

# Constrained Control of Multi-Vehicle Systems for Smart Cities and Industry 4.0: from Model Predictive Control to Reinforcement Learning

Part 4 - Distributed control architecture: a joint distributed reinforcement learning and model predictive control approach

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UNIVERSITÀ DELLA CALABRIA  
DIPARTIMENTO DI  
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ENERGETICA E GESTIONALE  
DIMEG

Dr. Giuseppe Franzè, June 04 2023

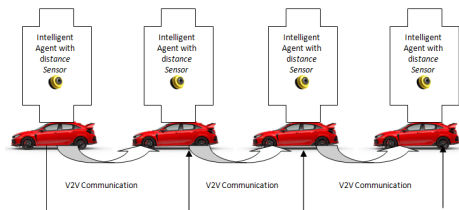
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## Autonomous vehicles in urban road networks -

Develop an efficient path planning strategy for constrained autonomous vehicles moving in the cluttered environments of urban road networks under time-varying operating scenarios

## Multi-vehicle configuration -

The autonomous vehicles are topologically organized as a platoon



## Key aspects -

- the computation of routing decisions for mitigating traffic congestion phenomena
- a constrained control strategy in charge of adequately exploiting the routing decisions

## Modeling -

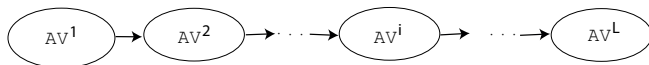
- **Topological configuration:** Platoon of  $L$  autonomous vehicles
- **Dynamics:**

$$x^i(t+1) = A^i x^i(t) + B^i u^i(t), \quad i = 1, \dots, L,$$

- $x^i(t) = [p^i(t)^T, x_{np}^{iT}]^T : x_{np}^i \in \mathbb{R}^{n_i-2}$  accounts for the non-spatial components
- $u^i \in \mathbb{R}^{m_i}$  : the control input
- **Set-membership constraints:**

$$x^i(t) \in \mathcal{X}^i := \{x^i \in \mathbb{R}^{n_i} : x^{iT} x^i \leq \bar{x}^2\},$$

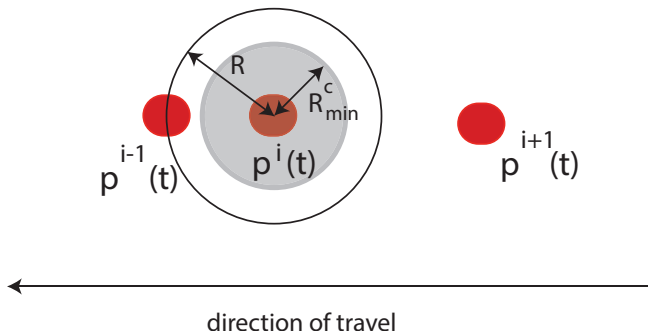
$$u^i(t) \in \mathcal{U}^i := \{u^i \in \mathbb{R}^{m_i} : u^{iT} u^i \leq \bar{u}^2\}, \quad \forall t \geq 0, \bar{x}, \bar{u} \in \mathbb{R}^+$$



### Perception capabilities -

- vehicles are equipped with a **perception module** with a detection radius  $R > 0$
- **field of view**:  $360^\circ$
- **minimum curvature radius**:  $R > R_{min}^c$

**Detected region**: the ball  $\mathcal{B}(p^i(t), R)$  centered at the current vehicle planar position  $p^i(t)$



### Information exchange -

At each  $t$ ,  $AV^i$  sends to  $AV^{i+1}$  its predicted future state trajectory  $\hat{\mathbf{x}}^i(t)$

### Operating scenario -

- **Link:** a portion of a road comprised between two junctions
- **Urban road network:**  $M$  links and  $F$  junctions, where sequence of routing decisions have to be taken into account during the travel

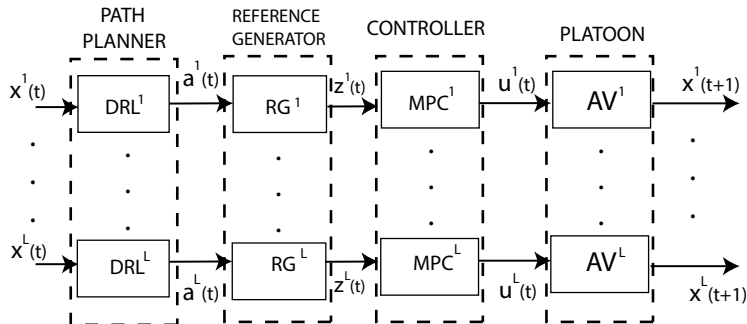
### Platoons in Urban Environments (PL-UE) -

Given a platoon of constrained autonomous vehicles and a target  $x_f = [x_f^1, x_f^2, \dots, x_f^L]^T$ , design a distributed state-feedback control policy

$$\begin{aligned}u^1(t) &= g(x^1(t), x_f^1), \\u^i(t) &= g(x^i(t), \hat{\mathbf{x}}^{i-1}(t), x_f^i), \quad i = 2, \dots, L,\end{aligned}$$

satisfying state and input constraints and such that, starting from an admissible initial condition  $x(0) = [x^1(0), x^2(0), \dots, x^L(0)]^T$ , the team is driven towards  $x_f$  regardless any junction occurrence

# Proposed solution: an overview

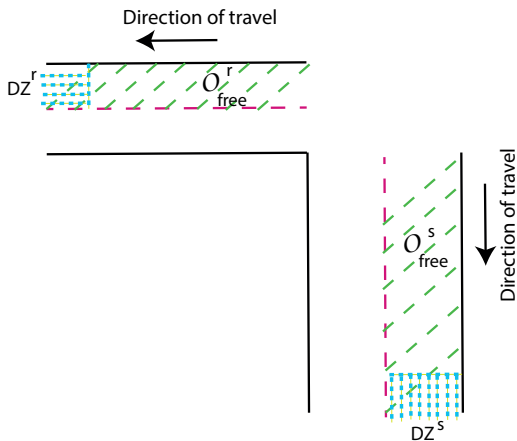


## Basic units -

- **Path Planner** : based on distributed reinforcement learning arguments, **receives** the measurement  $x^i(t)$  and **generates** the action  $a^i(t)$  (link road selection) to be executed by the vehicle  $AV^i$  until the next junction
- **Reference Generator** : **translates**  $a^i(t)$  into a set-point  $z^i(t)$  for the underlying control logic
- **Controller** : **computes** an admissible command  $u^i(t)$  in a distributed receding horizon fashion

## Characterizing aspects -

- the environment is *a-priori* known, i.e., the link-free region  $\mathcal{O}_{free}^j$
- a decision zone  $DZ^j$  is defined for each link:  $H^j p > b^j, p \in \mathbb{R}^2$ ,





### Customization -

- **Geometric constraints:**

$$(\mathcal{X}_c)_j^i \triangleq \mathcal{X}^i \cap \mathcal{O}_{free}^j := \{x^i \in \mathbb{R}^{n_i} : x^{iT} x^i \leq \bar{x}_j^2\} \quad i = 1, \dots, L, \quad j = 1, \dots, M$$

- **Set-point computation:**

$$z^i(t) := \arg \max_{z^i \in (\mathcal{X}_c)_{a^i(t)}^i} \|x^i(t) - z^i\|^2$$

### Decision zone -

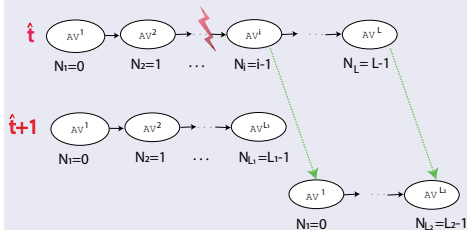
Exploited for routing decision purposes

## Model Predictive Controllers -

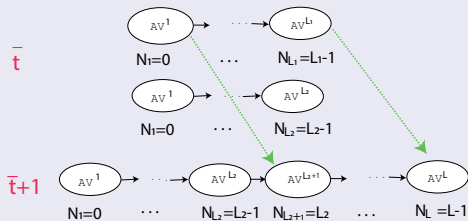
- 1 the leader  $AV^1$  implements a receding horizon controller with  $N_1 = 0$
- 2 each follower  $AV^i$  uses an MPC controller with  $N_i = i - 1, i = 2, \dots, L$

## Time-varying leader-follower topology -

The current platoon can be **split** in two or more leader-follower configurations



Two platoons  $LF^1$  and  $LF^2$  can be **grouped** by reconnecting the leader of  $LF^1$  to the leaf node of  $LF^2$



# Distributed Reinforcement learning scheme

## Task -

The routing problem is recast as a path search on a connected graph by exploiting as a threshold the decision zone

## Vehicle density -

Given a road link  $j$ , the vehicle density is

$$\rho^j(t) \triangleq \frac{\mu^j(t)}{\bar{d}^j l^j}$$

with  $\mu^j(t) \in \mathbb{Z}^+$  the number of vehicles on the link  $j$ ,  $\bar{d}^j \in \mathbb{R}^+$  the average road width and  $l^j$  the road length

## Multi-agent RL model -

$$\langle V, \Sigma, \Lambda, \Phi \rangle$$

- $V = \{AV^1, \dots, AV^h\}$  : the set of vehicles
- $\Sigma := \{\sigma^i\}_{i=1}^h$  : the set of RL states
- $\Lambda$  : the admissible vehicle actions: e.g., “turn right”, “turn left”, “go straight on”;
- $\Phi : \Sigma \times \Lambda \times \Sigma \rightarrow \mathbb{R}$  : the global reward function

## Notation -

- $x^i(t+k|t), u^i(t+k|t) : k - th$  predicted state and predicted control
- $\mathbf{x}^i(t) := \{x^i(t+k|t)\}_{k=0}^{N_i}, \mathbf{u}^i(t) := \{u^i(t+k|t)\}_{k=0}^{N_i} : \text{predicted state and predicted control sequences}$
- $\mathbf{x}^{i*}(t) := \{x^{i*}(t+k|t)\}_{k=0}^{N_i} : \text{optimal state trajectory}$
- $\hat{\mathbf{x}}^i(t) := \{\hat{x}^i(t+k|t)\}_{k=0}^{N_i} : \text{assumed state trajectory}$

## Information exchange -

The sequence  $\hat{\mathbf{x}}^i$  is transmitted to  $AV^{i+1}$  which in turn hypothesizes that  $AV^i$  will implement during the update prediction time interval

$$[(t+1) + k, (t+1) + k + N_i]$$

## Ingredients -

- **Input sequence parametrization:**

$$u^i(\cdot|t) = \begin{cases} u^i(t+k|t), & k = 0, \dots, N_i - 1 \\ K^i x^i(t+k|t), & k \geq N_i \end{cases}$$

with  $K^i \in \mathbb{R}^{m_i \times n_i}$

- **Cost-to-go:**

$$J^i(x(t|t), x_f^i, \mathbf{u}^i(t)) := \sum_{k=t}^{t+N_i-1} \left[ \|x^i(t+k|t) - x_f^i\|_{R_x^i}^2 + \|u^i(t+k|t)\|_{R_u^i}^2 \right] + \|x^i(t+N_i|t) - x_f^i\|_{P^i}^2$$

with  $R_x^i = R_x^{iT} > 0$ ,  $R_u^i = R_u^{iT} \geq 0$ ,  $P^i = P^{iT} \geq 0$

- **Terminal constraint:**

$$x^i(t+N_i|t) \in \Xi^i \subset \mathbb{R}^{n_i}$$

## Positively invariance -

The pair  $(\Xi^i, K^i)$  is such that  $u^i(\cdot) = K^i x^i(\cdot)$  a **stabilizing** state feedback law and  $\Xi^i$  a **positively invariant** region for the state evolutions of the closed-loop system:

$$x^i(t+N_i|t) \in \Xi^i \Rightarrow (A^i + B^i K^i)^{t+N_i+k} x^i(t+N_i|t) \in \Xi^i, \forall k \geq 0$$

## Time-varying terminal sets -

- the LF formation moves towards  $x_f$
- the vehicle must take into account routing decisions  $z^i(t)$  at each junction

## Prescriptions -

- Given the equilibrium  $\bar{x}_{t-1}^i$ , the positively invariant region  $\Xi^i(t)$  is such that

$$\bar{x}_{t-1}^i \in \Xi^i(t-1) \cap \Xi^i(t)$$

- Terminal set-membership requirement:

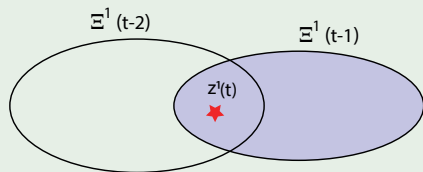
$$x^i(t + N_i|t) \in \Xi^i(t-1) \cup \Xi^i(t)$$

- Leader admissible set constraint:

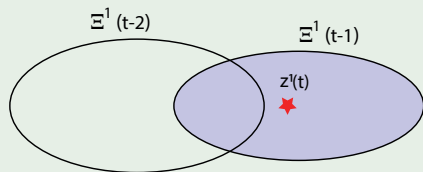
$$\text{Proj}_p(\Xi^1(t)) \subseteq \mathcal{B}(p^1(t), R), \forall t \geq 0$$

- Follower admissible sets:  $\Xi^i(t)$ ,  $i \geq 2$ , take advantage of transmitted  $\Xi^{i-1}(t-1)$

## Leader positively invariant set computation -

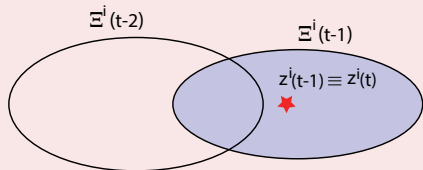


$$\left\{ \begin{array}{l} \Xi^1(t) := \arg \min_{\Xi^1} \text{dist}(x_f^1, \Xi^1) \text{ s.t.} \\ \bar{x}_{t-1}^1 \in \Xi^1(t-1) \cap \Xi^i(t) \\ \text{Proj}_p(\Xi^1(t)) \subseteq \mathcal{B}(p^1(t), R), \forall t \geq 0 \end{array} \right.$$

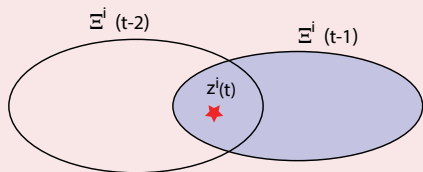


$$\left\{ \begin{array}{l} \Xi^1(t) := \arg \min_{\Xi^1} \text{dist}(z^1(t), \Xi^1) \text{ s.t.} \\ \bar{x}_{t-1}^i \in \Xi^1(t-1) \cap \Xi^1(t) \\ \text{Proj}_p(\Xi^1(t)) \subseteq \mathcal{B}(p^1(t), R), \forall t \geq 0 \end{array} \right.$$

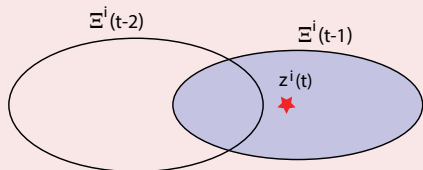
## Follower positively invariant set computation -



$$\begin{cases} \Xi^i(t) := \arg \min_{\Xi^i} \text{dist}(\Xi^i, \Xi^{i-1}(t-1)) \text{ s.t.} \\ \bar{x}_{t-1}^i \in \Xi^i(t-1) \cap \Xi^i(t) \end{cases}$$



$$\begin{cases} \Xi^i(t) := \arg \min_{\Xi^i} \text{dist}(x_f^i, \Xi^i) \text{ s.t.} \\ \bar{x}_{t-1}^i \in \Xi^i(t-1) \cap \Xi^i(t) \end{cases}$$



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### Predecessor assumed state sequence -

Given

- the current state condition:  $x^i(t|t) \in \mathcal{X}^i$
- the received optimal state sequence of  $AV^{i-1}$  at the previous time instant  $t - 1$  :  
 $\mathbf{x}^{(i-1)*}(t - 1)$

The vehicle  $AV^i$  will implement

$$\hat{\mathbf{x}}^{i-1}(t) = \begin{cases} x^{(i-1)*}(t - 1 + k|t - 1), k = 1, \dots, N_{i-1} \\ (A^{i-1} + B^{i-1}K^{i-1})^{t-1+k}x^{(i-1)*}(t - 1 + N_{i-1}|t - 1), k = N_{i-1} + 1, \dots, N_i \end{cases}$$

### Formation constraints -

- Preserve the platoon configuration
- Maintain a safe distance from a vehicle and its predecessor
- Requirements in terms of desired relative distance and bearing between  $AV^i$  and  $AV^{i-1}$

## Leader optimization problem - $\mathcal{P}_L(t)$ :

$$\min_{K^1, \Xi^1} \sum_{k=t}^{\infty} \left[ \|x^1(t+k|t) - x_f^1\|_{R_x^1}^2 + \|u^1(t+k|t)\|_{R_u^1}^2 \right] \text{ subject to}$$

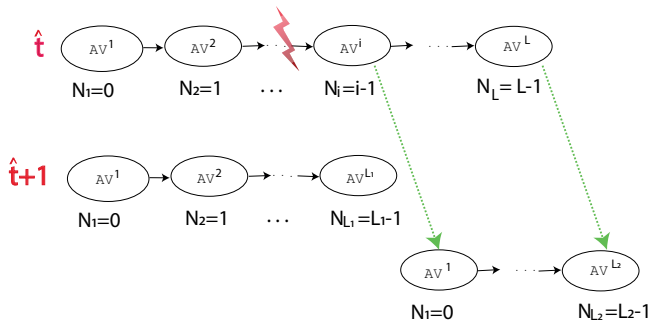
$$\begin{aligned} x^1(t+k+1|t) &= A^1 x^1(t+k|t) + B^1 u^1(t+k|t), \forall k && - \text{Dynamics} \\ u^1(t+k|t) &= K^1 x^1(t+k+1|t) \in \mathcal{U}^1, \forall k && - \text{Saturation} \\ x^1(t+k|t) &\in \mathcal{X}^1, \forall k && - \text{Geometric} \\ \bar{x}_{t-1}^1 &\in \Xi^1(t-1) \cap \Xi^1(t) && - \text{Terminal set} \end{aligned}$$

## Follower optimization problem - $\mathcal{P}_F^i(t)$ :

$$\min_{\mathbf{u}^i(t)} J^i(x(t|t), x_f^i, \mathbf{u}^i(t)) \text{ subject to}$$

$$\begin{aligned} x^i(t+k+1|t) &= A^i x^i(t+k|t) + B^i u^i(t+k|t) && - \text{Dynamics} \\ u^i(t+k|t) &\in \mathcal{U}^i, k = 0, 1, \dots, N_i - 1 && - \text{Saturation} \\ x^i(t+k|t) &\in \mathcal{X}^i, k = 0, 1, \dots, N_i - 1 && - \text{Geometric} \\ x^i(t+N_i|t) &\in \Xi^i(t) && - \text{Terminal set} \\ \alpha_{min}^c &\leq \|p^i(t+k|t) - \hat{p}^{i-1}(t+k|t)\| \leq \alpha_{max}^c, k = 0, 1, \dots, N_i && - \text{Formation} \end{aligned}$$

$\alpha_{max}^c \in \mathbb{R}^+$  and  $\alpha_{min}^c$  are guaranteed bounds depending on formation requirements



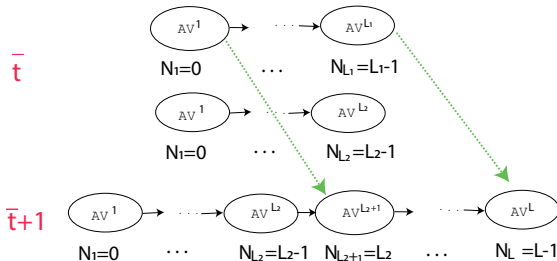
## Modus operandi -

- At  $\hat{t}$ , the routing decision sequence  $\{a^1(\hat{t}), a^2(\hat{t}), \dots, a^L(\hat{t})\}$  is such that

$$\exists i : a^k(\hat{t}) \neq a^{k+1}(\hat{t}), \quad k = i, \dots, L-1$$

- Two platoon configurations:  $LF^1$  and  $LF^2$  :
  - $LF^1$  keeps the same control horizon lengths  $N_i = i - 1, i = 1 \dots, L_1$
  - $LF^2$  modifies the control horizon length as  $N_{L_2} = 0, N_i = i - L_2, i = L_2 + 1, \dots, L$
  - A re-numbering procedure is implemented:  $\{AV_i^1\}_{i=1}^{L_1}$  and  $\{AV_i^2\}_{i=1}^{L_2}$

# Platoon queuing



## Modus operandi -

- $LF^1$  is added to  $LF^2$  or *viceversa* at the leaf node of the first platoon
- horizon lengths are set as  $N_h = N_{L_1} + h, h = 1, \dots, L_2$

## Leaf node detection -

- $AV^i$  makes available a data packet  $Count^i = \{Count^i.child(i), Count^i.N_i\}$ , the integer  $child(i)$  accounts for its follower:  $child(i) = 0$  implies no follower
- $Count^i$  can be acquired by  $AV^h$  if  $\alpha_{min}^c \leq \|p^h(\bar{t}) - p^i(\bar{t})\|_2 \leq \alpha_{max}^c$

## Regrouping -

$LF^1$  platoon is able to regroup with  $LF^2$  when  $Count^{L_2}$  is detected by  $AV^1$  of  $LF^1$

## Simulation of Urban MObility®(SUMO) -

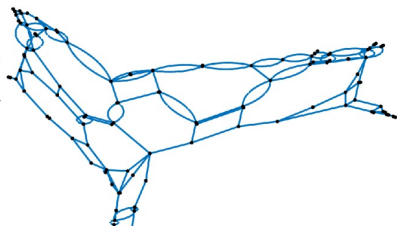
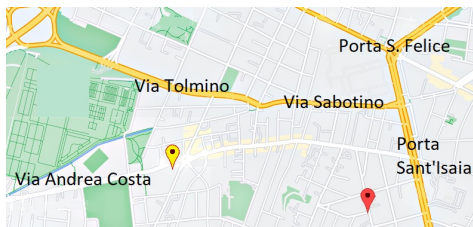
- Open source highly **portable, microscopic and continuous** multi-modal traffic simulation package
- Handle **large urban road networks**
- Traffic is described by means of **departure times and routes** with certain duration
- Particularly suitable to simulate different classes of vehicles: **cars, trains,buses**
- Simulation engine is based on an **hybrid description** : discrete-time and continuous-space.
- Enjoy **collision avoidance** capabilities, **multi-lane roads** under lane changing, **junction-based right-of-way** rules, **lane-to-lane** connections
- Data expressed in **XML formats**

## Ingredients -

- The TraCI4Matlab®toolbox allowing the **communication** between any MATLAB application and the SUMO traffic simulator
- The TraCI application level protocol is based on the **client-server paradigm**
- The application developed in MATLAB plays the role of a **client** and it can access and modify the simulation environment provided by SUMO that acts as a **server**

## Co-design procedure -

- Urban road network information are properly **encoded** into MATLAB objects
- A MATLAB graph is obtained by exploiting SUMO XML files: **junctions (nodes)**, **roads (edges)**
- A set of **APIs** get, modify or add information about vehicles within the simulation environment
- **Neural network weights optimization** by the MATLAB Reinforcement Learning toolbox® and the MATLAB Deep learning toolbox®



## Simulation setup -

- Matlab R2022a environment by using the SUMO 1.11.0 toolbox
- Lenovo IdeaPad L340-15RH laptop equipped with an Intel Core i9 processor under a 64-bit operating system

## Vehicle platoon and constraints -

- Platoon:  $L = 11$  vehicles
- Point mobile robot dynamics:
  - $x^i = [p_x^i, p_y^i, v_x^i, v_y^i]^T : (p_x^i, p_y^i)$  the vehicle planar coordinates and  $(v_x^i, v_y^i)$  velocity components
  - $u_i = [a_x^i, a_y^i]^T \in \mathbb{R}^2$  the acceleration vector

$$A^i = \begin{bmatrix} I_2 & T_s I_2 \\ 0_2 & I_2 \end{bmatrix}, B^i = \begin{bmatrix} \frac{T_s^2}{2} I_2 \\ \frac{T_s}{2} I_2 \end{bmatrix},$$

- $T_s = 1 [s]$  the sampling time
- Input constraints

$$\|u^i(t)\|^2 \leq 5 [m/s^2], \forall t \geq 0$$

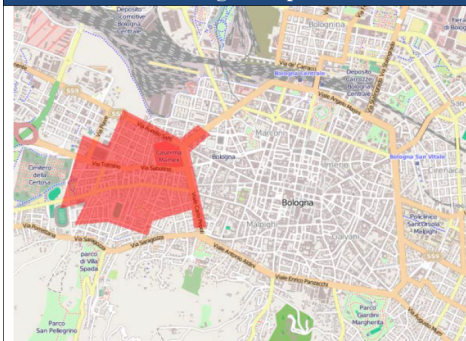
- Formation constraints

$$\alpha_{min}^c \leq \|p^i(t+k|t) - \hat{p}^{i-1}(t+k|t)\| \leq \alpha_{max}^c, \alpha_{min}^c = 3 [m], \alpha_{max}^c = 10 [m]$$

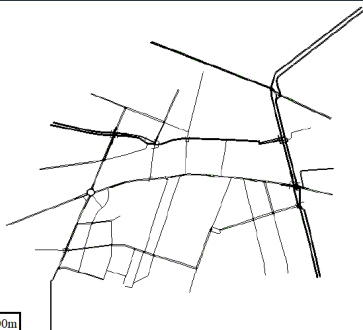
## Urban road network -

The district around the “Andrea Costa” road in Bologna (IT)

Bologna Map



SUMO Road Network



## Traffic data -

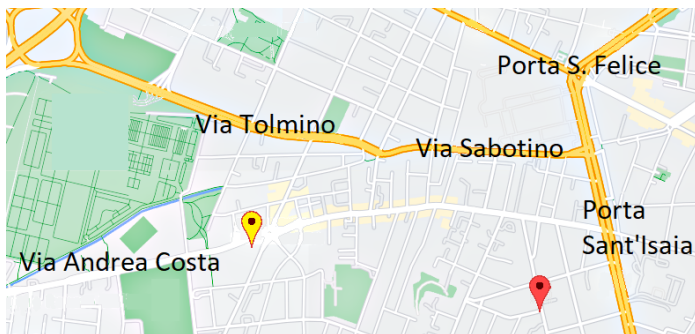
- Peak hour (8:00 am - 9:00 am) generated during a football match on March 3rd 2010
- Daily flow one week before



## Opening scenario -

The reference initial position is located at the latitude  $44^{\circ}29'23.6''N$  and longitude  $11^{\circ}18'30.2''E$  coordinates. Within a local tangent plane East-North-Up reference frame, the AVs initial positions have been set as reported in Table with  $v_x^i = 0$  and  $v_y^i = 0, \forall i$ . The aim consists in driving all the vehicles towards the parking area located close to the target position in the south-east side.

$AV^1$	$AV^2$	$AV^3$	$AV^4$	$AV^5$	$AV^6$	$AV^7$	$AV^8$	$AV^9$	$AV^{10}$	$AV^{11}$
70.0195	63.955	57.8905	51.826	45.7615	39.697	33.6325	27.54	21.503	15.44	9.374
386.4212	384.08	381.743	379.403	377.064	374.725	372.39	370.047	367.707	365.368	363.029

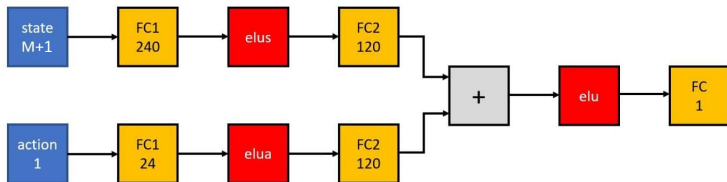


## Simulation knobs -

- Vision radius:  $R = 5 [m]$ ,
- Curvature radius:  $R_{min}^c = 2 [m]$ ,
- Cost-to-go function weights:  $R_x^i = I_{n_i}$ ,  $R_u^i = I_{m_i}$ ,  $i = 1, \dots, L$
- Decision zone:  $3.5 [m]$  before each junction

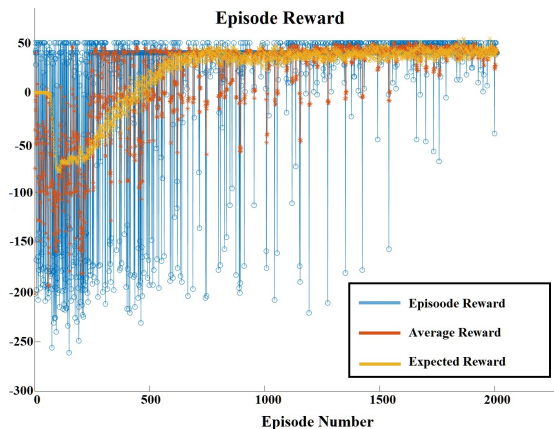
## Neural network architecture -

The state input layer dimension is twice the number of edges of the considered URN:  $2M = 358$



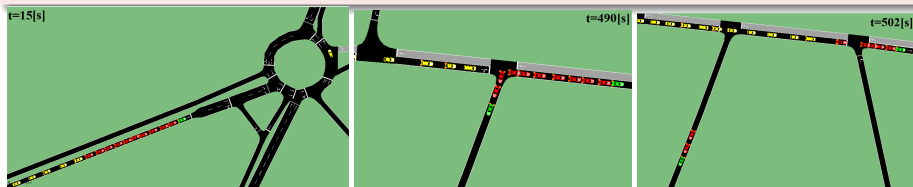
## Neural network training -

- SUMO data set
- MATLAB Reinforcement Learning Toolbox built-in routines
- 2000 traffic episodes: each one defined as a finite sequence of time instants starting from the initial positions until the target  $x_f$  (parking area)



## Splitting mode (1/3) -

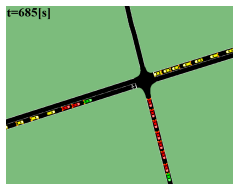
- traffic data in the time interval 8 : 00am – 8 : 30am
- initially the platoon configuration is kept constant
- at  $t = 482$  [s], the greedy actions  $a^i(482) \neq a^1(482)$ ,  $i = 9, 10, 11$ , suggest to  $AV^i$ ,  $i = 9, 10, 11$ , to **turn on the right** while the others continue on the **straight line**
- two platoons  $LF^1 = \{AV^1, \dots, AV^8\}$  and  $LF^2 = \{AV^9, AV^{10}, AV^{11}\}$  arise



(a)

(b)

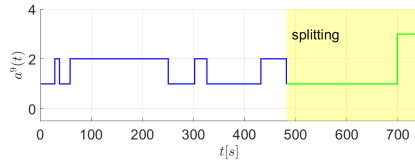
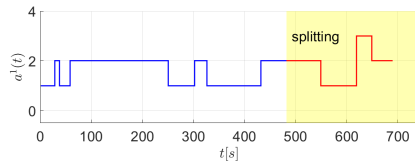
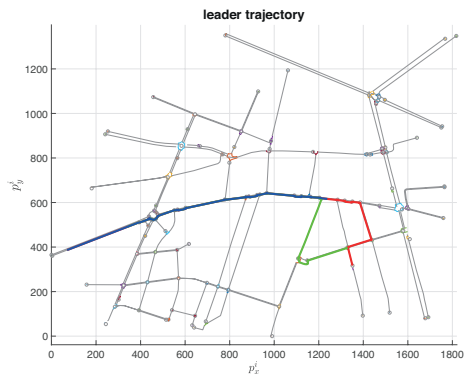
(c)

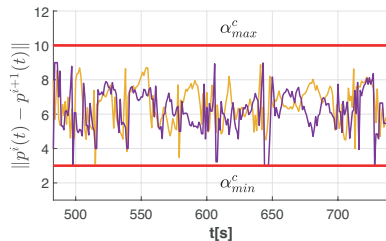
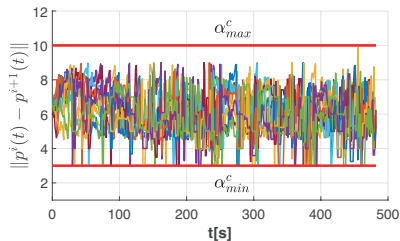
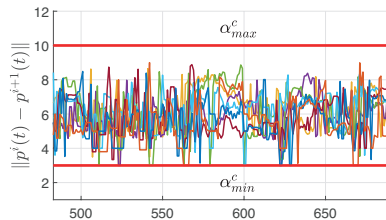
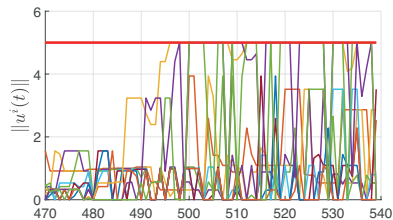


(d)

## Splitting mode (2/3) -

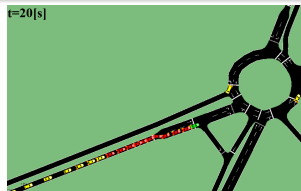
Routing actions behavior: the leader routing decision (green line) differs from  $AV^9$  (red line)



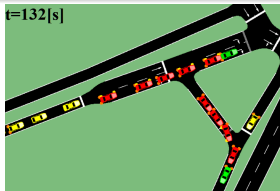


## Queuing mode (1/3) -

- traffic data in the time interval 8 : 30am – 9 : 00am
- at  $t \approx 123$  [s] the vehicles  $AV^i$ ,  $i = 6, \dots, 11$ , make **different routing decisions** with respect to the leader  $AV^1$  and two new platoons arise
- at  $t \approx 350$  [s]  $LF^2$  queues to  $LF^1$
- the platoon reaches the target at  $t \approx 715$  [s]



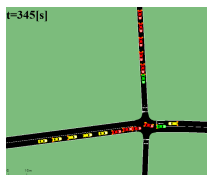
(e)



(f)



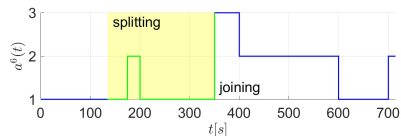
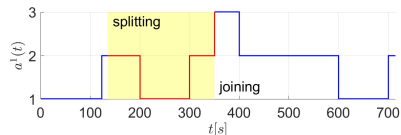
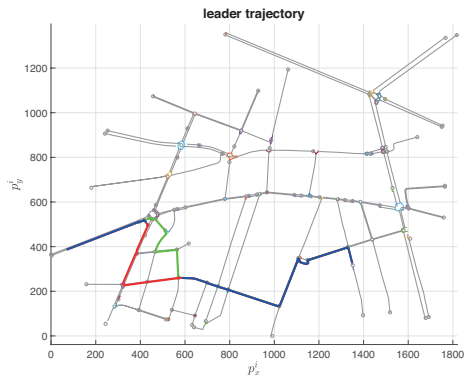
(g)



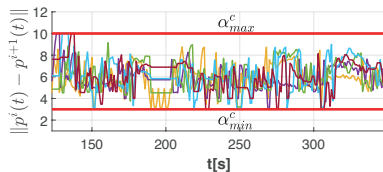
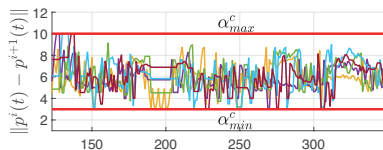
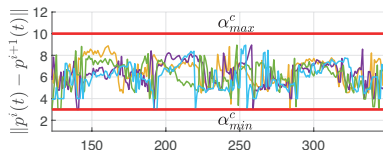
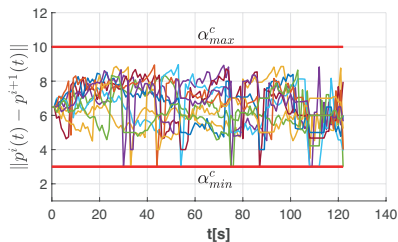
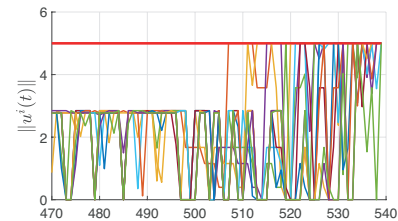
(h)

## Queuing mode (2/3) -

Routing actions behavior: the leader routing decision (green line) differs from  $AV^6$  (red line)







## Comparisons -

- Contrast the proposed distributed  $Q$ -learning scheme with the Dijkstra algorithm
- Three operating scenarios:
  - 1 splitting mode
  - 2 queuing mode
  - 3 within the time window  $8 : 00am - 8 : 30am$ , the initial platoon configuration is initialized by imposing that all the vehicles have zero velocities, the leader is located at  $(x_1^1(0), x_2^1(0)) = (874.54, 979.20)[m]$  and the target is  $x_f = [1611.76, 883.38, 0, 0]^T [m]$

Simulation	Route percentage	<b>DRL-MPC</b>	<b>Dijkstra</b>
scenario 1	20% of route length	200 [s]	210 [s]
	50% of route length	400 [s]	500 [s]
	100% of route length	760 [s]	850 [s]
scenario 2	20% of route length	180 [s]	170 [s]
	50% of route length	400 [s]	450 [s]
	100% of route length	745 [s]	800 [s]
scenario 3	20% of route length	124 [s]	120 [s]
	50% of route length	280 [s]	250 [s]
	100% of route length	610 [s]	655 [s]

## Conclusions -

- The path planning problem for vehicle platoons subject to routing decisions and operating in urban road networks has been addressed
- A new control architecture based on the joint exploitation of distributed reinforcement learning and model predictive control properties has been developed by fully taking advantage of the routing decisions
- Time-varying topologies of the multi-vehicle system have been considered according to the minimization of traffic congestion criteria

## Future directions -

- Extend the proposed approach to take care of obstacle avoidance requirements
- Customize the strategy to comply with roadways including more than one lane so that grid vehicle topologies can be used
- Improve the performance of the distributed reinforcement learning algorithm by accelerating the distributed learning phase to get faster vehicle planning decision responses

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**THANKS FOR THE ATTENTION!!**