Sensor Placement and Diagnosability Analysis at Design Stage

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Abstract. Adequate sensors are a necessary condition for fault diagnosability. Sensor placement for diagnosis task is to study where to put the sensors so that they are the minimal set to diagnose certain faults. This paper presents a method of sensor placement based on diagnosability analysis using the simulation model in the CAD environment. The fault signature matrix is determined by the projections of different operation modes on observable variables. The minimal sensor set for detecting faults and for discriminating the faults relies on adequate sensors to provide redundancy relations so that the discrepancies of the predictions and the observations can be detected. This paper presents a method of sensor placement based on diagnosability analysis using the simulation model in the CAD environment.

1 INTRODUCTION

There are demands on the automobile industry to consider vehicle maintenance and diagnosis at the early stage of design. The IDD (Integrated Diagnosis and Design) project, a V framework EU project, aims at the definition of a new design process for automotive systems. The goal is to integrate the process of diagnostic development (FMEA, diagnosability analysis, etc.) in the early phase of the design process. Sensor placement for diagnosis task is to study where to put the sensors so that they are the minimal set to diagnose certain faults. The diagnosis principles reveal that diagnosability of the faults relies on adequate sensors to provide redundancy relations so that the discrepancies of the predictions and the observations can be detected. This paper presents a method of sensor placement based on diagnosability analysis using the simulation model in the CAD environment.

A. Short View of Sensor Placement Methods

The criteria for making sensor location decisions vary depending on the goals of the task. In control theory, the sensor network is to provide necessary information for the control of the process or the system, so the study starts from the observability/controllability of the variables. Madorn and Veverka [1] addressed sensor placement for a linear process. Their method makes use of the Gauss-Jordan elimination to identify a minimum set of variables that need to be measured in order to observe all important variables while simultaneously minimizing the overall cost of sensors. Others consider sensor failures and their effect on the observability of variables. [1] introduced the concept of reliability of the estimation of a variable, which gives the probability of estimating a variable for any given sensor network and specified sensor failure probabilities. [2] discussed the redundancy of sensor network, i.e. more sensors than the minimum to ensure the observabilities of variables when some sensors fail. Similar work can be found in [3].

This paper is in the model-based diagnosis domain. The goal of sensor placement is to achieve the diagnosability, i.e. the sensor network can detect and discriminate the faults of the components in a system. Sensor failure is not considered in this paper. The observables from the point of view of diagnosis are the variables that can be measured by sensors. This differs from the concept of control theory, where the observable variables are the measurable variables plus the unmeasured variables deducible from the measurable variables. Though the terms used in the two domains are similar, the work in this paper has little relation to the work described in the previous paragraph.

Existing work of sensor placement in diagnosis is based on the Analytical Redundancy Relation (ARR) in [4]. In [5], AND-OR Graph is drawn to show the dependency relation of a potential sensor and components (faults). Then the HFS (Hypothetical Fault Signature) matrix is built to analyse the redundant relation. HFS makes a correspondence among the additional sensor, the resulting redundant relation and the involved component. The next step is to build an EHFS (Extended Hypothetical Fault Signature) matrix. This latter matrix takes into account the addition of several sensors at one time. If one independent redundant relation is added by one additional sensor and involves one component, the fault on this component is discriminable. Though [4] presents appropriate conclusions, it is not easy to use in practice because its complexity is beyond what an expert can handle for even a small system.

B. Our Approach

Our analysis is based on diagnosability analysis [6]. Intuitively, if the faulty behavior and the normal behavior have disjointed projections on some observables, the fault can be detected; similarly, if two faults have disjointed projections on some observables, the two faults can be discriminated.

Inspired by the signature of ARR, the projections of different modes on the observables can be encoded into \{0,1\}, based on whether the fault modes have the same or different values to the right mode. This gives a signature for each mode. Therefore, if different modes have different signatures, those modes can be discriminated.

The values of exogenous variables and/or inputs play a role in diagnosability, i.e. a fault can only be detected under certain domains. [2] implies that the domains of the variables can be partitioned into several areas that have different diagnosability. “Causal scope” is used to represent the partitioned domains. A physical sensor working under different causal scopes can be mapped to several virtual sensors.
Then the fault signatures are built on the virtual sensors. The minimal sensor set for detecting faults and for discriminating the faults can be computed from the fault signature matrix. It is an initial work that the domains of variables are taken into consideration for sensor placement.

When the sensor placement analysis is conducted in the CAD environment, the nature of the simulation function of the CAD environment actually does the work of constraint propagation. Thus we avoid the complex manual method to build the dependency relation between variables and components as in [7]. Therefore, the method developed in this paper can be conducted fully automatically inside CAD environment.

Section 2 discusses the diagnosability analysis; section 3 presents the approach of sensor placement; section 4 is a small demonstration; and section 5 contains the conclusions.

2 DIAGNOSABILITY ANALYSIS

From the diagnosis principle, a fault modifies the normal behavior of a component, thus generates a discrepancy on the outputs of the component. The discrepancy propagates by the components links until the discrepancy is detected by sensors on the observable points. Diagnosability includes two notions[1]: Fault Detectability is whether and under which circumstances the possible faults considered can be distinguished from the right modes; Fault Discriminability is whether and under which circumstances the faults (or the classes of faults) can be distinguished. Discriminability is a stronger definition.

The diagnosability is computed from the projections of different modes on the observables [8]:

\[
\text{SID}_{o-cause} = \text{PROJ}_{o-cause}(\text{OPC}) \backslash \text{PROJ}_{o-cause}(\text{PROJ}_{obs}(\text{MODEL}_{mode1} \cap \text{OPC}) \cap \text{PROJ}_{obs}(\text{MODEL}_{mode2} \cap \text{OPC}))
\]

where \(\text{PROJ}\) is projection operation, \(\text{OPC}\) represents operating conditions, and \(\text{V}_{obs}\) is observables set. \(\text{V}_{obs}\) are divided into two sets: \(\text{V}_{o-cause}\) which is the set of exogenous or “causal” variables in \(\text{V}_{obs}\), and \(\text{V}_{obs} \backslash \text{V}_{o-cause}\) which is the set of rest variables in \(\text{V}_{obs}\). \(\text{Model}_{mode1}\) and \(\text{model2}\) are the two modes. \(\text{SID}_{o-cause}\) is the range of \(\text{V}_{o-cause}\) that the two modes are discriminable. It is calculated in this way: calculate the projections on the \(\text{V}_{obs}\) for the two modes (by “\(\text{PROJ}\)’s”); calculate the conjunction of the two projections (by “\(\cap\)” between the two “\(\text{PROJ}\)’s”); calculate the subtraction of the projection on \(\text{V}_{o-cause}\) of the original modes and the projection of the conjunction on \(\text{V}_{o-cause}\) (by “\(\backslash\)”). \(\text{OPC}\) is the variable to describe different states of the system and its control. Examples are engine idle, clutch engaged, cold engine. For diagnosability, it is meaningful to compare the two modes only when the operating conditions are the same. In modeling, it is difficult to distinguish \(\text{OPC}\) and \(\text{V}_{o-cause}\) as long as \(\text{OPC}\) variables are observable. It is solely a matter of convenience.

The principle in (1) is that, if the projections on \(\text{V}_{obs}\) are disjoint, the discrepancy of the two modes can be observed, which is called deterministically discriminable (DD). [9] also defines two other categories of discriminability: possibly discriminable (PD), and non-discriminable (ND). We are only concerned with DD in this paper.

The goal of our project, as introduced in Section 1, is to integrate diagnostic tasks at system design time. The system model is available in the CAD environment. Matlab/Simulink is the target platform in our project because of its popularity in automobile industry. Design engineers normally have excellent knowledge of fault modes and analyze the fault effects by simulation. The approach developed in this paper assumes that the knowledge of fault modes is available.

**Figure 1.** values propagate through component links.
sections of \( q_1 \) and \( q_2 \) on \( u \) under a causal scope \( v_u = [v_x, v_y] \) are disjoint as \( U1 \cap U2 = \emptyset \), modes \( q1 \) and \( q2 \) are discriminated by \( u \) under scope \( v_u \).

The proof is quite straightforward and eliminated here. Proposition 2 says that if the qualitative models for two behavior modes take the same value for \( V_{o\text{-caus}} \), the necessary and sufficient condition for DD is that the projections on \( V_{obs\text{-caus}} \) are disjointed.

3 SENSOR PLACEMENT ANALYSIS

3.1 Minimal Sensor Set

Our approach is inspired by ARR which uses a fault signature matrix to discriminate faults [9]. The difference is that our fault detectability matrix is built on projections. More specifically, since the variables in \( V_{o\text{-caus}} \) always take the same values for different modes, the fault signature matrix is built on \( V_{obs\text{-caus}} \). In order to present the values of \( V_{o\text{-caus}} \) as the condition of diagnosability, we define:

Definition 2 (V-sensor): A virtual sensor is a physical sensor associated with a causal scope.

V-sensor is written as \( VS(v, \text{scope}) \), where \( v \) is the physical variable that the sensor measures; the scope is a qualitative value of \( V_{o\text{-caus}} \) (can be multi-dimension).

Using definition 2, a physical sensor may be mapped to several V-sensors if the causal scopes are different. For example, assume a sensor \( S_i \) measures variable \( v_1 \). Then its V-sensors can be \( VS_1(v_1, \text{scope}1) \) and \( VS_2(v_1, \text{scope}2) \), where \( \text{scope}1 \) and \( \text{scope}2 \) are different causal scopes. A fault can be detectable under a certain causal scope but not the others. Using V-sensor gives us a more precise view on sensor placement.

Definition 3 (Fault Detectability Signature): Given a vector of V-sensors \( VS = \{VS_1, VS_2, ..., VS_n\} \), the detectability signature of fault \( f_j \) is a binary vector \( FS_j = \{s_{1j}, s_{2j}, ..., s_{nj}\} \) in which \( s_i \) is given by:

\[
FS_j = \{0, 1\}
\]

where \( (VS_i, FS_j) \rightarrow s_{ij} = 1 \) if \( f_j \) causes discrepancy at \( VS_i \); \( s_{ij} = 0 \) if \( f_j \) causes no discrepancy at \( VS_i \).

Computing Fault Detectability Signature: discrepency is computed by the intersection of the values of VS, at mode \( f_j \) and at right mode \( r \), i.e.:

- if \( \text{value}(VS_i, f_j) \cap \text{value}(VS_i, r) = \emptyset \), \( s_{ij} = 1 \)
- if \( \text{value}(VS_i, f_j) \cap \text{value}(VS_i, r) \neq \emptyset \), \( s_{ij} = 0 \)

It is obvious that for the right mode, all the elements in the vector are 0. Table 2 is an example of fault detectability signature matrix.

<table>
<thead>
<tr>
<th>Table 1. fault detectability signature matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>( VS_1(v_1, \text{scope}1) )</td>
</tr>
<tr>
<td>( VS_2(v_2, \text{scope}2) )</td>
</tr>
<tr>
<td>( VS_3(v_3, \text{scope}3) )</td>
</tr>
<tr>
<td>( VS_4(v_4, \text{scope}4) )</td>
</tr>
<tr>
<td>( VS_5(v_5, \text{scope}5) )</td>
</tr>
</tbody>
</table>

Computing minimal sensor set to detect fault \( f_j \): It is easy to know that one \( s_{ij} = 1 \) is sufficient to detect the fault. The minimal sensor set (MSS) to detect fault \( f_j \) is \( MSS_{ij} = \{VS_i\} \) with \( s_{ij} = 1 \).

All MSS_{ij} defines a set MSS_{j} = {MSS_{ij}}.

Example 1: (Get MSSS (MSS Sets) for detecting a fault)

From table ??, the MSS for detecting F1 is MSSS1 = \{\{VS_2\}, \{VS_4\} \} the MSSs for F2, F3, F4 and F5 respectively are MSSS2 = \{\{VS_2\}, \{VS_3\} \}, MSSS3 = \{\{VS_1\}, \{VS_3\} \}, MSSS4 = \{\{VS_1\}, \{VS_2\} \}, MSSS5 = \{\{VS_2\}, \{VS_3\} \}.

Detectability is about discriminating a fault from normal behaviour. A detectable fault may or may not be discriminated from other faults. To discriminate multiple faults, we have proposition 2.

Proposition 2 If the two faults have different fault detectability signatures, they are discriminable.

Proof: If the two modes have different signatures, there exists at least one V-sensor taking value 0 in one mode and value 1 in the other. The projections on \( V_{obs} \) of the two modes, for the causal scope defined by the V-sensor, are thus disjoint (by definition ??). These modes are thus discriminable.

Example 2: (faults with different signatures are discriminable)

Using table ??, for F1 the signature is FS1 = \{0, 1, 1, 1, 0\} and F2’s signature is FS2 = \{0, 1, 1, 0, 1\}. Since FS1 \( \neq \) FS2, the two faults are discriminable. For F1 and F5, since their signatures are equal, the two faults are not discriminable.

Remark of Proposition 2: Proposition 2 gives sufficient conditions of discriminability because the fault modes are compared with only the normal mode in the fault detectability signature. If we compare fault modes not only with the normal mode, but also with each other, we need more values than \{0, 1\} to describe their relations. If so, we can get the necessary and sufficient condition for discriminability. To do this, the projections of different modes (including the right modes) have to be compared by pairs and assigned different values if they are disjoint. In this case, the following proposition 3 does not hold. The following proposition 4 can be modified to hold. But in the following context, we still use \{0, 1\} based fault detectability signature.

If we have \( n \) V-sensors, we can get \( 2^n \) possibilities of the fault detectability signature, including the right mode with a zero vector as the signature. Thus the maximum number of faults to be discriminable by \( n \) V-sensors has a limitation:

Proposition 3: given \( n \) V-sensors, the maximum number of faults to be discriminable is \( 2n - 1 \).

If we have \( m \) faults, we can determine from proposition ?? how many V-sensors we need to discriminate them:

**Corollary 1** The minimum number of V-sensors to discriminate \( m \) faults is equal to \( \log_2(m + 1) \).

**Proof:** If \( n \) is the minimum number of V-sensors to discriminate \( m \) faults, then \( n \) satisfies:

\[
2^{n-1} - 1 < m \leq 2^n - 1
\]

We can get \( n - 1 < \log_2(m + 1) \leq n \)

Thus, \( n = \lfloor \log_2(m + 1) \rfloor \)

Proposition 4 gives the selections of minimal sensor sets.
Proposition 4: For m faults, select \(\log_2(m+1)\) rows from the fault detectability signature matrix to form a new matrix. If the m column vectors in the new matrix are different and non-zero, the corresponding V-sensors on the row are a MSS to discriminate the group of faults.

Example 3: (Get MSSS for discriminating two faults) The fault matrix of F1 and F2 are \(\{0,1,1,1,0\}, \{0,1,1,0,1\}\). To discriminate the faults, we need at least two V-sensors. We select two rows in the matrix, that the new matrix has different non-zero column vectors. We have several choices here: \(\{2,4\}, \{2,5\}, \{3,4\}, \{3,5\}, \{4,5\}\). Thus we get the MSSS for F1 and F2 are MSSS=\(\{\{VS_2, VS_4\}, \{VS_3, VS_5\}\}\).

Example 4: (Get MSSS for discriminating multiple faults) considering F1 through F4, we need three V-Sensors to discriminate 4 faults. We then select three rows in the matrix, that the column vectors are non-zero and different. We get only two solutions MSSS=\(\{\{VS_2, VS_3, VS_4\}, \{VS_5, VS_6, VS_7\}\}\).

Notice that the MSSS used in this paper are on V-sensors. The correspondent physical sensors are the real physical MSSS. If we get several physical MSSS, we can use other criteria to select the best one. [?] discussed other criteria, e.g. cost.

3.2 Sensor set for Recovery Actions

Due to the scarcity of sensors, the diagnosis requirement is sometimes relaxed from discriminating each individual fault to detecting a group of faults. The criterion to group the faults is their common recovery action. Since the faults have the same recovery action, it is not necessary to discriminate them other than just to detect them.

We assign a signature for the group of faults in this way: if all the faults have the same signature at a sensor, the group takes the same signature. If the faults have different signatures, a question mark is used to show the ambiguity. Then the signature of a group is treated as the one of a fault.

Table 2. fault detectability matrix for fault group

<table>
<thead>
<tr>
<th></th>
<th>G</th>
<th>F4</th>
</tr>
</thead>
<tbody>
<tr>
<td>VS₁₇₁</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>VS₁₇₂</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>VS₁₇₃</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>VS₁₇₄</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>VS₁₇₅</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Example 5: (Sensor Set for Recovery Action) Uses the data in table ???. If F1, F2, F3 are in the same recovery group, we determine the signature of this group. We reduce the columns for F1, F2, and F3 into one column G. The value for each sensor depends on the individual values. For VS₁, the three give 0, so (VS₁, G) is 0. For VS₂ and VS₃, some fault gives 1, some gives 0, we put a question mark. So the signature for G is \(\{0,1,1,?,?\}\), see table 2. So we have four solutions \(\{\{VS_2, VS_4\}, \{VS_3, VS_5\}\}\). F5 can’t be discriminated with F1, so we just consider how to discriminate G from F4.

4 DEMONSTRATION

A simple air conditioning system has three components: blower; distribution; and cabin (figure 2). Figure ?? is the model in Matlab/Simulink.

P is pressure, f is airflow rate, E is the electrical power driving the blower. When the blower works, air will pass through the system. We only consider the behavior at a stable point. The right behavior mode is shown in Table 3.

Table 3. qualitative model for AC system

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blower</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>P0</td>
</tr>
<tr>
<td>125 250</td>
<td>000 042</td>
</tr>
<tr>
<td>2345 3317</td>
<td>004 042</td>
</tr>
<tr>
<td>3317 4060</td>
<td>006 074</td>
</tr>
<tr>
<td>4060 4700</td>
<td>007 085</td>
</tr>
</tbody>
</table>

Distribution

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>f1</td>
</tr>
<tr>
<td>200 2345</td>
<td>000 042</td>
</tr>
<tr>
<td>2345 3317</td>
<td>004 042</td>
</tr>
<tr>
<td>3317 4060</td>
<td>006 074</td>
</tr>
<tr>
<td>4060 4700</td>
<td>007 085</td>
</tr>
</tbody>
</table>

Cabin

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>P2</td>
<td>f2</td>
</tr>
<tr>
<td>213 301</td>
<td>004 042</td>
</tr>
<tr>
<td>301 370</td>
<td>006 074</td>
</tr>
</tbody>
</table>

We consider two fault modes. One is the lower efficiency of the blower, which causes a change on output flow rate and pressure. Another fault is the leak at distribution, which causes a lower output flow rate and pressure at the outputs of distribution. For this system, the pressures are measurable, but not the flows. By simulating the fault modes, we get the fault signature as table 4. We have two solutions: \(\{\{VS_0(P2, [125 250][0 0]), VS_1(P2, [250 375][0 0])\}\}, \{VS_0(P2, [125 250][0 0]), VS_4(P2, [375 500][0 0])\}\). Physically they are correspondent to the sensors on P2. The \(V_{cause}\) is E,P0. The discriminable causal scopes are E=\(\{125 250\}, \{250 375\}\). We can distinguish faults within these scopes by observing the fault signature.

Some faults are so-called dynamic faults which are detectable only at the dynamic process. It is possible to use our approach for dynamic faults. As in [?], pseudo variables, which are the derivatives of “flow” or “effort” variables (as in bond graph modelling approach), are added to model the dynamic. Treating the pseudo variables as the normal variables, the approach developed here can be used for the dynamic faults. Actually, this demo system is a dynamic system. Due to the length of the paper, the demonstration for the dynamic faults is not covered.

5 CONCLUSION

This paper considers sensor placement based on discriminability analysis at the design stage. The approach we have presented gives us not only the minimal sensor set but also the causal scopes for fault detection.
detectability and discriminability. Using the causal scope concept, we can make more precise conclusions on sensor placement. The sensor placement analysis takes place in the simulation environment and is an automatic approach with no structural knowledge needed. This approach is practical for analysis of real world applications.

REFERENCES