control buttons at the top and bottom. Click "GO" in the lower-right corner and a pulse travels along the transmission line at the top of the simulation menu of Fig. 3.

BOUNCE offers nine built-in circuit templates including the simple circuit of Fig. 3 and the series and branching transmis-

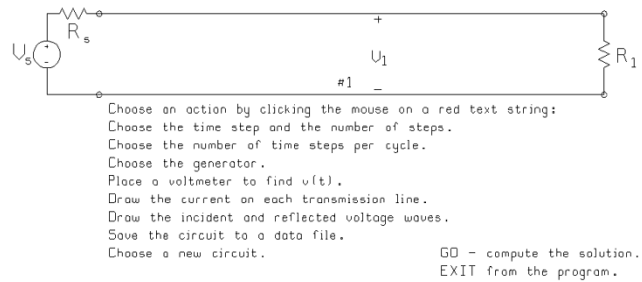


Fig. 2. BOUNCE's main menu.

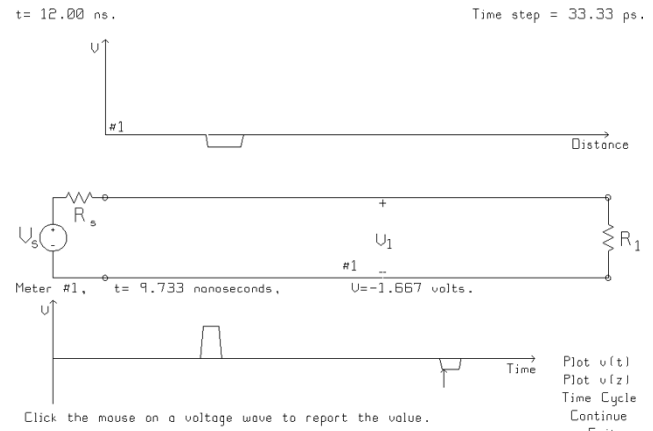


Fig. 3. BOUNCE's simulation menu.

sion line circuits shown in Fig. 4. Built-in circuits obviate the need to describe the circuit in an input file or to use a graphical interface to construct a circuit. BOUNCE offers two or three transmission lines in series, two lines with a shunt load, and lines with up to three branches. The last two circuit choices in Fig. 1, "quarter wave transformer" and "power splitter," demonstrate these devices in an ac circuit context, using built-in designs.

Once a circuit has been chosen, the main menu of Fig. 2 appears. At the top, there are buttons to choose the generator (step, pulse, ramp, pulse train, or sinusoid), the transmission line parameters (length, characteristic resistance, wave speed), and the load (resistor, series resistance-capacitance ( $RC$ ) or resistance-inductance ( $RL$ ), parallel  $RC$ , or  $RL$ ). The center of the main menu of Fig. 2 shows a schematic of the circuit. At the bottom, there are eight "program control" buttons. In the right-hand corner, the "GO" button runs the simulation, and the "EXIT" button terminates the program. The User's Guide [9] provides a complete description of BOUNCE's menus and controls.

BOUNCE solves the circuit by subdividing time into steps of length  $\Delta t$ . Each transmission line is subdivided into "cells" of length  $\Delta z = u\Delta t$ , where  $u$  is the speed of propagation on the line. The length of each line  $L$  is approximated with the nearest integer number of cells  $L \approx N\Delta z$ . BOUNCE's length approximation is a good lesson in the compromises associated with time-domain methods in computational electromagnetics. A long time-step gives lively animation but with coarse approximation of the transmission line lengths. Conversely, a small time-step slows down the animation but provides finer length approximation.

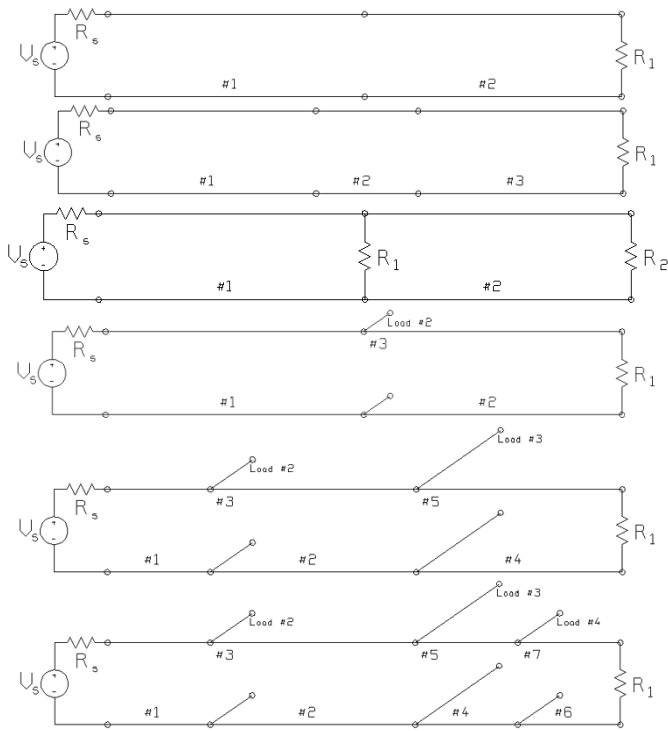


Fig. 4. Various circuit templates in BOUNCE, in addition to the simple circuit of Fig. 2.

### III. DEMONSTRATING BASIC BEHAVIOR

The first classroom demonstration addresses the basic behavior of transmission lines. The fundamental concept of “traveling wave” is demonstrated by showing a step function traveling across the screen using a line with a matched load. The demonstration is repeated with a pulse generator, a ramp, and a sinusoid to reinforce the idea that any function  $f(t)$  can propagate on the line as a voltage wave,  $v(z, t) = f(t - z/u)$ . The “movie” showing a sinusoidal wave advancing from left to right across the screen and being absorbed by a matched load is of primary importance in a fields-and-waves course. Students should verify the transit time for the wave across the transmission line.

#### A. Simple Reflection

Unmatched loads are then demonstrated. This topic can be introduced with a short pulse as the source, which separates the incident and the reflected wave, as shown in Fig. 3. The generator has an internal resistance of  $50\ \Omega$  and produces a pulse with a duration of  $0.5\ \text{ns}$ , and an open-circuit voltage of  $10\ \text{V}$ . The line is  $200\ \text{cm}$  in length, has  $50\text{-}\Omega$  characteristic resistance, and speed of propagation  $29.979\ \text{cm/ns}$ , which is equal to the speed of light in free space. The load is  $25\ \Omega$ . The time step is  $33.33\ \text{ps}$ , corresponding to 200 cells modeling the transmission line. Since BOUNCE’s approximate solution is somewhat dependent on the time step, the value is reported in the upper-right corner of the simulation menu.

BOUNCE uses “voltmeters” to report the voltage as a function of time at various locations on the transmission lines. In Fig. 3, an observer or “voltmeter” has been put at  $110.5\ \text{cm}$  from the source, and BOUNCE graphs the voltage at this location as a

function of time at the bottom of the screen. If the voltmeter location is between cell boundaries on the transmission line, then BOUNCE uses linear interpolation to approximate the voltage at the voltmeter location. This approximation can lead to rounded corners on step-function leading edges.

BOUNCE runs the simulation for  $N_t$  time steps, called the time cycle, and then pauses, showing the simulation menu of Fig. 3, to let the user examine the voltage-versus-distance and voltage-versus-time waveforms. A long time cycle is useful for watching the overall behavior of waves traveling back and forth on the various lines. A very short time cycle, even one step, is useful for studying an individual interaction at a load or a junction.

When the program pauses at the end of a time cycle, the user can “read back” values of the voltage from the curves shown on the screen. In Fig. 3, the mouse was clicked on the pulse reflected from the unmatched load. The program draws a small arrow at the chosen point and reports below the circuit schematic that the amplitude of this pulse is  $-1.667\ \text{V}$ . Students can verify that for a  $5\text{-V}$  incident pulse on a  $50\text{-}\Omega$  line with a  $25\text{-}\Omega$  load, the reflected pulse amplitude should indeed be  $-1.667\ \text{V}$ . Some students are astonished that a positive incident pulse can produce a negative reflected pulse.

It is instructive to change the generator to a step function and demonstrate that the resulting voltage on the transmission line is the sum of the incident voltage step of  $5\ \text{V}$  and the reflected step of  $-1.667\ \text{V}$ . The incident and reflected steps can be displayed explicitly by clicking the button “Draw the incident and reflected voltage waves.” The demonstration can be repeated with a  $100\text{-}\Omega$  load having a positive reflection coefficient, and with the important special cases of an open circuit and a short circuit.

The classroom demonstration should be followed by a simple homework exercise in which students draw a lattice diagram to trace several reflections back and forth. BOUNCE is used to verify the result at each step.

#### B. A Homework Problem Introducing Digital Circuits

A more difficult homework problem plants the idea that in digital logic, a poorly matched gate output driving a high-resistance gate input gives rise to a long series of reflections and an unsatisfactory “logical 1” at an observer. The generator is a  $5\text{-V}$ ,  $5\text{-}\Omega$  pulse generator with a pulse length of  $4.5\ \text{ns}$ . The line is  $20\text{-cm}$ -long,  $50\text{-}\Omega$  characteristic resistance, and wave speed  $20\ \text{cm/ns}$ , so that the  $1\text{-ns}$  transit time is only about one-fifth of the pulse length. The load is  $1000\ \Omega$ , approximately an open circuit for a  $50\text{-}\Omega$  line. The time step was set to  $5\ \text{ps}$ , modeling the transmission line with 200 cells. The observer is  $12\ \text{cm}$  from the source. The homework asks that the wave be traced by hand calculation with a lattice diagram through  $6.5\ \text{ns}$  to find the trailing edge of the pulse. Run BOUNCE for this circuit to advance the time to  $15\ \text{ns}$ . Click the mouse on “Plot  $v(t)$ ” to draw the rectangular graph of Fig. 5. BOUNCE draws rectangular graphs using the program “RPLOTT” [9]. RPLOTT gives the user full control of curve styles, axis limits, and so forth, to create graphs suitable for incorporation into lab reports or homework assignments.

Fig. 5 shows that the initial voltage at the observer is of amplitude  $4.545\ \text{V}$ , a satisfactory “logical 1.” But when the reflection

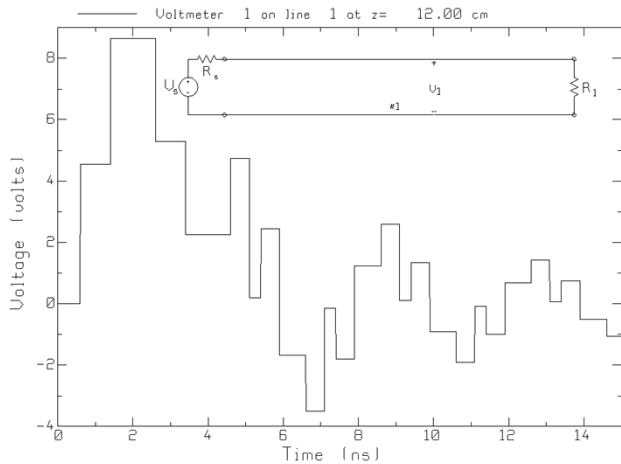


Fig. 5. A 4.5-ns pulse propagates on a 20-cm line with a poorly matched source and load.

from the 1000- $\Omega$  load arrives, the voltage jumps up to 8.658 V, a significant “over voltage” for a complementary metal-oxide-silicon (CMOS) input. At 3.405 ns, the voltage at the observer drops to 2.249 V because a reflection from the poorly nmatched source arrives, and then the voltage at the observer is too low for a “logical 1.” BOUNCE can be used to verify the individual interactions and then extend the waveform to 15 ns to see the “ringing down” of the energy trapped on the transmission line. Further homework can address the problem of suppressing the ringing using matching resistors by the methods described in [10].

### C. Transmission at a Junction

The first classroom demonstration continues using the “two lines in series” circuit template to verify the formulas for reflection and transmission at a junction of two lines. The default values are a step generator of amplitude 10-V and 25- $\Omega$  internal resistance, driving two lines in series, the first of length 1.5 m and the second of length 1 m, both with a propagation speed of 30 cm/ns. The first line has a characteristic resistance of 50  $\Omega$ , and the second 100  $\Omega$ , terminated with a 50- $\Omega$  load. At the junction, the reflection coefficient is one-third, and the transmission coefficient is  $\frac{4}{3}$ . The simulation is run to show that the unmatched source gives rise to an incident step of 6.667 V and that transmission through the junction increases the amplitude to 8.889 V. Again, students are sometimes astonished that the voltage increases on transmission through the junction.

[12] describes a follow-up homework exercise that sometimes leads to student complaints that BOUNCE gets incorrect results. The problem differs from the above in that the transmission lines are both 2 m in length with equal wave speed of 30 cm/ns and is designed so that the reflected pulses from the load and the generator arrive at the junction simultaneously. Students have difficulties in calculating the junction interaction. The generator, with an internal resistance of 25  $\Omega$ , produces a 10-V, 1-ns pulse. Line 1 has a characteristic resistance of 50  $\Omega$ , and line 2, 100  $\Omega$ . The load resistor is 50  $\Omega$ . Students are asked to draw a lattice diagram and find the voltage on the two transmission lines at  $t = 22$  ns, long enough for three trips of 6.667 ns over each transmission line. The time step for BOUNCE was chosen

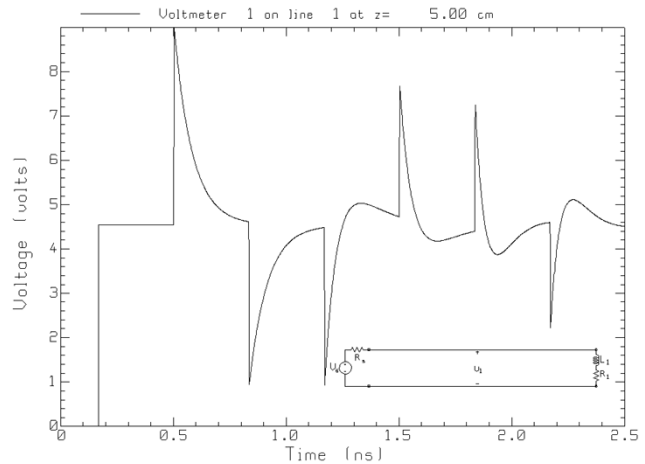


Fig. 6. Voltage at the center of a 50- $\Omega$  transmission line with a matched-load resistor in series with an 8-nH inductance.

as 26.667 ps so that the line lengths are accurately represented. When the source’s pulse first reaches the junction, it is partially transmitted and partially reflected. The reflected pulse reaches the generator, and the transmitted pulse reaches the load at the same instant of time, and both are reflected. They arrive back at the junction simultaneously. Tracing the pulses on each transmission line to  $t = 20$  ns leads to a pulse arriving at the junction from the left of amplitude  $-0.7407$  V and from the right of amplitude  $-2.963$  V. The  $-0.7407$ -V pulse incident from the left leads to a  $-0.2469$ -V pulse reflected to the left and a  $-0.9877$ -V pulse transmitted to the right. Also, the  $-2.963$ -V pulse incident from the right leads to a  $-1.9753$ -V pulse transmitted to the left of the junction and a  $0.9877$ -V pulse reflected to the right. Adding these contributions results in a  $-2.222$ -V pulse traveling to the left away from the junction, and a  $0$ -V pulse traveling to the right. BOUNCE animates the reflections and transmissions and shows that at  $t = 21.37$  ns, there is no voltage on line 2, and line 1 indeed carries a pulse of amplitude  $-2.222$  V.

## IV. REACTIVE LOADS

The second classroom demonstration with BOUNCE starts with reactive loads. BOUNCE permits loads to be series or parallel  $RL$  or  $RC$  circuits and thus can be used to demonstrate the waveforms found in many textbooks. Students learn to find the exponential response of  $RL$  and  $RC$  circuits in elementary circuit analysis [15], including the concept of initial conditions, final conditions, and time constant.  $L$  or  $C$  loads on transmission lines reinforce these concepts and extend them to a distributed circuit environment.

In the  $RL$  series load in Fig. 6, a 5-V step function generator of 5- $\Omega$  internal resistance drives a 10-cm line of characteristic resistance 50  $\Omega$  and propagation speed 29.979 cm/ns. Initially, the line is terminated with a matched load, and the voltage at the middle of the transmission line steps up cleanly to 4.545 V. If an inductance of 8 nH is added in series with the 50- $\Omega$  load, representing a typical inductance for a pin of an integrated circuit (IC) package [11], then BOUNCE can be used to calculate the voltage for an observer in the middle of the transmission line shown in Fig. 6. The time step is 1 ps, which subdivides the

line into 334 cells. Because of the approximation of the transmission line length, the exact numerical results are somewhat dependent on the time step that is used. The voltage at the load steps up to 4.545 V as the leading edge of the step passes the voltmeter location. Initially, the inductor has zero current and behaves as an open circuit. Since the reflection coefficient is initially equal to +1, the initial reflection from the load is a step up from 4.545 to 8.991 V. However, as the inductor “charges,” its current becomes constant in value, and the voltage across the inductor goes to zero. After being charged, the inductor behaves as a short circuit in series with a matched load. Then there is zero reflection, and the voltage at the observer declines exponentially back toward 4.545 V. The time constant is  $L/R$ , where  $R$  is the resistance seen from the inductor terminals, including the characteristic impedance of the line as a resistive element, in this case  $R_L$  in series with  $R_c$ . Thus, the first spike in Fig. 6 is easily obtained by hand calculation. Students should verify the time constant of the transient, using the “read back” feature of BOUNCE to get time and voltage values from the computer screen.

In this circuit, in which the generator is severely mismatched, the 4.545-V spike is reflected from the source with a reflection coefficient of  $(-45/55) = -0.818$ , and the upward spike inverts to a downward spike. As the inverted spike passes the observer location, the voltage drops to 0.953 V, then rises exponentially back toward 4.545 V. The momentary drop is re-reflected from the load inductance, giving a second downward spike at the middle of the line, then again reflecting from the mismatched source as an upward spike. As time passes, the observer sees pairs of spikes of gradually diminishing amplitude. These later spikes are much more difficult to calculate since an exponential wave arriving at the load is the driving function for an energy storage element. BOUNCE “automagically” solves the circuit to permit students to understand the physics without becoming bogged down in calculations.

## V. DIGITAL LOGIC DESIGN

The second classroom demonstration continues with a lesson in digital logic design. For logic design, it is crucial to know when lumped circuit analysis suffices, and conversely, when delay times on transmission lines become significant; then distributed circuit analysis must be used. Textbooks [10], [11] relate the rise time or switching speed at the output of a logic gate to the limiting transmission line length. Thus, if  $T_r$  is the rise time and  $u$  is the speed of propagation, then  $L_r = uT_r$  is the length of the rising edge on the transmission line. If the transmission line is longer than one-sixth of the length of the rising edge, then distributed circuit analysis is recommended [11]. The longest length of transmission line for lumped circuit analysis is only indirectly related to clock speed. Thus, as the clock speed increases the rise time must decrease in proportion. But some relatively “slow” logic families, measured in terms of gate propagation delay, have quite fast rise times.

### A. Lumped Versus Distributed Circuit Analysis

The purpose of this demonstration is to compare the behavior of an interconnect that is at the limiting length of  $uT_r/6$  with

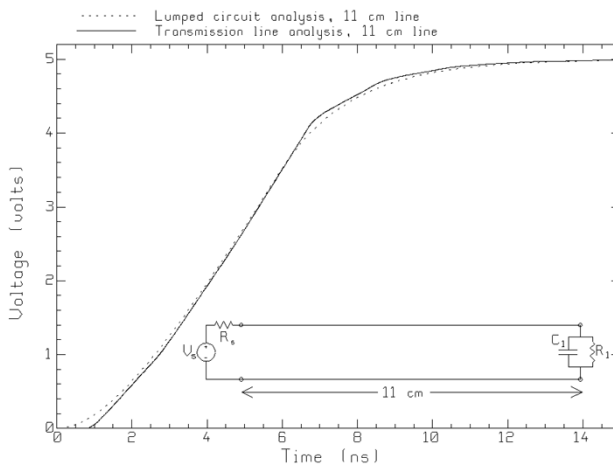


Fig. 7. Comparison of the lumped-circuit and distributed-circuit results for the voltage across a CMOS gate input for an 11-cm transmission line.

lumped circuit predictions, and also with a transmission line that is far too long for lumped analysis.

A CMOS logic gate drives an interconnect on a circuit board, and the interconnect is terminated with the input to another logic gate. “Logical 0” is 0 V and “logical 1” is 5 V. A typical output resistance for a gate is 45–165  $\Omega$  [11]. The “10%–90% rise” time is  $T_r = 4.7$  ns. The input resistance for a gate is very high, but the input capacitance is 3.5 pF. The line has a propagation velocity of  $u = 14$  cm/ns and a characteristic resistance of 50  $\Omega$ . Then the length of the rising edge is 65.8 cm, so that an 11-cm line is at the limit for lumped circuit analysis.

BOUNCE can be used to construct a reasonably realistic model of the CMOS gate driving the 11-cm line. A 10%–90% rise time of 4.7 ns is modeled with a step voltage rising linearly from 0 to 5 V in 5.9 ns. The mean value of approximately 100  $\Omega$  is used for the internal resistance of the output of the gate. The CMOS gate input is modeled as a 3.5-pF capacitor in parallel with a very large resistance. The time step was set at 5 ps. When BOUNCE is run and the source ramps up, the leading edge of the ramp is seen to propagate from the source to the capacitive load. Once the source reaches 5 V, the voltage is approximately the same at all points on the 11-cm transmission line at all instants of time. Thus, the transmission line behaves as if it were a simple capacitor. To graph the load voltage, click the mouse on the “Plot  $v(t)$ ” button in the simulation menu of Fig. 3. BOUNCE launches RPLOTT, which graphs  $v(t)$  as shown in Fig. 7. The rise time in Fig. 7 from 0 to 4.5 V is 7.92 ns.

The capacitance per unit length is  $c = 1/(R_c u)$ , so  $C = cL = 15.7$  pF for an 11-cm line. The lumped-circuit model consists of the 100- $\Omega$  source resistance in series with the sum of the line capacitance and the gate input capacitance of 3.5 pF. The inductance is neglected. Fig. 7 compares the lumped-circuit result with the distributed-circuit result from BOUNCE. The lumped-circuit curve is very similar to that found with BOUNCE, and the rise time is 8.08 ns, comparable to BOUNCE’s figure of 7.92 ns. If the inductance of the transmission line were also included in the lumped-circuit model, then the curve is flatter at the beginning, approximating the propagation delay in the distributed circuit result. Fig. 7 demonstrates that the rule “not

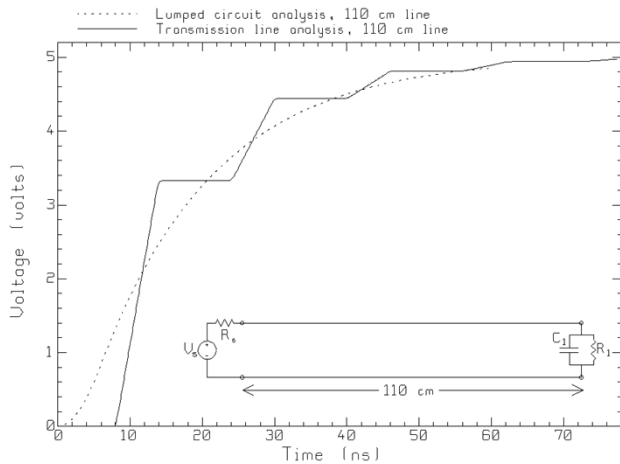


Fig. 8. Voltage at the load with a 110-cm transmission line.

more than one-sixth of the length of the rising edge” is effective for this problem and is somewhat conservative.

However, if the line length is increased to 110 cm, ten times the maximum length for lumped analysis, the transmission-line model predicts the ramp-step behavior shown in Fig. 8, which is quite different from the smooth curve predicted by lumped-circuit analysis. Watching the animation in BOUNCE shows that the voltage at various locations on the transmission line is quite different at most instants of time, and the line clearly does *not* behave as a simple capacitor. The voltage at the load has three distinct ramps as it rises from 0 to 5 V. For such a long transmission line, the propagation delay time plays a dominant role.

### B. Circuit With a Short Branch

Consider a faster CMOS logic family, with a rise time of 0.1 ns, and a correspondingly smaller input capacitance of 1 pF. A logic gate output with a 100- $\Omega$  internal resistance and a 5-V open-circuit voltage must be connected to a gate input 12 cm away. Also, at 3 cm distance from the gate output, there is a 2-cm branch to another gate input. The circuit is shown in Fig. 9. All three lines have a characteristic resistance of 50  $\Omega$  and a speed of propagation of 14 cm/ns. The amount of time required for the voltage at the load to rise to 4.5 V and remain above that level must be found.

Approximating the generator as a 5-V step function with zero rise time and neglecting the CMOS gate input capacitance gives insight into the behavior of the transmission line circuit. The time step was set at 5 ps. When BOUNCE is run, the generator launches a step of height 1.67 V onto the transmission line, which divides into steps of height 1.11 V on the two branches. On the short branch, the step reflects from the open circuit and returns to the junction, launching a step up to 1.85 V toward the load. The step on the branch reflects back and forth with decreasing amplitude, and each interaction at the junction launches another step toward the load. When the initial 1.11-V step arrives at load 1 at the end line 2, the open circuit reflects a step up from 1.11 to 2.22 V toward the junction. There, this step is partially reflected toward the observer and partially transmitted onto the branch. The branch then launches another series of steps through the junction back to the load. The solid curve in Fig. 9 shows the load voltage with open-circuit loads. The load

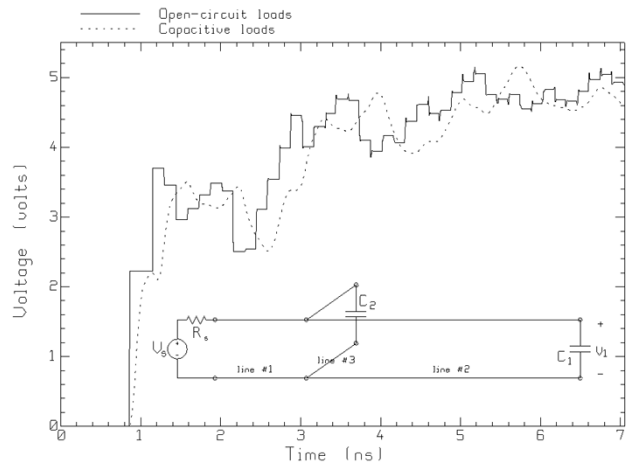


Fig. 9. Circuit with a branch excited by a step-function generator.

voltage gradually increases, with many small steps up and down due bouncing on the short branch, and larger steps, more widely spaced, because of reflections from the open-circuit load back to the branch. After 4.76 ns, the voltage steps above 4.5 V and remains above that level. But all the little steps up and down in Fig. 9 do not look much like the response of a real logic circuit.

The model can now be made much more realistic by including the logic gate’s rise time of 0.1 ns and including the 1-pF input capacitance of the gates terminating lines 2 and 3. The resulting response is shown in Fig. 9, dashed curve, and is much more like a storage scope trace than is the “steppy” response with open-circuit loads. The rise time and the input capacitance round all the corners to produce a smooth curve. The “rise time” from 0 to 4.5 V is 5.37 ns with the loading capacitors, longer than the 4.76 ns figure with the sharp step. The generator’s more gradual rise and the input capacitances slow down the response of the system.

## VI. TRANSITION TO THE SINUSOIDAL STEADY-STATE

“Fields and Waves” courses usually follow the presentation of transmission line transients with sinusoidal steady-state behavior. The relation of transient to steady-state behavior is made clear in circuit analysis by writing a differential equation and then decomposing the solution into the “natural response” and the “forced response” [15]. The exponentially decaying terms of the natural response are the transient behavior associated with the turn-on of the generator, and the forced response is the steady-state solution. Phasor analysis using impedance is a convenient shortcut for finding the steady-state solution.

The third classroom demonstration using BOUNCE animates the turn-on transient, starting with the leading edge of the sinusoidal voltage as it emerges from the generator, and then showing the reflections from loads and from an unmatched source, building up to the sinusoidal steady-state. “Standing wave” behavior is demonstrated prior to the introduction of ac analysis of transmission lines using phasors.

Choose the simple generator-with-line-and-load circuit. The generator should be set to a 10-V sinusoid with 50  $\Omega$  internal resistance, at 300 MHz. The line length is 2 m; the characteristic resistance is 50  $\Omega$ ; and the speed of propagation is

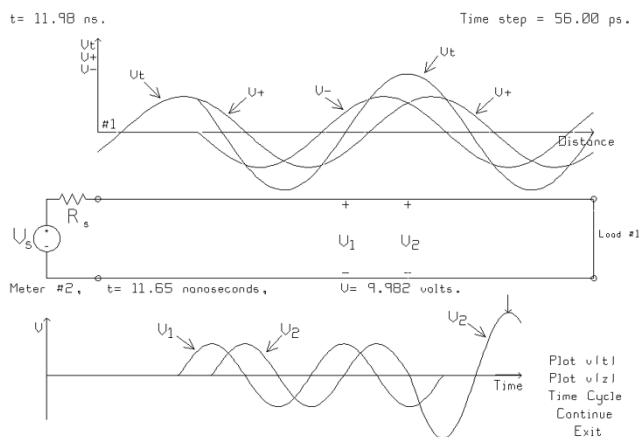


Fig. 10. Sinusoidal generator with a short-circuit load results in a pure standing wave on the transmission line.

300 m/ $\mu$ s. The user should make the load matched and choose a long time cycle. BOUNCE shows the wave progressing at constant speed from generator to load and being fully absorbed. This demonstration reminds students of fundamental “traveling wave” behavior. Ask students to imagine a graph of the *amplitude* of the sinusoid as a function of position; it is, of course, a horizontal straight line. The load should be changed to a short circuit. The reflection coefficient is now  $-1$ . As the reflected wave progresses from right to left across the screen, the nature of the voltage on the transmission line changes. The voltage now “marches in place” rather than progressing across the screen and is a “standing wave.” Again, students should envisage a graph of the amplitude as a function of position, namely the standing wave pattern.

Fig. 10 shows the voltages on the transmission line when BOUNCE pauses at  $t = 11.98$  ns, using a 56-ps time step. For this demonstration it is useful to decompose the transmission line voltages into the total voltage, the incident voltage wave, and the reflected voltage wave, by clicking the mouse on “Draw the incident and reflected waves” in the main menu of Fig. 2. In the animation, the incident wave and the total voltage are identical until the leading edge reaches the load. Then the reflected wave emerges, and the incident wave is seen as a red sinusoid traveling to the right, and the reflected wave as a green sinusoid traveling to the left. The net voltage is their sum, point by point. The demonstration shows neatly that a “pure” standing wave is composed of equal-and-opposite traveling waves.

At 11.98 ns, the reflected voltage has progressed almost all the way back to the source. At the  $V_1$  observer, there are two cycles of the incident wave, and then the reflected wave arrives and exactly cancels the incident wave. It is easy to see, in the animation, that the red and green curves are always equal-and-opposite at this location. Conversely, at the  $V_2$  observer, there are one and a half cycles of the incident wave, and then the reflected wave arrives exactly in phase. The amplitude of the sinusoidal voltage at this location doubles, and again it is easy to see that the red “incident” sinusoid and the green “reflected” sinusoid are always of equal value (i.e., in phase) so that they add up.

The demonstration can be repeated with an open circuit load to show that the  $V_1$  location becomes a standing-wave max-

imum, and the  $V_2$  location, a minimum. It is instructive, too, to terminate the line with a 10-pF capacitance, to see that the voltages at the two observers are approximately equal, and to see that neither is at a standing-wave maximum or minimum. Also, the line can be terminated with an unmatched load, such as  $100\ \Omega$ . The voltage on the line at steady state is an oscillating pattern that can be thought of as the sum of a pure standing wave and a pure traveling wave. The envelope of this oscillation is a standing-wave pattern with nonzero minima.

The companion program called “TRLIN” [16] provides sinusoidal steady-state analysis of transmission line circuits in a format similar to BOUNCE. TRLIN can be used to calculate and display the standing-wave patterns for these demonstrations, in comparison to the animated, time-domain graphics in BOUNCE. Like BOUNCE, TRLIN provides students with a “laboratory” for checking ac steady-state homework problems.

For some problems, it is instructive to run BOUNCE and TRLIN side by side. BOUNCE provides a circuit template for a quarter-wave transformer at 300 MHz, matching a  $50\text{-}\Omega$  line to a  $100\text{-}\Omega$  line. Running BOUNCE for this problem shows that the input to the transformer is not matched for the leading edge of the generator’s sinusoid. Displaying the incident and reflected voltage waveforms separately shows an initial reflected wave that is largely extinguished by the first reflection from within the transformer section. Then TRLIN can be used to demonstrate the match in the steady state and to find the bandwidth of the match. A second example demonstrates a power splitter, using a quarter-wave transformer to match a  $50\text{-}\Omega$  line to two  $50\text{-}\Omega$  lines in parallel, at 300 MHz. Again, BOUNCE shows the transient to the sinusoidal steady state and the time-dependent voltages at steady state. TRLIN shows the envelopes of these voltages and can be used to find the bandwidth of the match.

## VII. THE BOUNCE ALGORITHM

BOUNCE uses a *shift-and-add* algorithm to advance the time and display the transmission line voltages. BOUNCE chooses a time step  $\Delta t$  and then subdivides each transmission line into  $N_i$  cells of length  $\Delta z_i = u_i \Delta t$ , where  $u_i$  is the wave speed on transmission line  $i$ . Thus, the length is approximated as  $L_i \approx N_i \Delta z_i$ , introducing “modeling error” dependent on the choice of time step. The voltage on line  $i$  is the sum of a positive-going wave and a negative-going wave

$$V_i(z_j, t_k) = V_i^+(t_k - \frac{z_j}{u_i}) + V_i^-(t_k + \frac{z_j}{u_i}) \quad (5)$$

where the cell boundaries are  $\{z_j = j \Delta z_i : j = 0, \dots, N_i\}$  and time step  $k$  is  $t_k = k \Delta t$ . The positive-going wave voltages at the cell boundaries on line  $i$  are stored as  $\{V_{i,j}^+ : j = 0, \dots, N_i\}$ . To advance time to the next step

$$\begin{aligned} V_i^+(z_j, t_{k+1}) &= V_i^+(t_k + \Delta t - \frac{z_j}{u_i}) = V_i^+(t_k - \frac{z_j - u_i \Delta t}{u_i}) \\ &= V_i^+(t_k - \frac{z_{j-1}}{u_i}) \end{aligned} \quad (6)$$

which calls for replacing  $v_{i,j}^+$  with  $v_{i,j-1}^+$ . The voltage values are shifted one cell “to the right” along the transmission line. Similarly, to update the negative-going wave to time step  $k +$

1, the cell voltages are shifted one cell “to the left.” Then to find the net voltage at cell boundary  $j$ , the positive-going and negative-going waves are added

$$v_{i,j} = v_{i,j}^+ + v_{i,j}^- \quad (7)$$

This solution is exact for transmission lines of length  $N_i \Delta z_i$ , but approximate for the given problem since the lengths of the lines have been approximated as  $L_i \approx N_i \Delta z_i$ . The shift-and-add algorithm is *not* a one-dimensional finite-difference time-domain (FDTD) method.

When  $v_{i,j}^+$  are shifted to the right, the end value  $v_{i,N_i}^+$  “pops out” of the transmission line and is the input to the junction or load connected to the right-hand end of the line. Also, when the  $v_{i,j}^-$  are shifted to the left, a new value must be supplied for  $v_{i,N_i}^-$  as the input to the leftward-moving sequence. If the load is a simple resistor  $R_L$ , then the new value is  $v_{i,N_i}^- = \Gamma v_{i,N_i}^+$ , where the reflection coefficient is  $\Gamma = (R_L - R_c)/(R_L + R_c)$ . When the load is a series or parallel  $RL$  or  $RC$  circuit,  $v_{i,N_i}^-$  is found by solving a first-order differential equation by Euler’s method.

A junction of three transmission lines can be used to illustrate how BOUNCE computes interactions at junctions. The right-hand end of line 1 branches into the left-hand end of line 2 and line 3. When the three transmission lines are updated to time step  $t_k$ , then voltage  $v_1^+$  pops out of line 1 into the junction, and similarly, voltages  $v_2^-$  and  $v_3^-$  pop out of lines 2 and 3 into the junction. From  $\{v_1^+, v_2^-, v_3^-\}$ , the junction must be solved for the voltages  $\{v_1^-, v_2^+, v_3^+\}$ . The transmission coefficient from line 1 onto line 2 or line 3 is  $T_1 = 2R_A/(R_A + R_{c1})$ , with  $R_A = R_{c2} \parallel R_{c3}$ . Similarly,  $T_2$  is the transmission coefficient from line 2 to line 1 or 3, and  $T_3$  from line 3 to 2 or 1. Also, the reflection coefficients from lines 1, 2, and 3 are  $\Gamma_1 = (R_A - R_{c1})/(R_A + R_{c1})$ ,  $\Gamma_2$ , and  $\Gamma_3$ , respectively. If  $v_1^+$  were nonzero but  $v_2^- = v_3^- = 0$ , then the voltages entering the three transmission lines would be  $v_1^- = \Gamma_1 v_1^+$ ,  $v_2^+ = T_1 v_1^+$ , and  $v_3^+ = T_1 v_1^+$ . If all three voltages that “pop out” are nonzero, then the voltages emerging from the junction are

$$\begin{pmatrix} v_1^- \\ v_2^+ \\ v_3^+ \end{pmatrix} = \begin{bmatrix} \Gamma_1 & T_2 & T_3 \\ T_1 & \Gamma_2 & T_3 \\ T_1 & T_2 & \Gamma_3 \end{bmatrix} \begin{pmatrix} v_1^+ \\ v_2^- \\ v_3^- \end{pmatrix} \quad (8)$$

Voltage  $v_1^-$  is loaded into the right-hand end of line 1 and will propagate to the left on subsequent time steps. Voltage  $v_2^+$  enters the “left hand” end of line 2 and will propagate to the right; and  $v_3^+$  is loaded into line 3 and propagates to the right.

Animation is implemented as follows. Graphics are drawn with black lines on a white background. At time  $t_k$ , the voltage on each transmission line is drawn by joining point  $(j \Delta z_i, v_{i,j})$  to point  $((j+1) \Delta z_i, v_{i,j+1})$  for  $j = 0, \dots, N_i - 1$  on each transmission line. To advance to time  $t_{k+1}$ , the voltage curve for time  $t_k$  is “undrawn” by drawing it in white, which makes the curve disappear into the background. Then the new traveling-wave voltage vectors are calculated with the shift-and-add algorithm, and the new voltage curve at time  $t_{k+1}$  is drawn in black. The speed of the animation thus depends on the speed of the underlying processor. BOUNCE was written on a 266 MHz

Pentium, and the animation proceeds at a satisfactory speed with about 100 cells on each transmission line. On an 800-MHz Pentium, the animation is too fast, and the time step needs to be reduced by one-third so that there are about 300 cells on each line. On an 1800-MHz Pentium, about 600 cells per line produces effective animation.

## VIII. CONCLUSION

BOUNCE is a simple, easy-to-use program for animating transients on transmission line circuits. This paper has described classroom demonstrations for fundamental ideas, such as traveling wave, reflection from a load, transition to the sinusoidal steady state, and reflection from capacitive and inductive loads. BOUNCE is used to introduce rise time and matching considerations in digital circuit design and to show the change from lumped-circuit analysis for short transmission lines to distributed circuit analysis for longer lines. The document “Basic Demonstrations with BOUNCE” [13] is available on the internet and describes the various demonstrations more fully than the capsule descriptions in this paper. Teaching transients with BOUNCE and then sinusoidal steady analysis with TRLINE [11] dovetails nicely.

BOUNCE provides students with the opportunity to investigate problems that involve interactions too complex to be readily traceable by hand calculation. Although the voltage with step excitation and open-circuit loads in Fig. 9 might be calculated with a mammoth lattice diagram, when the input rise time and the gate capacitance are included in the model, the calculation becomes impractical. BOUNCE readily finds the result and focuses attention on the physics of the problem.

BOUNCE’s animation helps students who are having trouble “seeing” waves and interactions on transmission lines. Watching the instructor run BOUNCE “live” in the classroom convinces students that BOUNCE is easy to use and encourages them to try it at home. BOUNCE has proven a useful teaching tool in classroom demonstrations to reinforce the lecture material, and for students to use as a “laboratory” for verifying lattice-diagram solutions one interaction at a time. The program encourages advanced students to investigate more complex problems than those included in textbooks. It has been well received by students, particularly in computer engineering classes.

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