Joint Iterative Channel Estimation and Data Detection for MIMO-CDMA Systems over Frequency-Selective Fading Channels

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I. Abstract

In this paper, we present an iterative joint channel estimation and data detection technique for multiple-input multiple-output (MIMO) code-division multiple-access (CDMA) systems over frequency-selective fading channels. Based on the expectation-maximization (EM) algorithm, the proposed iterative receiver achieves a performance close to the optimum maximum-likelihood (ML) receiver. In addition, the performance of the proposed receiver is optimized through weight coefficients using the minimum mean-square error (MMSE) criterion. Compared to the single-user bound, our results show that the proposed receiver can mitigate the multiple-access interference and attain the full system diversity. Furthermore, our simulation results confirm that the proposed receiver is near-far resistant and offers fast convergence in severe near-far scenarios.

II. Introduction

In multiple-input multiple-output (MIMO) code-division multiple-access (CDMA)systems, channel estimation plays a crucial role in determining the system performance [1]. In order to achieve the promised performance gain of MIMO systems, the channel coefficients must be known or estimated perfectly at the receiver [2]. Recently, there has been an increasing interest in iterative joint channel estimation and data detection algorithms [3], which take the advantage of the detected data symbols in the channel estimation process. In particular, there has been a growing interest in expectation-maximization (EM) based joint detection and estimation (JDE) techniques because of its ability to achieve accurate estimation without wasting the system resources [4]. Therefore, an extensive effort has been focused on employing the EM algorithm in JDE techniques for MIMO systems [5]. For example, the authors in [6] proposed various EM-based channel estimation techniques for MIMO systems. Choi [7] has proposed a general framework for EM-based JDE schemes considering different types of MIMO channels (e.g., Rician or Rayleigh fading). As well, many research efforts have considered the JDE problem for MIMO systems considering flat [5] and frequency-selective channels [8]. In this paper, we propose a JDE technique based on the EM algorithm for MIMO CDMA systems assuming asynchronous transmission over frequency-selective fading channels. The proposed JDE receiver structure is derived, where we show that it can bring an optimum balance between a single-user matched filter detector and a parallel interference cancellation (PIC) based detector. To achieve a fast convergence to the optimum solution, we initialize the EM estimates based on reliable estimates of space-time minimum mean-square error (MMSE) separate detection and estimation (ST-MMSE-SDE) receiver [9]. The rest of the paper is organized as follows. The system model is described in the following section. In Section IV, the EM-based ST receiver is discussed. In Section V, we derive a closed-form expression for the optimized weight coefficients. In Section VI, we discuss the initialization of the EM algorithm. Simulation results and discussions are then presented in Section VII. Finally, conclusions are drawn in Section VIII.

III. System Model

Throughout our work, we consider a transmit diversity scheme with U=2 transmit antennas at the mobile user and V multiple receive antennas at the base station. We also consider the original space-time spreading (STS) scheme proposed in [10] for an asynchronous K-user system over a slow frequency-selective fading channel with L resolvable paths. The channel coefficients are, therefore, considered fixed within a block of M codewords, where each codeword has a period of $T_s = 2T_b$ and T_b denotes the bit period. The received complex low-pass equivalent signal at the v^{th} receive antenna is given by

$$r^v(t) = \sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{m=0}^{M-1} h_{1l}^{k,v}\Big[b_{k1}[m]c_{k1}(t-mT_s-\tau_k-\tilde{\tau}_l) +b_{k2}[m]c_{k2}(t-mT_s-\tau_k-\tilde{\tau}_l)\Big]+h_{2l}^{k,v}\Big[b_{k2}[m]c_{k1}(t-mT_s-\tau_k -\tilde{\tau}_l)-b_{k1}[m]c_{k2}(t-mT_s-\tau_k-\tilde{\tau}_l)\Big]+n^v(t), \quad (1)$$

where $b_{k1}[m]$ and $b_{k2}[m]$ are the odd and even data streams of the k^{th} user within the m^{th} codeword interval. The codes

$c_{k1}(t)$ and $c_{k2}(t)$ represent the k^{th} user's spreading sequences with processing gain $2N$, where $N = T_b/T_c$ is the number of chips per bit, and T_c is the chip duration. In (1), $h_{ql}^{k,v}$, $q = 1, 2$, is the attenuation coefficient corresponding to the k^{th} user, l^{th}path from the q^{th} transmit antenna to the v^{th} receive antenna, where $h_{ql}^{k,v} = \sqrt{\frac{E_k}{2}}\alpha_{ql}^{k,v}$, $\alpha_{ql}^{k,v}$ is the corresponding fading channel coefficient and E_k is the k^{th} user transmit energy. These attenuation coefficients are modeled as independent complex Gaussian random variables with zero mean and variance σ_k^2, where $\sigma_k^2 = \frac{E_k}{2}\sigma_h^2$, and $\sigma_h^2 = \frac{1}{L}$. The noise $n^v(t)$ is Gaussian with zero mean and variance N_o. At the receiver side, the received signal at each receive antenna is chip-matched filtered, sampled at a rate $1/T_c$, and accumulated over an observation interval of $(2N + \tau_{max} + L - 1)$ chips corresponding to the m^{th} symbol of the received data block for the K-user system. The $(\tau_{max} + L - 1)$ samples are due to the maximum multipath delay (i.e., delay of the l^{th} path) corresponding to the user with the maximum transmit delay, τ_{max}. Let $\mathbf{y}^v[m]$ denote the observation vector at the v^{th} receive antenna containing all samples related to the STS symbols transmitted by the K users within the observation interval. Then, we have

$$\mathbf{y}^v[m] = (\mathbf{C}[0]\mathbf{B}(m) + \mathbf{C}[-1]\mathbf{B}(m-1) + \mathbf{C}[1]\mathbf{B}(m+1)) \times \mathbf{h}^v + \mathbf{n}^v[m], m = 1, \ldots, M-2, \quad (2)$$

where $\mathbf{C}[0]$, $\mathbf{C}[-1]$, and $\mathbf{C}[1]$ include the code sequences corresponding to the current, previous and following STS symbols of the K-user system within the observation interval respectively. In (2), $\mathbf{B}(m)$, $m = 0, \ldots, M-1$, represents the users data matrix within the m^{th} period, defined as

$$\mathbf{B}(m) = diag\{\mathbf{B}_1(m), \mathbf{B}_2(m), \ldots, \mathbf{B}_K(m)\},$$

where

$$\mathbf{B}_k(m) = \mathbf{I}_L \otimes \tilde{\mathbf{b}}_k(m), \quad \tilde{\mathbf{b}}_k(m) = \begin{bmatrix} b_{k1}[m] & b_{k2}[m] \\ b_{k2}[m] & -b_{k1}[m] \end{bmatrix},$$

$k = 1, \ldots, K, m = 0, \ldots, M-1$, and $\mathbf{I}_L$ is an identity matrix of L-dimension. Also, $\mathbf{h}^v$ is a $(2LK \times 1)$ channel coefficients vector defined as

$$\mathbf{h}^v = [\mathbf{h}_1^{vT}, \mathbf{h}_2^{vT}, \ldots, \mathbf{h}_K^{v\,T}]^T,$$

where $\mathbf{h}_k^v = [h_{11}^{k,v}, h_{21}^{k,v}, \ldots, h_{2L}^{k,v}]^T$, and T denotes the transpose operation. Finally, in (2), $\mathbf{n}^v[m]$ is a $[(2N + L - 1 + \tau_{max}) \times 1]$ vector representing the additive-white-Gaussian noise (AWGN) samples at the v^{th} receive antenna, each with zero mean and variance N_o. After sampling the received signal, the matched filter output at the v^{th} receive antenna , $\mathbf{y}^v[m]$, is correlated with the code matrix, $\mathbf{C}[0]$, as follows

$$\mathbf{y}_c^v[m] = (\mathbf{R}[0]\mathbf{B}(m) + \mathbf{R}[-1]\mathbf{B}(m-1) + \mathbf{R}[1]\mathbf{B}(m+1)) \times \mathbf{h}^v + \mathbf{n}_c^v[m], \quad (3)$$

$m = 1, \ldots, M-2$, where $\mathbf{R}[0] = \mathbf{C}^H[0]\mathbf{C}[0]$, $\mathbf{R}[-1] = \mathbf{C}^H[0]\mathbf{C}[-1]$, $\mathbf{R}[1] = \mathbf{C}^H[0]\mathbf{C}[1]$, and H denotes Hermitian transpose. In (3), $\mathbf{n}_c^v[m]$ is modelled as $N_c(\mathbf{0}, \mathbf{R}[0])$. Similar to [11], let

$$\mathbf{R}[0] = \mathbf{F}[0]^T\mathbf{F}[0] + \mathbf{F}[1]^T\mathbf{F}[1], \quad (4)$$

and

$$\mathbf{R}[-1] = \mathbf{F}[0]^T\mathbf{F}[1], \quad (5)$$

where $\mathbf{F}[0]$ is a lower triangular matrix, and $\mathbf{F}[1]$ is an upper right triangular matrix with zero diagonal. If $\mathbf{y}_c^v[m]$ is passed through a filter with impulse response $(\mathbf{F}[0] + \mathbf{F}[1]z)^{-T}$ [11], then

$$\mathbf{y}_w^v[m] = \sum_{\Delta m=0}^{1} \mathbf{F}[\Delta m]\mathbf{B}(m - \Delta m)\mathbf{h}^v + \mathbf{n}_w^v[m], \quad (6)$$

where $\mathbf{n}_w^v[m]$ is a complex Gaussian vector with zero mean and covariance matrix $N_o\mathbf{I}_{2LK}$, and $\mathbf{I}_{2LK}$ is an identity matrix of dimension $2LK$. Note that both $\mathbf{y}_c^v[m]$ and $\mathbf{y}_w^v[m]$ have the same information about the transmitted data. Due to the whitening noise property of (6), our subsequent analysis will be based on $\mathbf{y}_w^v[m]$.

IV. Space-Time Receiver Structure

In this section we describe the details of our proposed EM-Based ST Receiver. The receiver is derived by first decomposing the observed data into its signal components using the approach presented in [12] for the estimation problem of superimposed signals. Then, using the decomposed signal components which are coupled with the corresponding unknown channel coefficients of different users, we define the complete data set. Subsequently, we alternate between the E-step and the M-step of EM algorithm until the convergence is achieved. Now starting with the approach proposed in [12], we decompose the whitening filter output, $\mathbf{y}_w^v[m]$, into a sum of K statistically independent components, i.e.,

$$\mathbf{y}_w^v[m] = \sum_{k=1}^{K} \mathbf{g}_k^v(m), \quad (7)$$

where $\mathbf{g}_k^v(m) = \sum_{\Delta m=0}^{1} \mathbf{F}_k[\Delta m]\mathbf{B}_k(m-\Delta m)\mathbf{h}_k^v + \mathbf{n}_{wk}^v[m]$, $\mathbf{F}_k[\Delta m]$ is $2LK \times 2L$ matrix including the $2L$ columns corresponding to the k^{th} user in the matrix $\mathbf{F}[\Delta m]$, $\mathbf{n}_{wk}^v[m]$ is a complex Gaussian vector with zero mean and covariance matrix $\beta_k^v N_o \mathbf{I}_{2LK}$ and β_k^v is a non-negative value satisfying the constraint $\sum_{k=1}^{K} \beta_k^v = 1$. Our goal is to obtain the users' data estimates using the EM algorithm. First, we define the EM algorithm parameters:

1) *Observed data*, $\mathbf{y}_w$, which includes the outputs of the V whitening matched filters within a frame of M codes is given by $\mathbf{y}_w = [\mathbf{y}_w^{1\,T}, \mathbf{y}_w^{2\,T}, \ldots, \mathbf{y}_w^{V\,T}]^T$, where $\mathbf{y}_w^v = [\mathbf{y}_w^v[0]^T, \mathbf{y}_w^v[1]^T, \ldots, \mathbf{y}_w^v[M-1]^T]^T, v = 1, \ldots, V$.
2) *Parameters to be estimated*, $\mathbf{b}$, which include the transmitted data bits from the K users within the frame period $\mathbf{b} = [\mathbf{b}_1^T, \mathbf{b}_2^T, \ldots, \mathbf{b}_K^T]^T$, where $\mathbf{b}_k = [\mathbf{b}_k[0]^T, \mathbf{b}_k[1]^T, \ldots, \mathbf{b}_k[M-1]^T]^T, k = 1, \ldots, K$, and $\mathbf{b}_k[m] = [b_{k1}[m], b_{k2}[m]]^T, m = 0, \ldots, M-1$.

3) *Complete data*, $\mathbf{G}$, which are defined based on the complete data definition in [9], where the unknown channel coefficient vectors are included as a part of the complete data as follows: $\mathbf{G} = [\mathbf{G}^{1^T}, \mathbf{G}^{2^T}, \ldots, \mathbf{G}^{V^T}]^T$, where $\mathbf{G}^v = [(\mathbf{g}_1^v, \mathbf{h}_1^v), (\mathbf{g}_2^v, \mathbf{h}_2^v), \ldots, (\mathbf{g}_K^v, \mathbf{h}_K^v)], v = 1, \ldots, V$, and $\mathbf{g}_k^v = [\mathbf{g}_k^v(0), \mathbf{g}_k^v(1), \ldots, \mathbf{g}_k^v(M-1)], k = 1, \ldots, K$.

Since the components of $\mathbf{G}$ given $\mathbf{b}$ are statistically independent, the complete log-likelihood function is given by

$$\Phi(\mathbf{G}|\mathbf{b}) = \sum_{v=1}^{V} \sum_{k=1}^{K} \Phi(\mathbf{g}_k^v, \mathbf{h}_k^v | \mathbf{b}_k), \quad (8)$$

where

$$\Phi(\mathbf{g}_k^v, \mathbf{h}_k^v|\mathbf{b}_k) = \Phi(\mathbf{g}_k^v|\mathbf{h}_k^v, \mathbf{b}_k) + \Phi(\mathbf{h}_k^v|\mathbf{b}_k). \quad (9)$$

The second summand in (9) can be ignored as it is independent of $\mathbf{b}$. Therefore, (9) is reduced to

$$\Phi(\mathbf{g}_k^v, \mathbf{h}_k^v|\mathbf{b}_k) \propto -\sum_{m=0}^{M-1} \Big(\mathbf{g}_k^v(m) - \sum_{\Delta m=0}^{1} \mathbf{F}_k[\Delta m] \times \mathbf{B}_k(m-\Delta m)\mathbf{h}_k^v\Big)^H \Big(\mathbf{g}_k^v(m) - \sum_{\Delta m=0}^{1} \mathbf{F}_k[\Delta m] \times \mathbf{B}_k(m-\Delta m)\mathbf{h}_k^v\Big). \quad (10)$$

By ignoring the terms in (10) which are independent of $\mathbf{b}$, the conditional likelihood in (10) can be simplified to

$$\Phi(\mathbf{g}_k^v, \mathbf{h}_k^v|\mathbf{b}_k) \propto \sum_{m=0}^{M-1} 2Re\{\mathbf{h}_k^{v^H}\mathbf{B}_k(m)\mathbf{F}_k[0]^T\mathbf{g}_k^v(m) + \mathbf{h}_k^{v^H}\mathbf{B}_k(m)\mathbf{F}_k[1]^T\mathbf{g}_k^v(m+1) - \mathbf{h}_k^{v^H}\mathbf{B}_k(m)\mathbf{R}_{kk}[-1] \times \mathbf{B}_k(m-1)\mathbf{h}_k^v - \mathbf{h}_k^{v^H}\mathbf{B}_k(m+1)\mathbf{R}_{kk}[-1]\mathbf{B}_k(m)\mathbf{h}_k^v\} - \mathbf{h}_k^{v^H} \times \mathbf{B}_k(m)\mathbf{R}_{kk}[0]\mathbf{B}_k(m)\mathbf{h}_k^v, \quad (11)$$

where $\mathbf{R}_{kk}[-1] = \mathbf{F}_k[0]^T\mathbf{F}_k[1]$, and $\mathbf{R}_{kk}[0] = \mathbf{F}_k[0]^T\mathbf{F}_k[0] + \mathbf{F}_k[1]^T\mathbf{F}_k[1]$. At the i^{th} iteration, the E-step of the EM algorithm is implemented by taking the expectation of the complete log-likelihood function defined in (8) with respect to the observed data vector, $\mathbf{y}_w$, and the current EM data estimates, $\mathbf{b}^i$, i.e.,

$$\mathcal{Q}(\mathbf{b}|\mathbf{b}^i) = \sum_{k=1}^{K} \mathcal{Q}_k(\mathbf{b}_k|\mathbf{b}^i), \quad (12)$$

where

$$\mathcal{Q}_k(\mathbf{b}_k|\mathbf{b}^i) = \sum_{v=1}^{V} E\left[\Phi(\mathbf{g}_k^v, \mathbf{h}_k^v|\mathbf{b}_k)|\mathbf{y}_w, \mathbf{b}^i\right] \quad (13)$$

From (11), the expectation of the individual log-likelihood function is reduced to

$$\mathcal{Q}_k(\mathbf{b}_k|\mathbf{b}^i) = \sum_{v=1}^{V}\sum_{m=0}^{M-1} 2Re\Big\{E\Big[\mathbf{h}_k^{v^H}\mathbf{B}_k(m)\mathbf{F}_k[0]^T\mathbf{g}_k^v(m) + \mathbf{h}_k^{v^H}\mathbf{B}_k(m)\mathbf{F}_k[1]^T\mathbf{g}_k^v(m+1) - \mathbf{h}_k^{v^H}\mathbf{B}_k(m)\mathbf{R}_{kk}[-1] \times \mathbf{B}_k(m-1)\mathbf{h}_k^v - \mathbf{h}_k^{v^H}\mathbf{B}_k(m+1)\mathbf{R}_{kk}[-1]\mathbf{B}_k(m) \times \mathbf{h}_k^v|\mathbf{y}_w, \mathbf{b}^i\Big]\Big\} - E\Big[\mathbf{h}_k^{v^H}\mathbf{B}_k(m)\mathbf{R}_{kk}[0]\mathbf{B}_k(m)\mathbf{h}_k^v|\mathbf{y}_w, \mathbf{b}^i\Big]. \quad (14)$$

To find the joint conditional expectation in (14), we evaluate $E\left[\mathbf{g}_k^v(m_s)|\mathbf{y}_w, \mathbf{b}^i, \mathbf{h}\right]$, $m_s \in \{m, m+1\}$, where $\mathbf{h} = [\mathbf{h}^1, \mathbf{h}^2, \ldots, \mathbf{h}^V]$. Then the subsequent expression is used to find $E\left[f(\mathbf{h}_k^v)|\mathbf{y}_w, \mathbf{b}^i\right]$, where $f(\mathbf{h}_k^v)$ denotes the resultant function in $\mathbf{h}_k^v$. By noting that the conditional probability density function, $P_c\left(\mathbf{g}_k^v(m_s)|\mathbf{y}_w, \mathbf{b}^i, \mathbf{h}\right)$ is Gaussian with mean [12]

$$E\left[\mathbf{g}_k^v(m_s)|\mathbf{y}_w, \mathbf{b}^i, \mathbf{h}\right] = \sum_{\Delta m=0}^{1} \mathbf{F}_k[\Delta m]\mathbf{B}_k(m_s - \Delta m)^i\mathbf{h}_k^v + \beta_k^v\Big(\mathbf{y}^v(m_s) - \sum_{j=1}^{K}\sum_{\Delta m=0}^{1} \mathbf{F}_j[\Delta m]\mathbf{B}_j(m_s-\Delta m)^i\mathbf{h}_j^v\Big), \quad m_s \in \{m, m+1\}, \quad (15)$$

The conditional expectation of the likelihood function in (14), after some algebraic manipulations, can be expressed as

$$\mathcal{Q}_k(\mathbf{b}_k|\mathbf{b}^i) = \sum_{v=1}^{V}\sum_{m=1}^{M} Re\Big\{\sum_{l=1}^{L}\sum_{l'=1}^{L} T_{11}^{kv}(l,l') + T_{12}^{kv}(l,l') + T_{21}^{kv}(l,l') + T_{22}^{kv}(l,l') - S^{kv}(l,l') + \beta_k^v(\mathbf{h}_k^v)^{i^H} \times \mathbf{B}_k(m)\Big(\mathbf{y}_{c,k}^v[m] - \sum_{j=1, j\neq k}^{K}\Big(\mathbf{R}_{kj}[-1]\mathbf{B}_j(m-1)^i + \mathbf{R}_{kj}[-1]^T \times \mathbf{B}_j(m+1)^i + \mathbf{R}_{kj}[0]\mathbf{B}_j(m)^i\Big)\left(\mathbf{h}_j^v\right)^i\Big)\Big\}, \quad (16)$$

where

$$T_{qw}^{kv}(l,l') = \Big((1-\beta_k^v)\Big(\rho_{qw,ll',m}^{kk^i}(-1) + \rho_{qw,ll',m}^{kk^i}(1) + \rho_{qw,ll',m}^{kk^i}(0)\Big) - \rho_{qw,ll',m}^{kk}(-1) - \rho_{qw,ll',m}^{kk}(1)\Big) \times \left(h_{ql}^{k,v}\right)^{i^*}\left(h_{wl'}^{k,v}\right)^i, q, w \in \{1,2\},$$

$$S^{kv}(l,l') = \sum_{l=1}^{L}\sum_{l'=1, l'>l}^{L} \rho_{11,ll',m}^{kk}(0)\left(h_{1l}^{k,v}\right)^{i^*}\left(h_{1l'}^{k,v}\right)^i + \rho_{12,ll',m}^{kk}(0)\left(h_{1l}^{k,v}\right)^{i^*}\left(h_{2l'}^{k,v}\right)^i + \rho_{21,ll',m}^{kk}(0)\left(h_{2l}^{k,v}\right)^{i^*}\left(h_{1l'}^{k,v}\right)^i + \rho_{22,ll',m}^{kk}(0)\left(h_{2l}^{k,v}\right)^{i^*}\left(h_{2l'}^{k,v}\right)^i,$$

where $\rho^{kk^i}_{qw,ll',m}(m_p)$, $m_p \in \{-1,0,1\}$, are defined in terms of the cross-correlation coefficients and the k^{th} user's current and next data estimates. Also, $\mathbf{y}^v_{c,k}[m]$ is a $(2L\times 1)$ vector including the outputs corresponding to the k^{th} user at the despreader output during the m^{th} symbol interval, defined as $\mathbf{y}^v_{c,k}[m] = \mathbf{F}_k[0]^T\mathbf{y}^v_w[m]+\mathbf{F}_k[1]^T\mathbf{y}^v_w[m+1]$ [13]. In (16), the conditional expectation of the attenuation coefficients given $\mathbf{y}_w$ and $\mathbf{b}^i$ is given by $(h^{k,v}_{ql})^i = E\left[\mathbf{h}^v|\mathbf{y}_w,\mathbf{b}^i\right]_{2L(k-1)+q+2(l-1)} = \left[(\mathbf{h}^v)^i\right]_{2L(k-1)+q+2(l-1)}$, where $[\cdot]_s$ denotes the s^{th} element of the assigned vector, and $(h^{k,v*}_{ql}h^{j,v}_{q'l'})^i = (h^{k,v}_{ql})^{i^*}(h^{j,v}_{q'l'})^i + (\mathbf{\Omega}^i_{hh})_{2L(k-1)+q+2(l-1),2L(j-1)+q'+2(l'-1)}$, where $q,q' \in \{1,2\}$, $l,l' \in \{1,\ldots,L\}$, $k,j \in \{1,\ldots,K\}$ and $\mathbf{\Omega}^i_{hh} = E\left[(\mathbf{h}^v-(\mathbf{h}^v)^i)(\mathbf{h}^v-(\mathbf{h}^v)^i)^H|\mathbf{y}_w,\mathbf{b}^i\right]$. Similar to [13], the conditional distribution of the channel vector, $\mathbf{h}^v$, given $\mathbf{y}_w$ and $\mathbf{b}^i$ is Gaussian with mean

$$(\mathbf{h}^v)^i = \left(\sum_{m=0}^{M-1}\mathbf{B}(m)^i\left(\mathbf{R}[0]\mathbf{B}(m)^i+\mathbf{R}[-1]\mathbf{B}(m-1)^i\right.\right.$$
$$\left.\left.+\mathbf{R}[1]\mathbf{B}(m+1)^i\right)+N_o\mathbf{\Sigma}^{-1}_{hh}\right)^{-1}\times\sum_{m=0}^{M-1}\mathbf{B}(m)^i\mathbf{y}^v_c(m), \tag{17}$$

and covariance

$$\mathbf{\Omega}^i_{hh} = N_o\left(\sum_{m=0}^{M-1}\mathbf{B}(m)^i\left(\mathbf{R}[0]\mathbf{B}(m)^i+\mathbf{R}[-1]\mathbf{B}(m-1)^i\right.\right.$$
$$\left.\left.+\mathbf{R}[1]\mathbf{B}(m+1)^i\right)+N_o\mathbf{\Sigma}^{-1}_{hh}\right)^{-1}, \tag{18}$$

where $\Sigma_{hh} = diag\{\Sigma_{h1},\Sigma_{h2},\ldots,\Sigma_{hK}\}$, $\Sigma_{hk} = \sigma^2_k\mathbf{I}_{2L}$. From (12), the M-step of the EM algorithm is performed by maximizing the individual likelihood functions $\mathcal{Q}_k(\mathbf{b}_k|\mathbf{b}^i)$, $k=1,\ldots,K$, as

$$\mathbf{b}^{i+1}_k = \arg\max_{\mathbf{b}_k}\mathcal{Q}_k(\mathbf{b}_k|\mathbf{b}^i). \tag{19}$$

V. Optimized Weights (β^m_k)

In (16), we notice that $Q_k(\mathbf{b}_k|\mathbf{b}^i)$ is a sum of V statistically independent terms given $\mathbf{b}$ and its EM estimate $\mathbf{b}^i$, which are related to the V receive antennas. Since the spatial channels corresponding to the links between transmit and receive antennas are considered independent, β^v_k can be separately optimized. In this case, we choose the weight coefficients to minimize the linear mean-square error between the true signal vector, $\mathbf{g}^v_k(m_s)$, and its estimate $(\mathbf{g}^v_k(m_s))^i = E\left[\mathbf{g}^v_k(m_s)|\mathbf{y}_w,\mathbf{b}^i\right]$, $m_s \in \{m,m+1\}$, after being projected on $\mathbf{F}_k[0]$ and $\mathbf{F}_k[1]$ respectively as

$$\beta^v_k = \arg\min_{\beta^v_k}E\left[\|E_g\|^2\right], \tag{20}$$

where

$$E_g = \mathbf{F}_k[0]^T\left(\mathbf{g}^v_k(m)-\mathbf{g}^v_k(m)^i\right)+\mathbf{F}_k[1]^T\left(\mathbf{g}^v_k(m+1)\right.$$
$$\left.-\mathbf{g}^v_k(m+1)^i\right), \tag{21}$$

and $\|\cdot\|$ denotes the vector norm. In order to simplify our analysis, we assume that $M\rightarrow\infty$, i.e. the random channel coefficients are assumed to be known at the receiver. It follows that

$$E\left[\mathbf{g}^v_k(m_s)|\mathbf{y}_w,\mathbf{b}^i\right] = \sum_{\Delta m=0}^{1}\mathbf{F}_k[\Delta m]\mathbf{B}_k(m_s-\Delta m)^i\mathbf{h}^v_k$$
$$+\beta^v_k\left(\mathbf{y}^v_w(m_s)-\sum_{j=1}^{K}\sum_{\Delta m=0}^{1}\mathbf{F}_j[\Delta m]\mathbf{B}_j(m_s-\Delta m)^i\mathbf{h}^v_j\right)$$
$$,m_s\in\{m,m+1\} \tag{22}$$

Substituting $\mathbf{g}^v_k(m_s)$ and $(\mathbf{g}^v_k(m_s))^i$, $m_s\in\{m,m+1\}$, in (21), we have

$$E_g = \mathbf{R}_{kk}[0]\left(\mathbf{B}_k(m)-\mathbf{B}_k(m)^i\right)\mathbf{h}^v_k+\mathbf{R}_{kk}[-1]$$
$$\times\left(\mathbf{B}_k(m-1)-\mathbf{B}_k(m-1)^i\right)\mathbf{h}^v_k+\mathbf{R}_{kk}[-1]^T$$
$$\times\left(\mathbf{B}_k(m+1)-\mathbf{B}_k(m+1)^i\right)\mathbf{h}^v_k+\mathbf{F}_k[0]^T\mathbf{n}^v_{wk}[m]+\mathbf{F}_k[1]^T$$
$$\times\mathbf{n}^v_{wk}[m+1]-\beta^v_k\mathbf{y}^v_{c,k}[m]+\beta^v_k\left(\sum_{j=1}^{K}\mathbf{R}_{kj}[0]\mathbf{B}_j(m)^i\mathbf{h}^v_j\right.$$
$$\left.+\mathbf{R}_{kj}[-1]\mathbf{B}_j(m-1)^i\mathbf{h}^v_j+\mathbf{R}_{kj}[-1]^T\mathbf{B}_j(m+1)^i\mathbf{h}^v_j\right) \tag{23}$$

where $\mathbf{R}_{kj}[-1] = \mathbf{F}_k[0]^T\mathbf{F}_j[1]$, and $\mathbf{R}_{kj}[0] = \mathbf{F}_k[0]^T\mathbf{F}_j[0]+\mathbf{F}_k[1]^T\mathbf{F}_j[1]$. Substituting $\mathbf{y}^v_{c,k}[m] = \sum_{j=1}^{K}\mathbf{R}_{kj}[0]\mathbf{B}_j(m)\mathbf{h}^v_j + \mathbf{R}_{kj}[-1]\mathbf{B}_j(m-1)\mathbf{h}^v_j + \mathbf{R}_{kj}[-1]^T\mathbf{B}_j(m+1)\mathbf{h}^v_j+\mathbf{n}^v_{c,k}[m]$, where $\mathbf{n}^v_{c,k}[m]$ is $(2L\times 1)$ vector including the noise samples corresponding to the k^{th} user at the v^{th} despreader output during the m^{th} symbol interval, and $\sqrt{\beta^v_k}\mathbf{n}^v_{c,k}[m] = \mathbf{F}_k[0]^T\mathbf{n}^v_{wk}[m]+\mathbf{F}_k[1]^T\mathbf{n}^v_{wk}[m+1]$ in (23), we have

$$E_g = \mathbf{R}_{kk}[0]\left(\mathbf{B}_k(m)-\mathbf{B}_k(m)^i\right)\mathbf{h}^v_k+\mathbf{R}_{kk}[-1]$$
$$\times\left(\mathbf{B}_k(m-1)-\mathbf{B}_k(m-1)^i\right)\mathbf{h}^v_k+\mathbf{R}_{kk}[-1]^T\left(\mathbf{B}_k(m+1)\right.$$
$$\left.-\mathbf{B}_k(m+1)^i\right)\mathbf{h}^v_k+\sqrt{\beta^v_k}\mathbf{n}^v_{c,k}[m]-\beta^v_k\left(\sum_{j=1}^{K}\mathbf{R}_{kj}[0]\right.$$
$$\times\left(\mathbf{B}_j(m)-\mathbf{B}_j(m)^i\right)\mathbf{h}^v_j+\mathbf{R}_{kj}[-1]\left(\mathbf{B}_j(m-1)\right.$$
$$\left.-\mathbf{B}_j(m-1)^i\right)\mathbf{h}^v_j+\mathbf{R}_{kj}[-1]^T\left(\mathbf{B}_j(m+1)-\mathbf{B}_j(m+1)^i\right)$$
$$\left.\times\mathbf{h}^v_j+\mathbf{n}^v_{c,k}[m]\right) \tag{24}$$

In our system model, we assume that the noise samples, $\mathbf{n}^v_{c,k}[m]$, and the channel coefficients, $\mathbf{h}^v_k$, are mutually independent, as well as $E\left[h^{k,v^*}_{ql}h^{j,v}_{q'l'}\right] = \sigma^2_k$ for similar channel coefficients and zero otherwise. In addition, $E\left[\mathbf{n}^v_{c,k}[m]^H\mathbf{n}^v_{c,k}[m]\right] = N_o\left(R_{kk,11}[0]+R_{kk,22}[0]+\ldots+R_{kk,2L2L}[0]\right)$, where $R_{kk,\zeta\zeta}[0]$, $\zeta\in\{1,2,...,2L\}$, represents the ζ^{th} diagonal element of $\mathbf{R}_{kk}[0]$. Using (24), (20) can be expressed as

$$\beta_k^v = \arg\min_{\beta_k^v} \Bigg\{ N_o \left(\beta_k^v - 2{\beta_k^v}^{\frac{3}{2}} + {\beta_k^v}^2\right)(R_{kk,11}[0] + R_{kk,22}[0] + \ldots + R_{kk,2L2L}[0]) - 8\sigma_k^2 \beta_k^v \left(P_{e_{k1}}^{v,i} + P_{e_{k2}}^{v,i}\right) \times \sum_{l=1}^{L} (r_{kk,0}(2l-1,2l-1) + r_{kk,0}(2l,2l) + r_{kk,-1}(2l-1,2l-1) + r_{kk,-1}(2l,2l) + r_{kk,1}(2l-1,2l-1) + r_{kk,1}(2l,2l)) + 4{\beta_k^v}^2 \sum_{j=1}^{K} \sigma_j^2 \left(P_{e_{j1}}^{v,i} + P_{e_{j2}}^{v,i}\right) \sum_{l=1}^{L} (r_{jj,0}(2l-1,2l-1) + r_{jj,0}(2l,2l) + r_{jj,-1}(2l-1,2l-1) + r_{jj,-1}(2l,2l) + r_{jj,1}(2l-1,2l-1) + r_{jj,1}(2l,2l)) \Bigg\}. \quad (25)$$

where $r_{jj,m_p}(\zeta,\zeta), j \in \{1,\ldots,K\}, \zeta \in \{1,\ldots,2L\}, m_p \in \{-1,0,1\}$, represents the ζ^{th} diagonal element of $\mathbf{R}_{jj}[m_p]^T\mathbf{R}_{jj}[m_p]$, and $P_{e_{jq}}^{v,i} = \frac{1}{2}\left(1 - E\left[b_{jq}[m+m_p] b_{jq}[m+m_p]^i\right]\right), j \in \{1,\ldots,k\}, q \in \{1,2\}, m_p \in \{-1,0,1\}$, where the probability of error, $P_{e_{jq}}^{v,i} = P\left(b_{jq}(m+m_p) \neq b_{jq}(m+m_p)^i\right)$. Assume that $\alpha_k = R_{kk,11}[0] + R_{kk,22}[0] + \ldots + R_{kk,2L2L}[0]$, and $\theta_l^j = r_{jj,0}(2l-1,2l-1) + r_{jj,0}(2l,2l) + r_{jj,-1}(2l-1,2l-1) + r_{jj,-1}(2l,2l) + r_{jj,1}(2l-1,2l-1) + r_{jj,1}(2l,2l)$. By differentiating (25) with respect to β_k^v and substituting $x = \sqrt{\beta_k^v}$, we have

$$\left(2N_o\alpha_k + 8\sum_{j=1}^{K} \sigma_j^2 \left(P_{e_{j1}}^{v,i} + P_{e_{j2}}^{v,i}\right) \sum_{l=1}^{L} \theta_l^j\right) x^2 + (-3N_o \times \alpha_k)x + \left(N_o\alpha_k - 8\sigma_k^2 \left(P_{e_{k1}}^{v,i} + P_{e_{k2}}^{v,i}\right) \sum_{l=1}^{L} \theta_l^k\right) = 0. \quad (26)$$

Solving(26) with respect to x results in two possible solutions for β_k^v. Assuming that the performance of the EM-based ST receiver with V=1 receive antenna will converge to the single-user (SU) bound with perfect channel state information (CSI) assuming transmission over frequency-selective fading channels. Then the probability of error of the odd and even data bits [10] are defined as

$$P_{e1} = \frac{1}{2} Q\left(Re\{\mathcal{G}(1,1) + \mathcal{G}(1,2)\} \sqrt{\frac{2}{\mathcal{G}(1,1)}} \right) + \frac{1}{2} Q\left(Re\{\mathcal{G}(1,1) - \mathcal{G}(1,2)\} \sqrt{\frac{2}{\mathcal{G}(1,1)}} \right)$$

and

$$P_{e2} = \frac{1}{2} Q\left(Re\{\mathcal{G}(2,2) + \mathcal{G}(2,1)\} \sqrt{\frac{2}{\mathcal{G}(2,2)}} \right) + \frac{1}{2} Q\left(Re\{\mathcal{G}(2,2) - \mathcal{G}(2,1)\} \sqrt{\frac{2}{\mathcal{G}(2,2)}} \right),$$

where $Q(x) = 1/\sqrt{2\pi} \int_x^\infty e^{-x^2/2} dx$,

$$\mathcal{G} = \left[(\mathbf{s}_1\mathbf{ch}_1 - \mathbf{s}_2\mathbf{ch}_2)(\mathbf{s}_2\mathbf{ch}_1 + \mathbf{s}_1\mathbf{ch}_2)\right]^H \times \left[(\mathbf{s}_1\mathbf{ch}_1 - \mathbf{s}_2\mathbf{ch}_2)(\mathbf{s}_2\mathbf{ch}_1 + \mathbf{s}_1\mathbf{ch}_2)\right],$$

and $\mathbf{s}_1$ and $\mathbf{s}_2$ include the multipath versions of the two code assigned for every user, and $\mathbf{ch}_q$, $q = 1,2$, include the multipath channel coefficients from the q^{th} transmit antenna to the receive antenna. Substituting with the EM channel estimates defined in (17) in the single-user bound, P_{e_1} and P_{e_2}, we obtain an approximation for $P_{e_{k1}}^{v,i}$, and $P_{e_{k2}}^{v,i}$.

VI. EM Initialization

Since the EM algorithm is sensitive to the initialization of the parameters to be estimated [14], as well as due to the high computational complexity of the joint estimation and detection in MIMO systems, our proposed EM-based ST receiver is initialized by reliable estimates, for example, by using the ST MMSE-SDE technique [9]. This guarantees that the performance of our proposed receiver converges to that of the optimum receiver with a fast rate. Furthermore, this will also ease the maximization of the individual likelihood functions $\mathcal{Q}_k(\mathbf{b}_k|\mathbf{b}^i)$ in (19). Instead of using the Viterbi algorithm [11], which increases the complexity of the proposed receiver, we consider that, at the first iteration, the ST MMSE-SDE estimates provide reliable estimation for the previous and following codewords, $b_{kq}[m-1] = b_{kq}[m-1]^0$, and $b_{kq}[m+1] = b_{kq}[m+1]^0$, $q \in \{1,2\}$, while for the subsequent iterations, we consider that $b_{kq}[m-1] = b_{kq}[m-1]^i$, and $b_{kq}[m+1] = b_{kq}[m+1]^{i-1}$. Consequently, the maximization of (19) is performed over 4 possibilities for the current k^{th} user data bits, $b_{k1}[m]$ and $b_{k2}[m]$, (i.e, {(1,1),(1,-1),(-1,1),(-1,-1)}) considering binary phase-shift keying (BPSK) transmission.

VII. Simulation Results

In this section, we examine the bit-error rate (BER) performance of the proposed EM-JDE receiver in MIMO CDMA systems. In all cases, we consider a system with two transmit and $V = 1,2$ receive antennas. We also consider an uplink asynchronous transmission of a data block of 40 codewords (M=40) over a frequency-selective fading channel. Without loss of generality, we consider a 5-user system, where all users are assigned Gold codes of length 31 chips. We assume the first user as the desired one. A training codeword of eight training bits is used for the initialization of the EM receiver.

As a reference, we include the BER performance of the single-user STS system assuming perfect channel estimation. Fig. 1 presents the BER performance of the proposed ST EM-JDE receiver using 2×1 antenna configuration. The results show the proposed JDE receiver achieves a performance close the single-user bound. We also notice that the full system diversity is attained at high signal-to-noise ratio (SNR) values. Similar argument applies to Fig. 2 where we consider a MIMO CDMA system with two receive antennas. In Fig. 3 we examine the near-far effect property of the proposed receiver for $V = 1$ receive antenna. We fix the received SNR of the first user γ_1 at 15 dB, while the interfering users have equal

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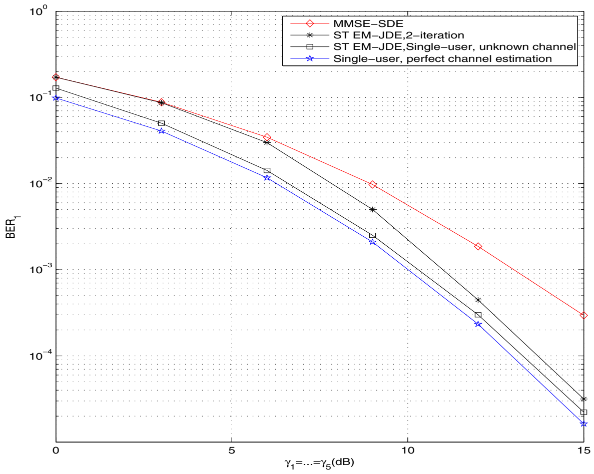

Fig. 1. BER performance of the first user considering ST EM-JDE receiver with V=1 receive antennas, $M = 40, p' = 8, L = 2$, 2 iterations.

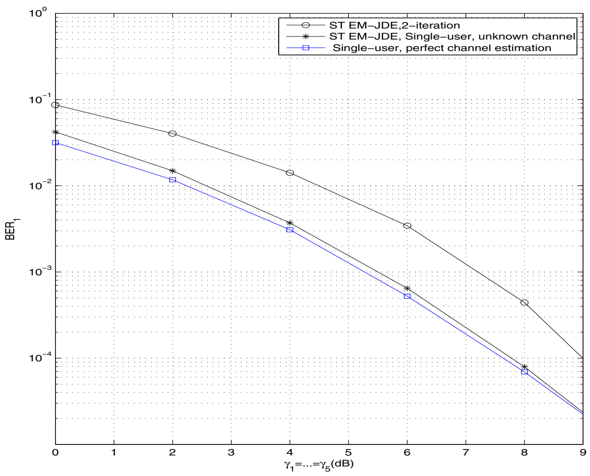

Fig. 2. BER performance of the first user considering ST EM-JDE receiver with V=2 receive antennas, $M = 40, p' = 8, L = 2$, 2 iterations.

SNR ratios relative to γ_1, varying from -10 to 60 dB. We also compare the performance of the ST EM-JDE receiver considering optimum β_k^v values (26) and equal β_k^v values ($\beta_k^v = 1/K$). The results show that the EM receiver with optimum β_k^v is near-far resistant.

VIII. Conclusion

We have developed an iterative joint detection and estimation receiver based on the EM algorithm for STS systems. Using Monte-Carlo simulations, we examined the performance of our proposed receiver in frequency-selective fading channels. It was shown that with few training bits, the receiver can achieve performance close to the single-user bound in few iterations. We have also shown that the proposed receiver attains the full system diversity through accurate channel

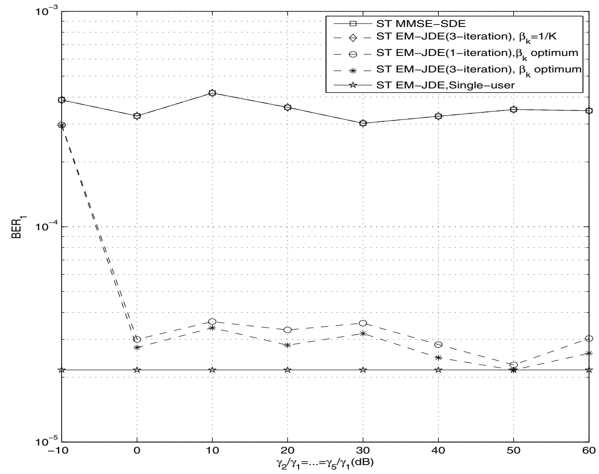

Fig. 3. BER behavior of the first user as a function of the MAI level with V=1 receive antenna, $M = 40, p' = 8, L = 2, \gamma_1$=15dB.

estimates.

References

[1] G. V. V. Sharma and A. Chockalingam, "Performance analysis of maximum-likelihood multiuser detection in space-time-coded CDMA with imperfect channel estimation", *IEEE Trans. Veh. Techn.*, vol. 55, no. 6, pp. 1824–1837, Nov. 2006.

[2] E.Telatar, "Capacity of multi-antenna Gaussian channels", *European Trans. on Telecomm.*, vol. 10, no. 6, pp. 585–595, 1999.

[3] Y. Huang and J. A. Ritcey, "Joint iterative channel estimation and decoding for bit-interleaved coded modualtion over correlated fading channels", *IEEE Trans. Wireless Comm.*, vol. 4, no. 5, pp. 2549–2558, Sep. 2005.

[4] T. K. Moon, "The expectation-maximization algorithm", *IEEE Signal Processing Mag.*, pp. 45–59, Nov. 1996.

[5] C. Cozzo and B. Hughes, "Joint channel estimation and data detection in space-time communications", *IEEE Trans. Comm.*, vol. 51, no. 8, pp. 1266–1270, Aug. 2003.

[6] X. Wautelet, C. Herzet, A. Dejonghe, J. Louveaux, and L. Vandendorpe, "Comparison of EM-based algorithms for MIMO channel estimation", *IEEE Trans. Commun.*, vol. 55, no. 1, pp. 216–226, Jan. 2007.

[7] J. Choi, "An EM based joint data detection and channel estimation incorporating with initial channel estimate", *IEEE Commun. Letters.*, vol. 12, no. 9, pp. 654–656, Sep. 2008.

[8] D. K. Chun So and R. S. Cheng, "Iterative EM receiver for space-time coded systems in MIMO frequency-selective fading channels with channel gain and order estimation", *IEEE Trans. Wireless Comm.*, vol. 3, no. 6, pp. 1928–1935, Nov. 2004.

[9] A. Kocian and B. H. Fleury, "EM-based joint data detection and channel estimation of DS-CDMA signals", *IEEE Trans. Comm.*, vol. 51, no. 10, pp. 1709–1720, Oct. 2003.

[10] B. Hochwald, T. Marzetta, and C. Papadias, "A transmitter diversity scheme for wideband CDMA systems based on space-time spreading", *IEEE J. Select. Areas Commun.*, vol. 19, no. 1, pp. 1451–1458, Jan. 2001.

[11] S. Verdu, *Multiuser Detection*, Cambridge University Press, 1998.

[12] M. Feder and E. Weinstein, "Parameter estimation of superimposed signals using the EM algorithm", *IEEE Trans. Acoustic, Speech and Signal Process.*, vol. 36, pp. 477–489, Apr. 1988.

[13] A. Kocian, I. Land, and B. H. Fleury, "Joint channel estimation, partial successive interference cancellation, and data decoding for DS-CDMA based on the SAGE algorithm", *IEEE trans. Commun.*, vol. 55, no. 6, pp. 1231–1241, Jun. 2007.

[14] J. Ylioinas and M. Juntti, "Iterative joint detection, decoding, and channel estimation in turbo-coded MIMO-OFDM", *IEEE Trans. on Vehicular Technology*, vol. 58, no. 4, pp. 1784–1796, May 2009.